

Department of Mechanical Engineering

**Modelling the performance of  
Air Source Heat Pump Systems**

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## Abstract

The objective of this project is the development of an air source heat pump-based heating system model which can be used to assess the impact of different methods of providing supplementary heating on heat pump performance.

Generalised heat pump performance is represented by regression models developed from publically-available independent test data of 30 heat pump units. An inverse model of a radiator-based distribution system is used to integrate the heat pump model within the IDEAS methodology (Murphy et al, 2011). IDEAS is a newly-developed dynamic approach for the estimation of building heating energy requirements. It allows perfect control of internal temperature through the use of inverse dynamics, and enables the constraints placed on air source heat pump performance by different supplementary heating configurations to be assessed independently of any particular method of practical control.

Two fundamental supplementary heating configurations are investigated: in the first, supplementary heating is delivered separately from the heat pump heating system; in the second, supplementary heating is integrated into the heat pump heating system. It was found that integrating supplementary heating placed constraints on heat pump performance; combined with radiator systems of less than optimal size, it lead to a 20% drop in system efficiency compared to separate supplementary heating.

The model is also used to compare the energy costs and CO<sub>2</sub> emissions of an air source heat pump system with optimally-sized radiators to those estimated for a gas condensing boiler. Emissions savings of 3 to 13% were found, depending on heating pattern used. Cost savings were dependent on an off-peak electricity tariff being used.

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<b>Section</b>	<b>Table of contents</b>	<b>Page</b>
1	Introduction	8
1.1	Project objectives	9
1.2	Project overview	10
1.3	Note on terminology	11
2	Factors affecting air source heat pump performance	12
2.1	Principles of the heat pump cycle	12
2.2	Practical heat pumps	14
2.3	Improvements to the basic practical heat pump cycle	16
2.4	Defrost cycles	17
2.5	Heating systems and seasonal performance factor	19
3	Literature review	24
4	Methodology	28
4.1	IDEAS methodology	28
4.2	ASHP system model - overview	32
4.3	Radiator system model	34
4.4	ASHP model	37
4.5	Climate data and internal gains calculations	40
4.6	Heat pump sizing methods	41
5	Results	44
5.1	Supplementary heating configuration	44
5.2	Intermittent versus continuous heating	47
5.3	Comparison of methods of deriving radiator heat transfer coefficients	50
6	Conclusions	52
7	Further work	54
Annex A	Description of the heat pump model's implementation in Simulink	55
Annex B	Calculation of radiator heat transfer coefficient	72
Annex C	Calculation of solar gain	76
	References	79

## List of figures

- 2.1 Idealised (reverse Carnot) heat pump cycle
- 2.2 Heat pump systems with different supplementary heating configurations
- 4.1 IDEAS model response to a step change in set point temperature
- 4.2 Simplified block diagram of heat pump model integrated into IDEAS
- 4.3 Manufacturers' and calculated radiator heat transfer coefficients
- 4.4 COP regression model
- 4.5  $\dot{Q}_{\text{factor}}$  regression model
- 5.0 Demonstration of ASHP model outputs
- 5.1 Heating system electricity use – ASHP capacity 6.3kW, continuous heating
- 5.2 Heating system electrical power consumption and return temperature over the first two weeks of December
- 5.3 Electricity use under intermittent and continuous heating patterns
- 5.4 Comparison of the methods of deriving radiator heat transfer coefficient
- A.1 Heat pump model integrated into IDEAS – Simulink block diagram
- A.2 Radiator subsystem block diagram – separate supplementary heating
- A.3 Heat pump system block diagram – separate supplementary heating
- A.4 Radiator subsystem block diagram – integrated supplementary heating
- A.5 Heat pump system block diagram – integrated supplementary heating
- B.1 Calculated radiator heat transfer coefficient
- B.2 Schematic of assumed radiator construction: view from above

## List of tables

- 4.1 Dimensions and thermal characteristics of building modelled
- 4.2 Sample of WPZ test data
- 5.1 Results of simulations of separate and integrated supplementary heating
- 5.2 Annual electricity costs under different tariffs and heating patterns – 4m<sup>2</sup> radiators

## 1. Introduction

Domestic heat pumps make use of the vapour compression cycle to absorb low grade heat from the external air and reject it at higher temperature inside a building. They can provide a highly efficient method of providing space and water heating: it is often stated that they can provide three times as much energy in the form of heat as they require in the form of electricity. They have the potential to reduce the costs, emissions and resource intensity of UK heating.

The use of heat pumps for domestic heating is common in several Northern European countries, as well as Japan and North America. A report published by the European Heat Pump Association (2009b) states that nearly 130,000 units were sold in Sweden in 2008 and around 80,000 in both Norway and Germany. The report also found that air source heat pumps (ASHPs) are gaining in popularity relative to their more-established ground source cousins; in 2008, sales of ASHPs in the “space heating only” market segment overtook sales of ground source heat pumps for the first time. Though ground source heat pumps can be more efficient than ASHPs, they are more costly to install and more restricted in their applicability.

The UK market for heat pumps, and ASHPs in particular, is relatively under- developed. A Scottish Parliament research report contained an estimate that in 2008 as few as 150 ASHPs were installed in the UK (Scottish Government Social Research, 2009). Government support for ASHPs, in the form of grants towards installation costs, has only been intermittently available since 2007 in the UK (2006 in Scotland).

However, against a background of the ambitious emissions reductions targets introduced in the Scottish and UK Climate Change Acts, interest is growing in their potential benefits. In his book *Sustainable Energy – Without the Hot Air*, David MacKay shows that a heat pump powered by electricity generated at a centralised, highly efficient gas power station represents a significantly smaller gas use than a condensing boiler (Mackay, 2009, p146-154). He argues that all fossil-fuel heating should be replaced with heat pumps, and estimates that doing so in conjunction with building insulation and heating system control improvements would reduce the primary energy used for heating by 75%. From 2012, long-term support for ASHPs may be provided by the Renewable Heat Incentive.

MacKay's arguments are based on heat pumps operating with an efficiency of 300 to 400%. However a recent UK field trial revealed that the efficiencies achieved in real installations varies widely (Energy Saving Trust, 2010). Most of the trial subjects were retrofit installations using radiator systems to distribute heat. A greater understanding of the reasons for this variation is required if the cost and emissions savings ASHPs could potentially provide is to be realised.

## **1.1 Project objectives**

The objective of this project is the development of an air source heat pump-based heating system model which can be used to assess the impact of different methods of providing supplementary heating on heat pump performance.

A key requirement for the model is that it works within the IDEAS methodology. IDEAS is a newly-developed, simplified dynamic approach to the modelling of building heating requirements, designed to integrate with and augment the UK government's SAP method for the energy rating of dwellings (Murphy et al, 2011). IDEAS is used for three main reasons:

- it has been calibrated against SAP, which in turn is based on extensive monitoring work of real houses;
- it incorporates inverse dynamics-based perfect control, meaning that heat pump performance can be assessed separately from the effect a particular control method; and
- the version used in this project has been implemented in the Matlab software package, and makes use of the Simulink graphical modelling environment. The flexibility this provides enables different heat pump system configurations to be examined.

