Errors in Signage.

Should be:

This is the difference

$$T_a = A_a + B_a \cos(\omega t - \phi_a) = A_a + B_a \sin\left(\omega t + \frac{\pi}{2} - P_s\right)$$
$$= A_a - B_a \sin\left(\omega t + \frac{3\pi}{2} - P_s\right)$$

where $A_a = \dfrac{\Sigma T_{a,i}}{12}$

$$S_1 = \Sigma T_{a,i} * \sin(\omega t)$$
$$C_1 = \Sigma T_{a,i} * \cos(\omega t)$$

$$B_a = \frac{-2}{12} * \sqrt{S_1^2 + C_1^2}$$

~~This is the difference!~~ Moore had $\frac{-2}{12}$

$$P_a = \tan^{-1}\left(\frac{S_1}{C_1}\right)$$

Send by Brian Bradley 99/01.

HOT-2000 ENERGY ANALYSIS PROGRAM

Soil Temperature Estimation Model

Prepared for: R-2000 Homes Program
Energy, Mines and Resources, Canada

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7 July 1986

1.0 Introduction

This soil temperature estimation model was developed to assist the HOT-2000 energy analysis computer program. HOT-2000 currently simulates the energy performance of air, water and ground source space heating heat pumps. To improve the accuracy of ground source heat pumps, it is necessary to know the temperature profile of the soil at a specified depth (dependant on heat pump placement).

To estimate soil temperatures for a given location, this model uses two weather parameters from the HOT-2000 weather files. These two parameters are the location's average monthy ambient dry air temperature and its annual heating degree days. From this information, the model can estimate the temperature of the soil for any given depth at any time of the year. The simplified weather requirements allow this model to be applied universally.

This paper outlines the methodology and derivation of the model and how it is able to estimate these soil temperatures at various depths. A statistical analysis of the estimated soil temperatures to the actual data is presented in the conclusion of the report.

2.0 Temperature Profile Function

The first step in developing the mathematical model was to decide on an appropriate time function to represent the temperatures of the soil during the year for any particular depth. It was found that a simple cosine function could be used to create this temperature profile. Test data acquired from Agricultural Canada Tech Bulletin 85 (1) confirmed that the temperature profile closely matched a cosine function.

A cosine curve needs only three variables to completely describe itself.

Referring to Fig.1, "A" represents the mean temperature. This is a temperature in which one half the cosine curve lies above and one half lies below. "B" is the amplitude of the curve, or a measure of how much the temperature oscillates. "P" is the phase shift angle, and measures the amount the curve is shifted to the right.

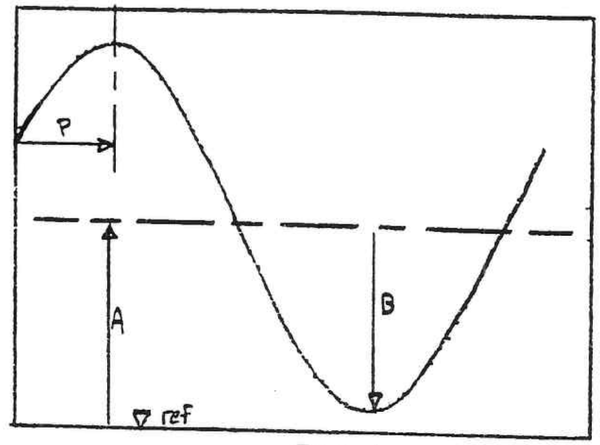

Fig.1

Thus the equation can be written as;

$$T = A + B * \cos(2 * Pi * n / 8766 - P) \qquad (1)$$

where:

T = temperature of soil at given depth and time of year (C)
A = mean temperature of soil (C)
B = soil temperature amplitude at given depth (C)
Pi = 3.1415927
n = number of hours into year (8766 hours/year)
P = phase angle at given depth (degrees)

With knowledge of A, B & P at a specified depth, the soil temperature for any time during the year can be estimated.