A second requirement is that the model must take account of the varying constraints placed on heat pump output and efficiency by the dynamic conditions in which it is operating. To determine these conditions, an inverse model of a radiator-based distribution system is developed.

## **1.2 Project overview**

The work undertaken for this project is broken down into five key stages. These form the main sections of this report.

1: Identification of the fundamental factors upon which heat pump performance depends. This involved gaining an understanding of the principles of the vapour compression cycle and important aspects of practical heat pump systems. This forms section 2 of the report

2: A literature review covering the findings and methodologies used in previous modelling studies of ASHP performance. The outcomes of field trials of ASHPs in the UK are also of interest. This forms section 3 of the report.

3: Familiarisation with relevant aspects of the IDEAS methodology, in order that the heat pump and radiator system model developed integrates with its approach. This forms section 4.1 of this thesis.

4: Development of the model. This is described in section 4.2 to 4.6.

5: Simulation of the effect on ASHP performance of different supplementary heating provision. Results and conclusions from these simulations are set out in sections 5 and 6.

### 1.3 Note on terminology

#### **Coefficient of performance (COP):**

COP is the usual method of expressing heat pump efficiency. An ‘instantaneous’ definition of COP is assumed throughout this thesis;

$$COP = \frac{\dot{Q}_{HP}}{\dot{W}_{HP}} \quad (1.1)$$

where  $\dot{Q}_{HP}$  is the rate of heat output of the heat pump and  $\dot{W}_{HP}$  is its power draw. This can be thought of as the special case of the classic definition in which Q and W are measured over an infinitesimally small time period. To simplify equations and figures the usual thermodynamical sign convention for work and heat transfers is also disregarded. The only exception to these points is the discussion of Carnot efficiency in section 2.1.

#### **Seasonal Performance factor (SPF)**

SPF will always refer to some defined time period (e.g. a year, season or month) and is defined

$$SPF = \frac{Q_{HP}}{W_{HP}} \quad (1.2)$$

where  $Q_{HP}$  is the total heat energy output of the heat pump over the period and  $W_{HP}$  is the total electrical energy it consumes. Where it applies to a season, winter is taken to be 1 December to 28 February; spring runs from 1 March to 31 May; and so on in three month blocks.

#### **System efficiency**

System efficiency is defined similarly to SPF, however relates to the heating system as a whole:

$$System\ efficiency = \frac{Q_{HS}}{W_{HS}} \quad (1.3)$$

Here  $Q_{HS}$  refers to the total heat energy delivered to the building by the heating system (heat pump, supplementary heating and circulation pump), and  $W_{HS}$  is the total electrical energy consumed by the heating system including distribution pumps.

## **2. Factors affecting air source heat pump performance**

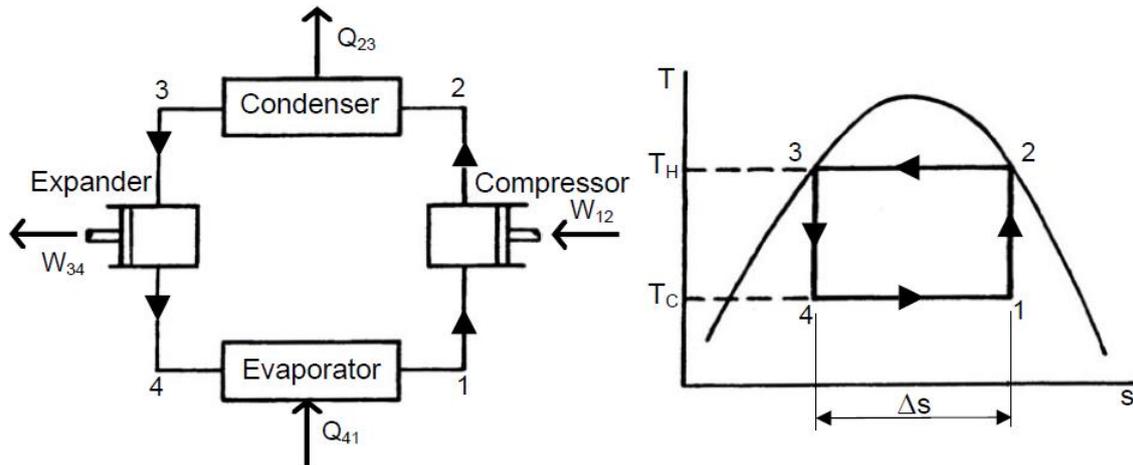
### **2.1 Principles of the heat pump cycle**

Heat pumps (for domestic heating at least) operate on the vapour compression cycle. They make use of one or more of a group of chemicals with useful thermodynamic properties known as refrigerants. A simplified air-to-water domestic heat pump system is depicted in figure 2.2 in section 2.5 (page 20).

Refrigerant enters the the evaporator as a liquid-vapour mixture at low pressure. In the case of an air-to-water heat pump, the evaporator is a coiled tube (“coil”) over which external air is drawn by a fan. The refrigerant is at lower temperature than the air, and so heat flows in from the air, vaporising the refrigerant. The refrigerant is then compressed to a higher pressure, raising its temperature. The hot refrigerant passes through one side of a heat exchanger known as the condenser. Water at slightly lower temperature passes through the other side. The refrigerant is cooled and condensed as the water is warmed. The water is used to provide space heating, domestic hot water or both. The condensed refrigerant is expanded to a lower pressure and temperature and returned to the evaporator.

The vapour compression cycle has its roots in the principles developed by Carnot in the early 19<sup>th</sup> century. He developed an expression for the maximum possible efficiency of a machine which, operating in a cycle, produced work through the transfer of heat from a hot to a cold source. A heat pump is the precise reverse of this; work is input to achieve a transfer of heat in the opposite direction. However Carnot theory is equally applicable in deriving an expression for the maximum possible efficiency that can be achieved by a heat pump.

**Figure 2.1: Idealised (reverse Carnot) heat pump cycle (Tuohy, 2010)**



Compression (1-2) and expansion (3-4) are reversible, adiabatic and isentropic.

Condensation (2-3) and evaporation (4-1) are reversible, isothermal and isobaric.

The efficiency of a heat pump is characterised by its coefficient of performance (COP)

$$COP = \frac{|Q|}{|W|}$$

where  $W$  is the net work done on the system. When the focus is on the heating rather than cooling function,  $Q$  is the heat transferred to the hot source. The maximum COP that can be achieved by a heat pump can be expressed as

$$COP_C = \frac{|Q_{23}|}{|W_{12}| - |W_{34}|} = \frac{T_H \Delta s}{(T_H - T_C) \Delta s} = \frac{T_H}{T_H - T_C}$$

This shows that for an ideal heat pump, efficiency varies widely depending on the temperatures,  $T_H$  and  $T_C$ , of the hot and cold sources source it is operating between.

A real ASHP might be expected to output hot water at  $45^\circ\text{C}$  when the ambient temperature is  $-5^\circ\text{C}$ .  $COP_C$  at these conditions is 6.36, whereas for a practical heat pump, a COP of 2 would be more realistic. In the real world real world then, heat pump COP affected by other factors, and these are discussed in the next section. However the efficiency of a real heat pump retains a strong dependence on  $T_H$  and  $T_C$ .

## 2.2 Practical heat pumps

A key criterion for achieving Carnot efficiency is that all of the processes making up the cycle must be reversible. This is not possible in reality; for example there will always be some heat loss to surroundings. More fundamentally, to be reversible processes must take place infinitesimally slowly. In order that a heat pump transfers a usable amount of heat whilst retaining reasonable dimensions, processes clearly need to happen much more quickly. This leads to energy losses due to friction, and also means that care must be taken when interpreting the temperatures that a real heat pump is operating between.

When calculating the limit on heat pump performance  $COP_C$  earlier, it was assumed that the temperatures the heat pump was operating between,  $T_C$  and  $T_H$ , were equal to ambient air temperature  $T_0$  and heat pump output temperature  $T_{out}$  respectively. In practice  $T_C$  must be a few degrees lower than  $T_0$ , and  $T_H$  generally<sup>1</sup> a few degrees higher than  $T_{out}$ , in order for a the transfer of heat to take place at a realistic rate.

$$T_C = T_0 - \Delta T_C$$

$$T_H = T_{out} + \Delta T_H$$

This is because the evaporator and condenser are heat exchangers, and in each there must be a temperature difference between the two sides in order for heat to be transferred. The size of  $\Delta T_H$  and  $\Delta T_C$  depends on (or dictates, depending on the viewpoint taken) the rate of heat transfer at the hot and cold sources. It is also worth stating that the refrigerant experiences a small pressure drop across both the condenser and the evaporator due to frictional forces acting against the fluid flow. This means that the temperatures of condensation and evaporation are not constant, and also contributes to the size of  $\Delta T_H$  and  $\Delta T_C$ .

Another practical consideration is that it is generally not considered economic to extract the work available from expansion. Consider a heat pump using refrigerant R407c, operating with an ambient air temperature of  $-5^\circ\text{C}$  and producing an output temperature of  $45^\circ\text{C}$ . By examining a pressure-enthalpy diagram it can be seen that expansion work could at best provide around one fifth of the work required for compression, assuming both expansion and

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<sup>1</sup>  $T_C$  and  $T_H$  are interpreted here as the representative temperatures of the refrigerant in the evaporator and condenser. In reality refrigerant temperature is not constant throughout the evaporator or condenser. In particular, if the condenser is a counterflow heat exchanger, and the vapour leaving the compressor has a high degree of superheating, it would be possible for  $T_H$  to be marginally lower than  $T_{out}$ . However given size constraints this seems unlikely, and is disregarded in most descriptions of heat pump cycles.

compression have an isentropic efficiency of 1. Assuming a more realistic isentropic efficiency of 0.8 for both, this falls to around one eighth. Heat pumps therefore make use of an expansion valve (basically a small orifice which restricts refrigerant flow) rather than attempting to extract work.

In the idealised heat pump cycle, refrigerant leaves the condenser as a saturated liquid and is then expanded. However expansion valves can be damaged if vapour bubbles form in the line from the condenser (known as flashing). To ensure this does not happen, in real heat pumps refrigerant must be subcooled to some degree.

Similarly, the idealised heat pump cycle requires a liquid-vapour mixture to be compressed (in order that refrigerant leaves the compressor as a saturated vapour). Not only would it be difficult to stop evaporation at the required point, but the presence of liquid in a compressor's suction line can cause damage to valves or other components. Generally, some degree of superheating is required before compression. Superheating increases the specific volume of the vapour, which increases compressor work. It also generally requires a controllable expansion valve. This only allows as much refrigerant into the evaporator as can be fully vaporised and superheated to the required level before reaching the compressor (known as dry expansion).

There are different methods for carrying out subcooling and for preventing liquid reaching from the compressor (in some arrangements, superheating is not necessary). However, a certain amount of divergence from the ideal cycle is always required, reducing efficiency.

Where a heat pump is designed so that the evaporator provides superheating and the compressor subcooling, the efficiency of the cycle is further reduced. In the case of subcooling, the compressed refrigerant is not being used at the highest possible temperature. In the case of superheating, a proportion of the evaporator must be used for heating a dry vapour, reducing the effectiveness of the heat transfer.

### 2.3 Improvements to the basic practical heat pump cycle

A number of modifications to the basic cycle depicted above are possible, which reduce some of the inefficiencies discussed.

Superheating and subcooling may be carried out by bringing the high temperature liquid from the evaporator briefly into thermal contact with the low temperature vapour from the condenser. This avoids the reduction in heat transfer described previously. Alternatively the need for superheating prior to the compressor can be removed by the use of an accumulator.

Other improvements are aimed primarily at reducing compressor work. One of these is to split compression into two stages, and introduce intercooling between the two. The cooling can be either be provided by an external source or, where expansion is also split into two stages, flash or shell and coil intercooling. The increased complexity and reliability issues introduced by a second compressor perhaps means intercooling is more relevant to larger-scale heat pump applications rather than domestic units. However an experimental domestic air source heat pump making use of economised vapour injection (a method of providing intercooling without requiring two compressors) has recently been trialled (Hewitt et al, 2011). The same article also describes development work being undertaken on an experimental ASHP which uses a turbine in place of an expansion valve, to reduce the net work input.

The net result of the practical considerations and improvements discussed explain much of the difference between actual and ideal heat pump COP. However a key point is that, like the COP of an ideal heat pump, the COP of a real heat pump varies widely depending on the temperatures of the hot and cold sources it is operating between. However the parameters which define a real cycle are complex, and the way they vary as source temperatures change is difficult to predict. Therefore a 'black-box' approach is often taken to modelling heat pump performance: rather than seeking to model the refrigerant cycle itself, equations representing the output and COP of a heat pump at particular source temperatures are developed empirically from test data.

## 2.4 Defrost cycles

The above discussion of factors affecting the performance of practical heat pumps applies whatever heat source is used. An important factor peculiar to air source heat pump performance is frosting of the evaporator.

The practical requirement for the evaporator coil to be several degrees below external air temperature means that condensation tends to form on it (indeed water run-off is an important consideration when installing an ASHP outdoor unit). Furthermore when external air temperature is below approximately 5°C, the surface of the evaporator coil is likely to be below 0°C. The result is a build-up of ice. This degrades heat pump performance by reducing both air flow through the coil, and potentially the heat transfer coefficient from the coil's surface. If frosting continues for an extended period, air flow through the coil can be completely blocked.