2.1 Soil Temperature Modeling Proceedures

HOT-2000 weather files store the average monthly ambient dry air temperatures for each location. The heating degree days are also stored. With this information the following procedure was adopted:

The procedure:

1. The average monthy ambient dry air temperatures are fitted to a cosine curve using a least squares technique as developed in Kusuda & Achenbach (2). For a complete description of this technique, refer to Appendix A. Thus an Aa, Ba & Pa value is obtained.

2. It is an incorrect assumption to assume the temperature profile of the soil at the surface of the ground matches the ambient dry air temperature profile. A difference exists for such factors as direct sunshine heating the ground faster than air and snow in the winter insulating the ground against severe cold air. Using three correlations involving degree days, an As, Bs & Ps value is obtained for the soil at the surface. The development of these correlations is described in section 2.4. As, Bs & Ps represent the modified values of the Aa, Ba & Pa values extracted from the air in step 1.

3. With the temperature profile defined at the ground surface, the next step is to determine the corresponding temperature profile for any depth. As the depth of the ground increases, the mean soil temperature approaches a constant value. Between the surface and a 3 meter depth, the temperature amplitude decreases and the phase shift angle shifts more to the right (increases). The diffusivity of the soil will determine how much the amplitude decreases and how much the phase shift increases. The diffusivity is a measure of how well the soil conducts heat. This differs at each location, depending on soil composition and presence of moisture. Correlations, again involving degree days are used to find the diffusivity of the soil at any particular degree day location. The development of these correlations is discussed in section 2.3.4. With this information found, the following equation can now be applied;

This step isn't for annual amplitude needed of ground surface.

$$T(x) = As + Bs*b * \cos(2 * Pi * n / 8766 - Ps - p) \qquad (2)$$

I want T(x=0m)

where:

T(x)	=	temperature of soil at depth x (C)
As	=	mean temperature of soil at surface (C)
Bs	=	soil temperature amplitude at surface (C)
" b	=	decrease in amplitude for depth x
Pi	=	3.1415927
n	=	number of hours into year (8766 hours/year)
Ps	=	phase angle at surface
" p	=	shift in phase angle for depth x

" - determined by diffusivity.

2.2 Test Data

Fourty-eight locations across Canada where selected from Bulletin 85. These locations were selected, as their monthly ambient dry air temperatures and degree day values were available from the HOT-2000 weather files. For each location, twelve monthly average soil temperatues for six different depths were obtained from Bulletin 85. The depths were 1cm, 10cm, 20cm, 50cm, 100cm and 150cm from the surface. A list of these locations with their corresponding degree days is presented in Appendix C.

I don't need section 2.3

2.3 Determining Location Soil Diffusivity

2.3.1 Calculated Diffusivity By Depth

For each of the 48 locations, a cosine curve was fitted to each of the six depths. Thus a value of A, B & P were obtained for each of the depth. From this information, the diffusivity of the soil at each location could next be calculated. This was calculated using the technique described in Kusuda & Achenbach (1). This method assumes homogeneous soil composition and permeability. In reality, soil is rarely homogeneous, but the effects of this assumption do not severely distort the average monthy derived soil temperatures.

Using this technique the diffusivity was calculated in two ways. One was from the decrease in cyclic temperature amplitude and the other by the increase in the phase angle. Both methods compared the changes against a given depth increase.

The 1cm depth was chosen as a reference depth and the amplitude decreases and phase angle increases of the remaining five depths were used to calculate five sets of diffusivities. The 1cm depth was chosen as a reference simply because it is the closest to the ground surface, which is used as the model reference depth (as described in section 2.1). Thus;

D1 = (Pi/8766) * ((X/ln(Bo/B))^2 (3)

and D2 = (Pi/8766) * ((X/(P-Po))^2 (4)

where:

D1 = amplitude diffusivity at given depth (m^2/h)
D2 = phase diffusivity at given depth (m^2/h)
Pi = 3.1415927
X = (ground depth - .01) (m)
Bo = amplitude of temperature curve at 1cm depth (C)
Po = phase angle of temperature curve at 1cm depth
B = amplitude of temperature curve at given depth (C)
P = phase angle of temperature curve at given depth

2.3.2 Fitting Inconsistant Diffusivity Data

The two values, D1 & D2, should theoretically be identical, but seldom were. This was due possibly to the fact that a cosine curve is only an approximation to the actual curve. Also, the actual soil from which the data is taken from is assumed to be homogeneous; yet it reality, seldom was. Thus they were kept separate and were defined as 'amplitude diffusivity' and 'phase diffusivity'.