ASHPs therefore incorporate mechanisms to heat the evaporator coil periodically, melting the ice. This process is known as a defrost cycle. The most common method of defrosting is to reverse the refrigerant flow, so that the evaporator coil temporarily becomes the condenser. A drawback of this method is that heat is removed from the building, and alternatives are sometimes used such as direct electric heating or the sensible heat defrosting method (Liang et al, 2010). The latter alternative, rather than reversing the cycle, involves bypassing the condenser and going directly from compression to expansion. This results in a highly superheated vapour which is used to heat the evaporator coil. The method avoids removing heat from the building, as instead all the heat is provided by compressor work.

Whichever method of defrosting is employed, the effect is to interrupt (or even reverse) the supply of heat to the distribution system whilst still consuming power. Performance is therefore reduced.

The rate of frosting and its effect on performance depends heavily on relative humidity as well as temperature. If air is very dry, a comparatively large amount of heat can be extracted before the dew point is reached. If air is moist however, extracting even small amounts of heat could result in condensation which may then freeze. For example, tests of a specific ASHP found that at an external air temperature of 0°C and a relative humidity of 85%,

frosting degraded heat output by one fifth after 35 minutes. At the same temperature and 65% relative humidity, the same performance reduction only occurs after 145 minutes (Guo et al, 2008). The effect of relative humidity is not always straightforward however: one study (Miller, 1982) reports that at 4.4°C, higher relative humidity actually delays the onset of frosting. This is because moist air has a higher enthalpy than dry air at the same temperature. This results in a better heat transfer to the coil, meaning that coil surface temperature remains higher.

The rate of frosting is further influenced by the characteristics of a particular heat pump. For example increasing the speed of the heat pump's fan reduces frosting (Miller, 1982). Research has shown that coil geometry (Cgawa et al, 1993), the use of special coatings (Lee et al, 2004) and even the use of an electric field (Tudor et al, 2005) have an effect.

The required frequency of defrost cycles is therefore difficult to predict. In older or more basic heat pumps, the frequency of defrost cycles may be 'time' or 'time and temperature' controlled. Here defrost cycles are initiated at set time intervals (in the case of time and temperature, only when external air temperature is below a set level). This often results in unnecessary or overdue defrost cycles, reducing system efficiency. Modern heat pumps are likely to sense the air pressure difference across the outside coil, or the difference in temperature between air and the coil outlet. Both will increase as frosting increases, and a defrost cycle is triggered once a certain difference is reached. This is intended to ensure that defrost cycles are only triggered when required.

## **2.5 Heating systems and seasonal performance factor**

### **2.5.1 Heating system configuration**

Air source heat pumps are available in a variety of configurations. The heat source may be external air, internal air in the case of mechanical ventilation and heat recovery systems, or both. They can provide space heating, domestic hot water, cooling or some combination of these. Finally they may use water as a heat distribution medium, or deliver heating or cooling directly to air.

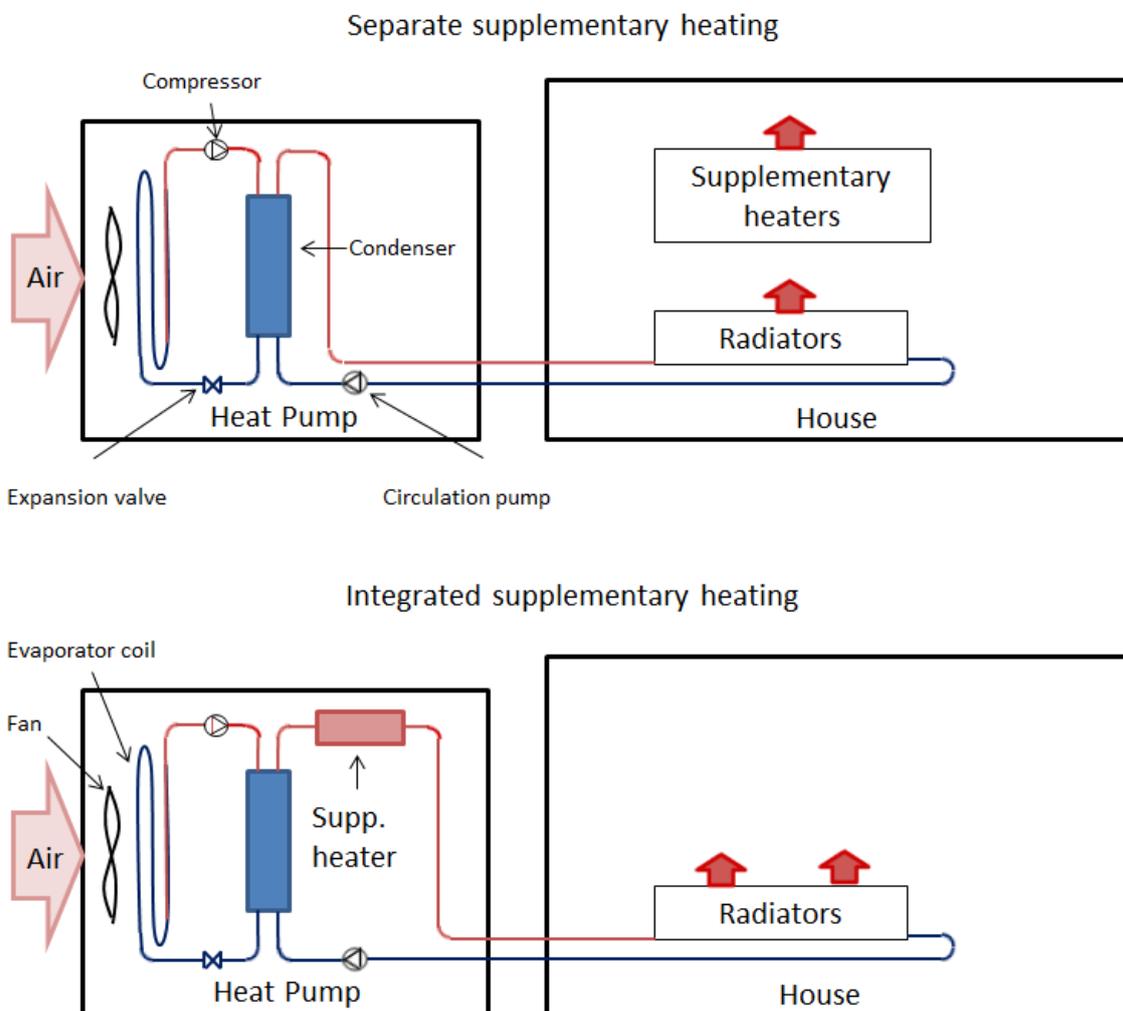
This project is concerned with the performance of retrofit ASHP installations in houses with insulation levels close to the average for UK housing stock. In these circumstances, ASHPs providing heating only and using a radiator-based wet distribution system are most likely, as the need for extensive and disruptive installation work can be minimised. Figure 2.2 is a simplified schematic of possible system configurations, on which the modelling work undertaken in this project is based. An outdoor ASHP unit is depicted, however performance is similar to indoor or split units and the results presented are relevant to all three types of unit.