Also, the calculated amplitude diffusivity tended to increase with increased ground depth. Theoretically, it should have remained constant, but continued to vary proportionally with ground depth. There is no definite reason why this was so. A possible explanation could be an increase in soil density with deeper ground depth. Thus the least squares method was applied to obtain a linear equation relating amplitude diffusivity with ground depth. The phase diffusivity was not affected by ground depth and an average phase diffusivity was assumed.

Therefore each location produced three values relating to diffusivity (**A1**, **B1**, **PH**). Two values, A1 and B1 defined a line to represent the amplitude diffusivity. Thus;

D(amp) = A1 + B1*X (5)

where:

D(amp) = amplitude diffusivity at given depth (m^2/h)
B1 = slope of D1 line (calculated by Least Squares) (m/h)
X = (soil depth) (m)
A1 = intercept of D1 line (calculated by L.S.) (m^2/h)

The third value, PH is the average of the phase diffusivities found at each depth. Thus phase diffusivity is represented as;

D(phase) = PH (6)

where:

D(phase) = phase diffusivity at given depth (m^2/h)
PH = sum(D2 values)/5 (m^2/h)

2.3.3 Calculating Diffusivity Coefficients

With the values D(amp) and D(phase) to account for the irregularities of D1 & D2, the coefficients 'b' and 'p' can be found to fit into equation (2). Solving Equations (3) and (4) gives;

B/Bo = e^(-X*sqrt(Pi/D(amp)/8766)) (7)
P-Po = -X*sqrt(Pi/D(phase)/8766) (8)

where:

B = temperature amplitude at given depth (C)
P = phase shift at given depth
Bo = temperature amplitude at 1cm depth (C)
Po = phase shift at 1cm depth
X = (ground depth - .01) (m)
Pi = 3.141529
D(amp) = calculated amplitude diffusivity at given depth (m^2/h)
D(phase) = calculated phase diffusivity (m^2/h)
8766 = number of hours in the year (h)

but let b = B/Bo and p = P-Po

and therefore;

b = e^(-X*sqrt(Pi/D(amp)/8766)) (9)
p = -X*sqrt(Pi/D(phase)/8766) (10)

thus;

T = Ao - b*Bo * cos(2*Pi*n/8766 - Po - p) (11)

where:

T	=	temperature of the soil (C)
X	=	(ground depth -.01) (m)
Pi	=	3.141529
Ao	=	average soil temperature at 1cm depth (C)
Bo	=	temperature amplitude at 1cm depth (C)
Po	=	phase shift at 1cm depth
b	=	diffusivity amplitude coefficient
P	=	diffusivity phase coefficient

2.3.4 Correlation of Diffusivity with Degree Days

The values A1, B1 & PH, defined in section 2.3.2 were compared against the location's degree days and its deep soil temperature (3 metre depth). Both of these parameters are found in HOT-2000 weather files. The correlation of diffusivity to deep soil temperature was not as consistant as degree days. The model can only deal with averages, thus using the 48 locations as statistical representations Therfore an exact fit was not required. Some data was eliminated because it varied too far from the normal. Figures 2-4 represent the log of these values vs degree days.

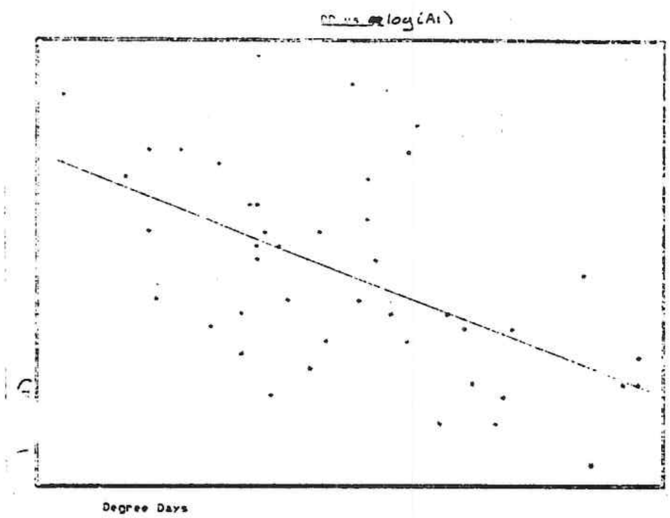

Figure 2
Degree Days vs log(A1)