ASHPs are typically not designed to meet the full peak heating load of a building, as this can lead to excessive cycling and reduced performance during warmer periods. This is largely because the thermal capacity of an ASHP, unlike that of a conventional boiler, declines at low external temperatures. An ASHP sized to meet the peak heating load of a building at low external temperature will have a higher capacity during warmer periods than a conventional boiler sized the same way. The provision of a supplementary heat source is therefore important, and supplementary heaters are depicted in figure 2.2. In the first configuration, the supplementary heating is a separate system. In the second, supplementary heating (often electric resistance heating) is integrated into the heat pump system. The latter appears to be a common arrangement. It has the advantages of reduced complexity and space requirements, and the supplementary heating can be used to prevent loss of heating during defrost cycles. However when used integrated supplementary heating affects heat pump performance.

As described in the previous section, the COP of an air source heat pump at any moment in time depends largely on the temperatures of the hot and cold sources it is operating between.

To ensure the seasonal performance factor is optimised, this temperature difference must be kept as small as possible. The cold source temperature is of course set by the external temperature. However the temperature of the hot source is set by the temperature of the return flow from the distribution system. When integrated supplementary heating is used, the return temperature will be increased, and COP will reduce. Potentially more significantly, heat pumps have a high pressure cut-out to prevent excessive refrigerant pressure. This may cause the heat pump to switch off if return temperatures exceed some maximum limit. Integrated supplementary heating systems make this more likely. Assessing the impact of this on long term performance is a key aim of this project.

**Figure 2.2: Heat pump systems with different supplementary heating configurations**



Other factors also have a major influence on return temperature and COP. The size and design of the distribution system, and the method employed to control its output, are particularly important.

Problems can arise where ASHPs replace conventional gas or oil boilers in a radiator-based heating system. The rate of heat output  $\dot{Q}$  of a radiator system can be represented by

$$\dot{Q} = hA \cdot \Delta T$$

where  $h$  and  $A$  are the heat transfer coefficient and total area of the radiators, and  $\Delta T$  is the temperature difference between the radiators and room. During installation, the area of a radiator system is chosen so that  $\dot{Q}$  equals the design heat loss rate of a house or room at a particular value of  $\Delta T$ . Radiator systems intended to work with conventional boilers are designed for a  $\Delta T$  of 50 or more, and have a relatively small total area. If the boiler is replaced by a heat pump, the area of radiators must be increased in order to ensure low return temperatures, a better COP and less use of supplementary heating.

In poorly insulated houses in particular, it may be difficult to increase the area of radiators enough to achieve satisfactory heat pump performance. An alternative solution is to install underfloor heating or fan assisted radiators (which have a higher heat transfer coefficient). However this involves additional expense and disruption.

### **2.5.2 Heating system control**

In order to maintain a particular temperature set point in a building, the heating system output must be controlled depending on the rates of heat loss and heat storage as well as internal and solar gains. The type of control employed influences the return temperature to the heat pump.

In the case of fixed capacity heat pumps with no means to control compressor speed, the rate of heat output when on is fixed: it is a function of the external temperature and return temperature only. The required modulation can therefore only be achieved by switching the heat pump on and off (“cycling”).

A variety of on-off control methods are possible, the most simple of which is based solely on room temperature as measured by a thermostat. Another method (which may be used in

conjunction with the first) aims to achieve the intended set point by maintaining a certain relationship between the return and external temperatures (Boait et al). Methods which take account of external temperature are often referred to as “weather compensation”.

Whichever method is employed, cycling tends to reduce system efficiency. At part loads, the average return temperature during the short bursts when the heat pump is on is higher than would be required if heat was delivered continuously. Additionally when heat pumps switch on, full output is not reached immediately, suggesting power is wasted.

Lower average return temperatures and better SPFs may be offered by ‘variable capacity’ heat pump models. These are able to modulate their output through means other than on-off cycling, leading to lower average return temperatures. Generally an inverter is used to control the speed of the compressor by varying the frequency of the supplied current. The European Heat Pump Association reported recently that “capacity modulating compressors... may already be considered a standard in air-source units” (European Heat Pump Association, 2009), though it should be noted the context is the air source heat pump market as a whole, rather than only domestic units. None the less at least one range of variable capacity domestic units are available. A number of experimental domestic-sized ground source and air source units have also been reported in the literature (Karlsson, 2007 and Hewitt et al, 2011). As well as the potential to minimise return temperature, variable capacity ASHPs offer:

- a higher COP at lower compressor speeds due to a reduction in  $\Delta T_H$  and  $\Delta T_C$ ;
- a reduction in frosting at lower compressor speeds, due to the reduction in  $\Delta T_C$ ; and
- a reduction in the amount of supplementary heating required, as variable capacity heat pumps can be sized for peak heat demand without risking excessive cycling.

Variable capacity heat pumps also have some drawbacks: there will be an efficiency loss in the inverter itself, and the non-sinusoidal wave form it produces may cause a reduction in motor efficiency. Also, as the heat pump is on for much longer, more energy is required to drive the heating system circulation pump. However the increasing prevalence of variable capacity heat pumps indicates that manufacturers at least believe the benefits outweigh the drawbacks.

Another aspect of heat pump control important to SPF is the heating pattern used. Timer control of heating is common in the UK, and many people do not heat their home during the day on weekdays or overnight. In buildings with conventional, responsive heating systems, these setback periods minimise energy use by reducing energy losses. However it is not clear whether this is the best way to operate heat pump systems. The increased rate of heat output needed to recover from setback periods results in higher return temperatures and reduced COPs. This means that even though heat losses are lower and less heating is required, electricity use could in some circumstances be increased. Indeed it is common for heat pump manufacturers to recommend that heating is used continuously. A comparison is therefore made in this project between the electricity used by a heat pumps operated to the standard SAP heating pattern, and one operated continuously.

### 3 Literature Review

The idea that ASHPs in the UK provide emissions but not necessarily cost savings when compared to gas boilers is becoming fairly well established<sup>2</sup>. As part of high level work to assess the impact on UK CO<sub>2</sub> emissions of new heat and power sources, Cockroft and Kelly concluded that ASHPs emissions are lower than those of a conventional gas boiler so long as the seasonal performance factor is at least 2<sup>3</sup> (Cockroft and Kelly, 2006). A later study by Kelly and Cockroft modelled the integrated performance of an ASHP installed in a UK home, and predicted a seasonal performance factor of 2.7. A reduction in emissions of 12% compared to gas was found, though energy costs are 10% higher (Kelly and Cockroft, 2008). Jenkins et al modelled the performance of an ASHP providing heating and cooling to an office building, and again found emissions were reduced (Jenkins et al, 2008).