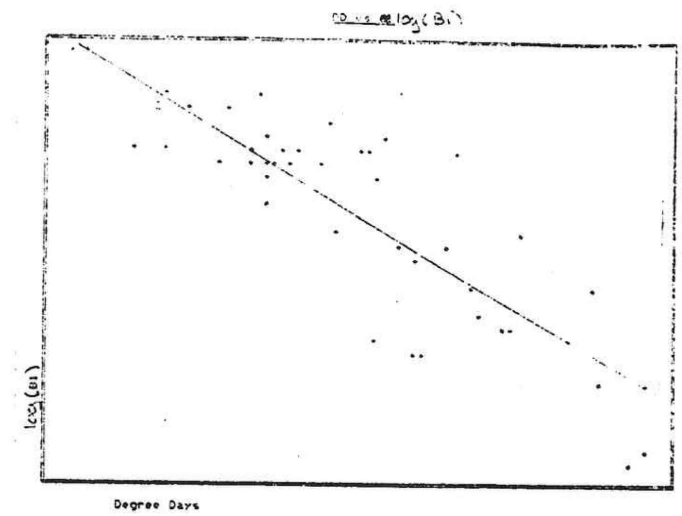

Figure 3
Degree Days vs log(B1)

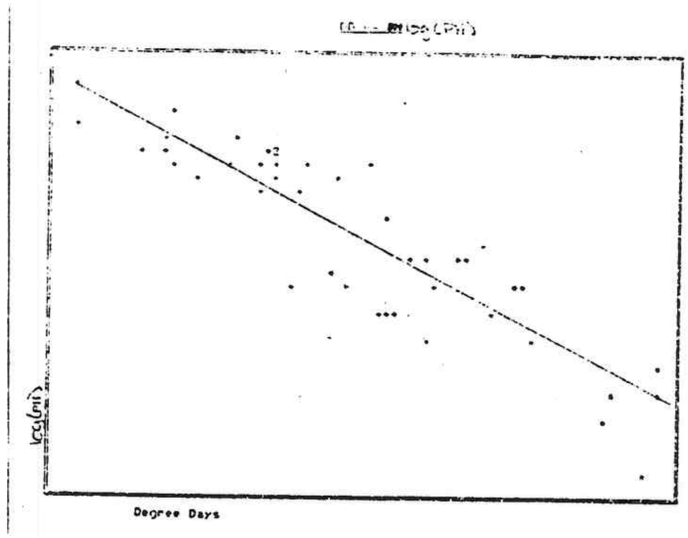

Figure 4
Degree Days vs log(PH)

The correlations found from figures 2-4 were as follows:

$$A1 = (\ 10^{(\ (-8.740e-5)*DD\ -\ 0.0271)}\ )/1000 \quad (12)$$
$$B1 = (\ 10^{(\ (-1.678e-4)*DD\ -\ 0.7918)}\ )/1000 \quad (13)$$
$$PH = (\ 10^{(\ (-9.371e-5)*DD\ -\ 0.5865)}\ )/1000 \quad (14)$$

where:

A1 = intercept of D1 line) (for use in D(amp)) (m^2/h)
B1 = slope of D1 line (for use in D(amp)) (m/h)
PH = phase diffusivity (D(phase)) (m^2/h)
DD = degree days of location

2.4 Correlation of Top Soil Temperature to Ambient Air Temperature

Equations ~~15,16,17~~ will estimate the soil temperature for any depth given a temperature profile at 1cm depth. However, this information is not available considering that the only two parameters available are the monthly average ambient dry air temperature and the location's heating degree days. The logical reference depth is 0cm or the surface of the ground. It is an incorrect assumption however to assume that the temperature at the surface of the ground will match the ambient air temperature. Factors such as snow cover and direct sunshine in the summer will cause the two temperatures to differ. Thus a correlation is needed to relate the ambient air temperature profile to the ground surface profile.

Using the known temperature profile at 1cm for each location from Technical Bulletin 85 and the calculated diffusivity of the soil from degree day correlations, the temperature profile at depth 0cm was calculated. Using the least squares method as was used in section 2.3.2, this temperature profile was fitted to a cosine curve. The ambient air temperature profile was also fitted to a cosine curve and the difference between the A, B & P values of the two curves are plotted against degree days in Figs. 5-7.

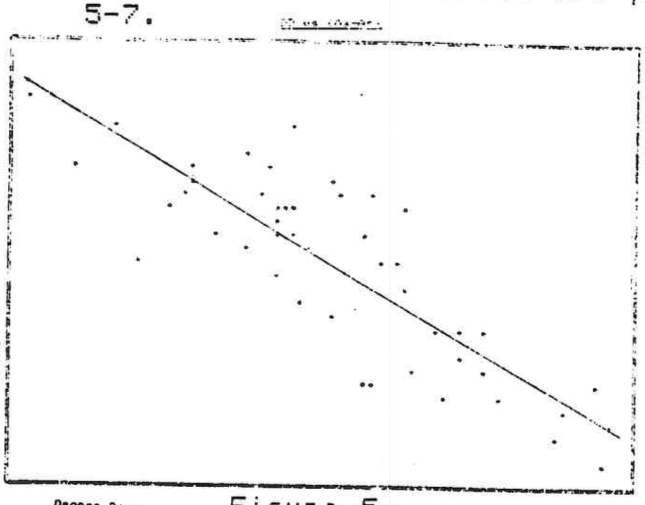

Figure 5
Degree Days vs Mean Air Temperature: Mean Soil Surface Temperature Difference

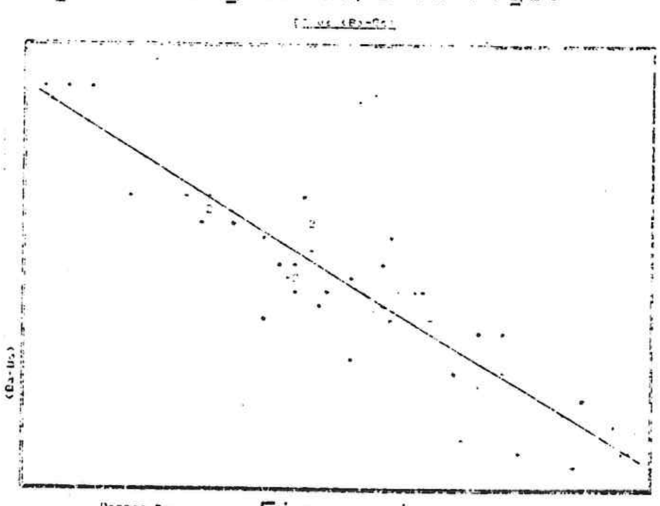

Figure 6
Degree Days vs Air Temperature Amplitude Soil Surface Temperature Amplitude Difference

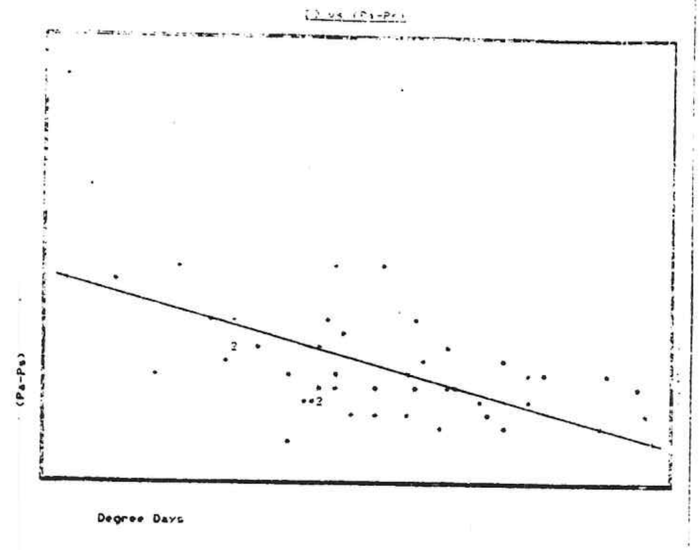

Figure 7
Degree Days Vs Air Temperature Phase Shift:Soil Surface Temperature Phase Shift Difference