Further evidence of the potential environmental benefits of ASHPs has been provided by a recent field trial. This involved the retrofit installation and monitoring of 29 ASHP in houses across the UK, and found an average seasonal performance factor of 2.2. It was concluded that a mid-performing ASHP results in a 9% reduction in CO<sub>2</sub> emissions compared to gas (Energy Saving Trust, 2010). In contrast to Kelly and Cockroft, it appears the trial found average ASHP running costs to be very slightly better than the predicted costs for a gas boiler<sup>4</sup>: it states that a ASHP with mid-range efficiency would repay the marginal cost of installation compared to a new gas boiler in 31 years. A potential reason for this may be that many of the field trial installations were supplied with electricity on “off-peak” tariffs, whereas Kelly and Cockroft assume a standard single-rate tariff. The cost advantages of different electricity tariffs are explored later in this project.

The findings of the EST field trial also demonstrate the variability of ASHP performance in retrofit installations. The SPFs achieved by individual installations ranged widely from 1.2 to 3.3. If ASHPs are to gain widespread acceptance as a viable and beneficial source of heating in the UK, the reasons for this variability need to be understood. The report identifies three main factors which contributed to the differences:

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<sup>2</sup> Approximately 80% of UK households are supplied with mains gas, and gas condensing boilers are generally accepted to represent both the lowest-cost and lowest-carbon form of conventional heating. Therefore it is often used as a baseline when examining the performance of newer heating technologies.

<sup>3</sup> Based on the UK electricity generation mix in 2005.

<sup>4</sup> based on 2010 energy prices

- errors in installation;
- differences in the use of controls by householders; and
- the quality and suitability of the heat pump system specified.

The heat pump performance data discussed in section 4.4 is consistent with the idea that heat pump quality varies significantly (if the COPs achieved in lab tests are taken to indicate quality). At each test point, the COPs achieved by different models covered a wide range. In a very few cases, heat pump models appear to have traded poor performance at low external temperatures, for example, with gains at higher temperatures. However in general, a heat pump with a higher than average COP at low temperature also has a higher COP in warmer conditions.

The EST report also confirms that heat pumps are more sensitive to the manner in which they are controlled than conventional heating systems. Furthermore, occupants and even installers do not always understand what the optimum control method for a particular heat pump system is.

Heat pump control issues are also reported by Boait et al. It was found that the high thermal mass and low distribution system efficiency of typical retrofit installations can compromise the controllability of heat pump systems. Analysis of simplified mathematical representations of a ground source heat pump installation with weather compensation suggested that, although set back periods can reduce energy use in some circumstances, they increase it in others (Boait et al, 2011). This is because the higher heating system flow temperature required to regain the setpoint reduces heat pump COP. It is not clear whether these results would be replicated for ASHPs and night-time set-back periods; unlike ground temperature, air temperature drops overnight.

Much of the research on air source heat pumps reported in the literature focuses on particular aspects of their operation, such as frosting. The only studies identified (in peer-reviewed journals) which model the integrated performance of ASHPs in UK buildings are the two mentioned above by Jenkins et al and Kelly and Cockroft. These have sought to model specific situations, rather than to examine the effect on heat pump performance of variations in control system or distribution system. However the methodology employed is of interest.

Neither study is concerned with the parameters of the heat pump cycle itself, and instead take a “black box” (or at least a “grey box”) approach. In both, regression models describing the heat pump performance across a range of input conditions are developed from test-bench data. Two regression models are required; one to describe the variation in COP across a range of conditions, and one to describe the variation in the rate of heat output. Different approaches have been used to account for the effect of cycling on performance.

The COP and output regression models developed by Kelly and Cockroft took the form of second order polynomials with return temperature and external air temperature as the predictor variables. Through the use of a model structure which took account of the thermal capacitances of the heat pump and heat distribution system, it was possible to dynamically model the cycling behaviour of the fixed capacity heat pump. The work was part of a field trial of ASHP heating systems in Central Scotland, and the range and trend of COPs output by the model matched well with data emerging from the field trial.

The approach used by Jenkins et al differed from Kelly and Cockroft’s in that heating and cooling loads of the office were modelled separately from the heat pump, which in this case is a variable capacity unit. Performance is again characterised by regression models of COP and output (or rather, maximum output in this case). The models take the form of polynomials taking the external air wet bulb temperature as the single variable. A third equation, representing compressor efficiency at part load, is used to adjust COP when the heat requirement of the office is less than the heat pump’s maximum output.

Both these studies made use of a sophisticated building thermal modelling tool developed at the University of Strathclyde, ESP-r, to determine heat requirements. The heat pump model developed by Kelly and Cockroft was fully integrated into ESP-r, enabling the impact of cycling on performance to be assessed in detail.

Others (e.g. Fisher and Rees, 2005 and Madani et al, 2011) have proposed more sophisticated methods of modelling heat pump units. The method employed by Fisher and Rees is of particular interest. It has similarities to the ‘grey-box’ approach used by Kelly and Cockroft, in that the form of the model imitates the physical processes it represents. However here the approach is extended to include the key components of the refrigerant cycle as well as system thermal capacitances. It is reported to enable confidence in results even at conditions outside

of the range for which calibration data was available. However, although the method is designed to minimise the data required for calibration to that generally published by manufacturers, there does not appear to be enough data publically available to allow the adoption of this method for ASHPs.

## **4. Methodology**

### **4.1 The IDEAS methodology**

The core of this project is the development of a model of air source heat pump performance which works within the IDEAS methodology. IDEAS is a newly-developed, simplified dynamic approach to the modelling of building heating requirements, designed to integrate with and augment the UK government's Standard Assessment Procedure (SAP) for the energy rating of dwellings (Murphy et al, 2011). The version used in this project has been implemented in the Matlab software package, enabling different heat pump system configurations to be examined easily.

In reduced form, SAP is used to calculate ratings for home Energy Performance Certificates under the European Energy Performance of Buildings Directive. The full version can be used to demonstrate compliance with UK building regulations (BRE, 2011).

In essence, the SAP methodology involves estimating the monthly space heating requirements of a building by multiplying its total heat transfer coefficient by the difference between the mean internal and external temperatures for the period, and subtracting the internal gains. Once the space heating requirement has been calculated, the energy needed to deliver this is calculated from the efficiency of the heating system.

In order to account for the dynamics of building thermal performance in what effectively a steady-state calculation, a detailed process is used to derive an appropriate value for mean internal temperature. In particular, adjustments designed to account for the effect of building thermal mass and the controllability of the heating system are applied. The process for deriving mean internal temperature was originally developed empirically using data from very large scale monitoring exercises in real houses. Over the years this initial work has been supplemented by modelling work and a small amount of additional monitoring.

SAP is generally held to produce reasonably accurate results across a wide range of typical UK house types. However it is designed first and foremost as a rating tool, and has limitations when used for design – particularly where innovative technologies are considered.