A perfect correlation is not possible given the inexact correlation between mean air temperature and soil temperature for reasons outlined earlier. Linear equations were fitted to the plots in Figures 5-7. The resulting equations were:

$$As = (9.189e-4)*DD - 1.438 + Aa \qquad (15)$$
$$Bs = (1.970e-3)*DD - 7.875 + Ba \qquad (16)$$
$$Ps = (2.128e-5)*DD - 0.0756 + Pa \qquad (17)$$

Since $Ba > Bs$, this assumes that $Ba > 0 \Rightarrow$ not consistent with appendix A signage.

where:

As = mean temperature at soil surface (C)
Bs = temperature amplitude at soil surface (C)
Ps = temperature phase shift at soil surface
Aa = mean temperature of ambient air (C)
Ba = temperature amplitude of ambient air (C)
Pa = temperature phase shift of ambient air
DD = degree days of location

7.0 Summary

To estimate the temperature profile for any location, the only data required is the monthly average ambient dry air temperature and the annual heating degree days.

With this information, the model will proceed to fit the ambient air temperature profile to a cosine curve as described in section 2.4 Mean temperature, temperature amplitude and phase shift of the air temperature cosine curve result in the variables Aa, Ba & Pa respectively.

Next the model converts the air temperature profile, to the ground surface temperature profile. This is achieved by converting Aa, Ba & Pa to As, Bs &Ps. These variables represent the temperature profile of the ground surface. The conversion is done by the following correlations: (see section 2.4 for derivation)

$$As = (9.189e{-4})*DD - 1.438 + Aa$$
$$Bs = (1.970e{-3})*DD - 7.875 + Ba$$
$$Ps = (2.128e{-5})*DD - 0.0756 + Pa$$

The ground diffusivity values A1, B1 & PH are determined for a location by the following correlations: (see section 2.3.4 for derivation)

$$A1 = (\ 10\hat{}(\ (-8.740e{-5})*DD - 0.0271)\ )/1000$$
$$B1 = (\ 10\hat{}(\ (-1.678e{-4})*DD - 0.7918)\ )/1000$$
$$PH = (\ 10\hat{}(\ (-9.371e{-5})*DD - 0.5865)\ )/1000$$

Next, the ground depth (m) from the surface (X) is used to calculate diffusivity values. (see section 2.3.2 for derivation)

$$D(amp) = A1 + B1*X$$
$$D(phase) = PH$$

With these values, the diffusivity 'b' and 'p' coefficients can now be found. Note that <u>X is now from the ground surface</u> rather than 0.01m below ground as was the case in section 2.3.3 when all calculations were done from 0.01m as a reference depth.

$$b = e\hat{}(\ -X*sqrt(\ Pi/D(amp)/8766\ )\ )$$
$$p = -X*sqrt(\ Pi/D(phase)/8766)$$

And thus the final soil temperature equation is;

Should be + as noted by Ian

$$T = As + b*Bs * cos(\ 2*Pi*n/8766 - Ps - p)$$

T(x)	=	temperature of soil at depth x (C)
As	=	mean temperature of soil at surface (C)
Bs	=	soil temperature amplitude at surface (C)
b	=	decrease in amplitude for depth x
Pi	=	3.1415927
n	=	number of hours into year (8766 hours/year)
Ps	=	phase angle at surface
p	=	shift in phase angle for depth x

This equation assumes that the ground is of constant permeability. Also the average monthly ambient dry air temperature is assumed to occur at the middle of the month.

An error analysis was conducted on the model in Appendix B. Of 1,728 test points at 0.5 - 1.5m the maximum error between estimated and actual temperature points was 3.6(C). The mean difference between the two was 0.0203(C) with a standard deviation of 1.0962. This is within acceptable limits of estimation, considering the model only requires heating degree days, ambient monthly dry air temperature and a user defined soil depth.

If n_i and n_f is the begining and the end of the period

$$\bar{T} = \frac{1}{n_f - n_i} \int_{n_i}^{n_f} \left\{ A_s + b B_s \cos\left(2 P_i n / 8766 - P_s - p \right) \right\}$$

$$\bar{T} = A_s' + \frac{b B_s'}{n_f - n_i} \frac{8766'}{2 P_i} \sin\left(\frac{2 P_i n}{8766} - P_s - p \right) \Big|_{n_i}^{n_f}$$

APPENDIX A

Fitting Temperature Data to a Cosine Curve

Method of fitting data to a cosine curve of the form:

this should be "+"

$$T = A + B * \cos(2 * Pi * n / 8766 - P)$$

using techniques outlined in Kusuda & Achenbach.

1. 12 monthy average temperatures must be given.

e.g.

jan	feb	mar	apr	may	jun	jul	aug	sep	oct	nov	dec
1.2	0.4	0.6	3.9	10.3	15.6	19.0	19.2	16.7	12.0	6.8	3.0

therefore t(1) = 1.2, t(2) = 0.4 ...etc

2. To fit monthly data to a cosine curve, it is assumed the temperature given corresponds to the middle of the month.

3.

$$A_a = \frac{\sum_{k=1}^{12} t(k)}{12}$$

$$S1 = \sum_{k=1}^{12} t(k) * \sin(2 * Pi / 12 * (k-0.5))$$

$$C1 = \sum_{k=1}^{12} t(k) * \cos(2 * Pi / 12 * (k-0.5))$$

* - note (k-0.5) represents the middle of the month.

$$B_a = (2/12) * sqrt (s1^2 + c1^2)$$

$$P_a = \arctan (s1/c1)$$

4. The variables A, B & P represent respectively the mean, amplitude and phase shift of the data.

APPENDIX B

Accuracy of Generated Soil Temperatures

The original 48 locations across Canada, used to generate the correlations, were compared against model estimated values for the depths of 0.5m , 1.0m and 1.5m. These depths were chosen, for the reason that most earth coupled heat pumps are placed in this depth regime. Thus 48 locations X 12 months X 3 depths gave a total of 1728 cases to analyse.

A statistical correlation comparing each measured temperature case to its corresponding estimated temperature was completed. The correlation, shown in Figure 8, yielded a slope of 0.96562 and an intercept of 0.2844. The coefficent of correlation was 0.9806. An ideal fit of measured temperature to predicted temperature would contain a slope of 1 and a correlation coefficent of 1.0.

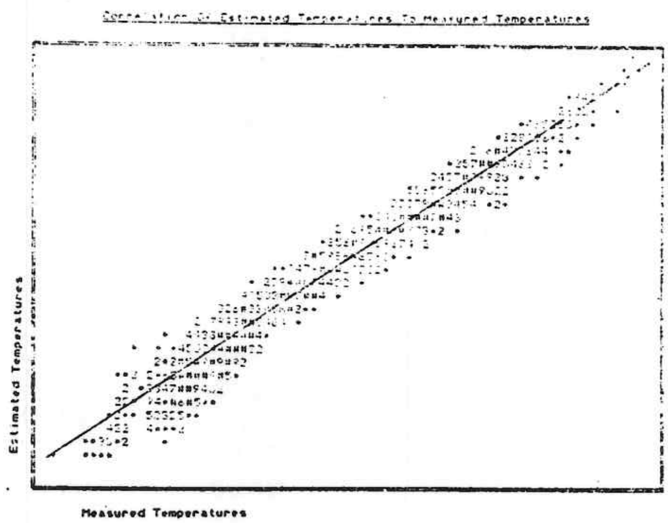

Figure 8

An analysis of the difference between each case gave a mean of -0.0203C with a standard deviation of 1.0962. The maximum difference was 3.61C.

REFERENCES

(1) Estimated Monthly Normals of Soil Temperatures in Canada , C.E. Ouellet, R. Sharp and D. Chaput. , Department of Agriculture, Ottawa; Tech. Bull. 85 Feb 1975

(2) Earth Temperature and Thermal Diffusivity at Selected Stations in the United States, T. Kusuda and P.R. Achenbach; ASHRAE No. 1914