Also, discrepancies have been reported between SAP results for low energy houses and the output of tools designed specifically for this type of building (Murphy et al, 2011).

IDEAS uses the same inputs in terms of building dimensions and thermal properties as SAP requires. However, through the addition of dynamic modelling, IDEAS is intended to enable easier assessment of the effects of new technologies on energy requirements and address discrepancies in areas such as low energy houses. At the core of the IDEAS method is the use of inverse dynamics-based ‘perfect’ control, which calculates the precise heat requirement needed at each timestep to exactly maintain the SAP standard intermittent heating pattern<sup>5</sup> (allowing for system responsivity, discussed below). IDEAS has been calibrated so that total monthly heating requirements match those produced by SAP. The calibration was carried out for housing types which were well represented in the extensive monitoring project upon which SAP was based.

A number of aspects of the IDEAS method are important to note. Firstly, the version used in this project is a fourth-order model which treats the building as a single zone. The four temperature nodes represent

- the internal air;
- the internal mass (furniture etc.);
- the internal structure (the ‘inner’ portion of the external wall structure); and
- the external structure (the ‘outer’ portion of the external wall structure)

The system output monitored by the (closed-loop) control is comfort temperature. It is assumed that internal air, internal mass and internal structure temperatures all contribute to comfort temperature. The ratio in which they contribute was one of the parameters adjusted during calibration: it is not known precisely what temperature was measured during the monitoring upon which SAP was based. It was found that the best correlation with SAP was given when all three were assumed to contribute equally to comfort temperature (i.e. in a ratio of 0.33 : 0.33 : 0.33).

In order for IDEAS to integrate with SAP, it must also take account of heating system responsivity. Responsivity relates to the controllability of a heating system. Radiator based

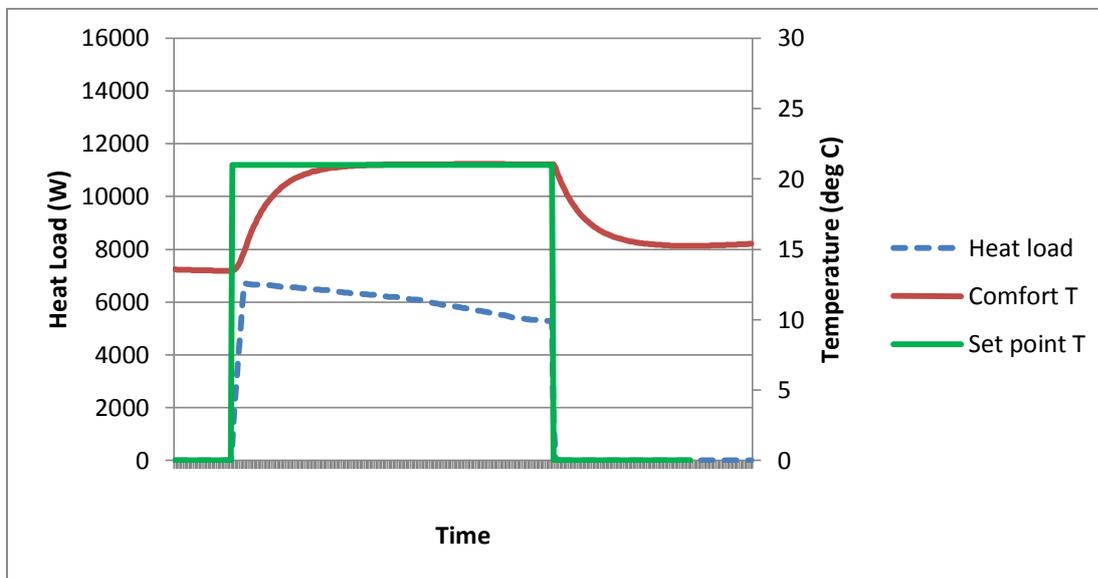
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<sup>5</sup> Building heated to a set point of 21°C for 16 hours each weekend day, and 9 hours on weekdays (split into a 2 hour heating period in the morning and 7 hours in the evening).

central heating systems are highly controllable, as they react quickly to changes in input, and are classified as having high responsivity. Heating systems with large thermal capacitance (for example, electric storage heaters and some types of underfloor heating) typically have low responsivity.

Responsivity is encapsulated in IDEAS through the time constant of the control response. This dictates the time the system takes to respond to a change in the desired system output, which in this case is comfort temperature. As shown in figure 4.1, when the comfort temperature set point of 21°C comes into force at the start of a heating period, actual comfort temperature increases to this level gradually, and only achieves it after a time period equal to the time constant. The time constant used in this project is 15 minutes, and is intended to reflect a building with a highly responsive heating system.

**Figure 4.1 IDEAS model response to a step change in set point temperature**



This approach to accounting for responsivity is necessarily an approximation in that, however low the temperature has dropped during a set back period, it recovers in the same amount of time. However it appears none the less to result in well-controlled heat demands, without excessively large spikes at the start of heating periods.

The dwelling model used in this project is a version of the IDEAS methodology implemented in the Matlab software package. It is one of the versions used for calibration against SAP. It represents a small house, built to approximately 1980 standards. The key thermal parameters

were taken from SAP tables, and are given in table 4.1. They correspond to un-insulated solid brick or cavity walls, 25mm of loft insulation, and double glazed windows. The full Simulink block diagram of the IDEAS model, and the Matlab code containing the model parameters, are included in Annex A.

**Table 4.1: Dimensions and thermal characteristics of building modelled.**

Total Floor Area	44.4 m <sup>2</sup>
Structure (external wall) area	81.8 m <sup>2</sup>
Glazed area	16.9 m <sup>2</sup>
Roof U value	1.0 W/m <sup>2</sup> K
Window U value	1.9 W/m <sup>2</sup> K
Floor U value	0.7 W/m <sup>2</sup> K
Wall U value	1.0 W/m <sup>2</sup> K
Air changes/hour	0.543

*Values have been rounded.*

## 4.2 ASHP heating system model - overview

The central aim of this project is to develop a model of air source heat pump performance that integrates with the IDEAS method. The model is designed to enable performance under different distribution and supplementary heating system configurations to be assessed. At this stage the model is subject to the perfect control provided by inverse dynamics built in to IDEAS. Further development would be required to examine the effect of ‘imperfect control’.

An important assumption is that the heat pump’s output can be modulated without affecting its COP. Given the apparent increase in the availability of variable capacity heat pumps, cycling may become less relevant. Though they may offer performance improvements at part load, there is not yet enough information available to make an allowance for this.

A simplified block diagram, representing the key components of the IDEAS and heating system model and the flow of information between them, is given in figure 4.2 below. The components (denoted by double outlines) of the IDEAS model are outlined in black. Those that have been added as part of this project are shown in red. Blocks with a single outline are input files. Full Simulink block diagrams and descriptions are included in Annex A.

The key components that have been developed in this project represent the radiator system, and the heat pump and supplementary heating unit. The radiator system component is an inverse model which calculates the return temperature which would result from delivering heat at the rate determined by the control component. When supplementary heating is assumed to be separate from the heat pump and radiator system, this rate is limited to the maximum output of the heat pump at the previous timestep.

Based on the return and external air temperatures, the heat pump component calculates COP and maximum heat pump output. The rate of heat output by the heat pump is assumed to be that determined by the control component or its maximum output, whichever is smaller. The power drawn by the heat pump is then calculated using the following equation:

$$\dot{W}_{HP} = \frac{\dot{Q}_{HP}}{COP}$$

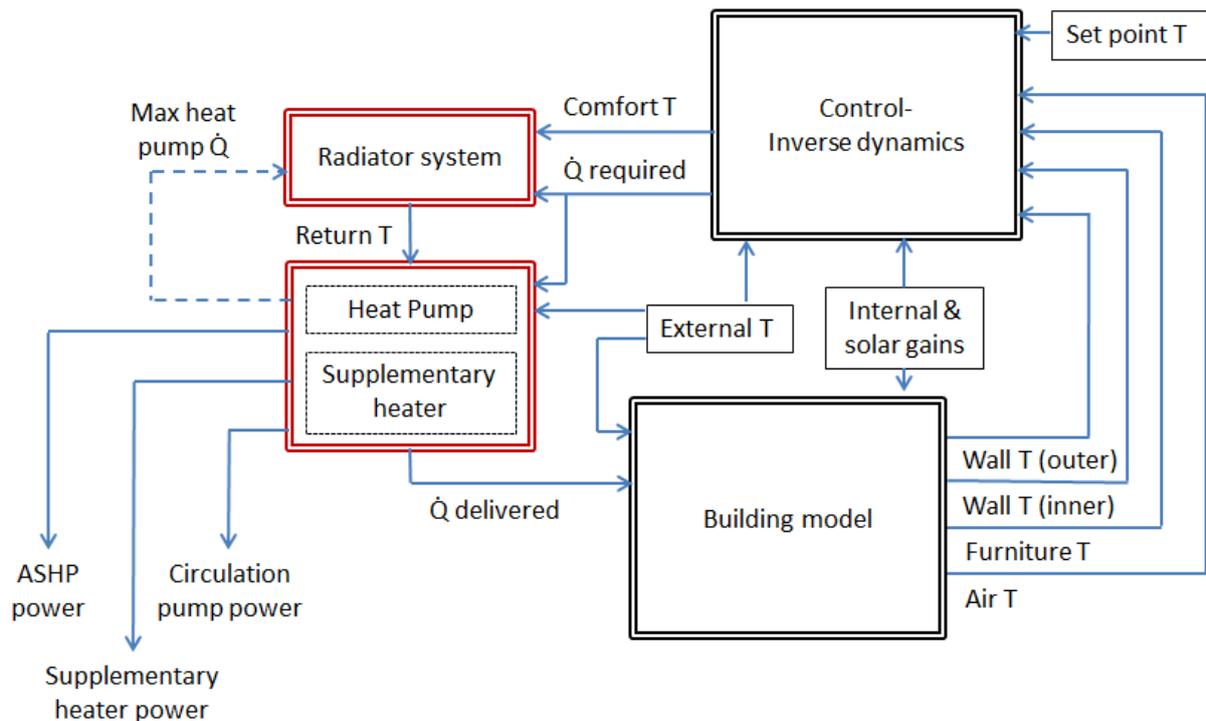
$\dot{W}_{HP}$  and  $\dot{Q}_{HP}$  are respectively the power required and heat delivered by the heat pump.

Any shortfall between the rate of heat output determined by the control component and the output of the heat pump is assumed to be met by the supplementary heating system with 100% efficiency.

It is assumed that the circulation pump consumes power at a constant rate whenever heat is required. Its power draw is also subtracted from the rate of heat required, as it is largely dispersed inside the house. The power draw of the pump has been set to 32.1W; when simulations are run using the SAP intermittent heating pattern this results in an annual consumption of 130kWh, which is the annual consumption assumed in SAP calculations. Whilst this power draw is lower than is reported to be typical, modern efficient circulation pumps appear to be capable of achieving it (OMV, 2001) and it is assumed a heat pump installation would include a new circulation pump.

The equations and parameters underpinning the radiator system and heat pump model components are described in the following sections. A description of the implementation of the model components in Matlab is given in Annex A.

**Figure 4.2: Simplified block diagram of heat pump model integrated into IDEAS**



*Note: T denotes temperature.*

### 4.3 Radiator system model

In order to integrate with IDEAS, an inverse model of the building radiator system is required, which calculates what return flow temperature would result from the delivery of heat at the rate  $\dot{Q}_{required}$  determined by the control component.

The output of a radiator system (ignoring distribution losses) can be expressed as

$$\dot{Q}_{required} = h_{rad}A_{rad}(T_{rad} - T_{room}) \quad (4.1)$$

where:

- $h_{rad}$  is the heat transfer coefficient of the radiators;
- $A_{rad}$  is the total area of radiators in the building; and
- $T_{rad}$  is the representative temperature of the radiators.

Equation (4.1) is a simplification, and leads to a difficulty in defining  $T_{room}$ . Heat is transferred from the radiator largely through convection, but also through radiation. Were it through convection only, ideally air temperature near the floor of the building should be used. Were it through radiation only, wall surface or furniture surface temperature might be more appropriate, depending on the layout of the room. Initial simulations used the air temperature calculated by the IDEAS model building component as  $T_{room}$ , however the resulting heat pump seasonal performance factors were lower than those found by others. This may be because the air temperature during heating periods is quite high, often over 30°C. In reality, air temperature at the low level radiators are generally installed at seems likely to be considerably lower than this. On this basis, comfort temperature is used in place of  $T_{room}$ .

Equation (4.1) can be inverted easily to output  $T_{rad}$ :

$$T_{rad} = \frac{\dot{Q}_{required}}{h_{rad}A_{rad}} + T_{room} \quad (4.2)$$

It is assumed that  $T_{rad}$  can be approximated as the mean of the heating system output and return temperatures, leading to the following relationship:

$$T_{out} = 2T_{rad} - T_{return} \quad (4.3)$$

The output of the heating system (the heat pump plus, if integrated, the supplementary heater) can be written

$$\dot{Q}_{HS} = \dot{m} c_p (T_{out} - T_{return}) \quad (4.4)$$

Here  $\dot{m}$  and  $C_p$  are the mass flow rate and specific heat capacity of the water in the distribution system. Substituting (4.3) then (4.2) into (4.4) and finally solving for  $T_{return}$  gives

$$T_{return} = \frac{\dot{Q}_{required}}{h_{rad}A_{rad}} + T_{room} - \frac{\dot{Q}_{HS}}{2\dot{m}c_p} \quad (4.5)$$

The distribution system model used in this project takes the general form of equation (4.5). However a number of the terms on the right hand side require comment.

If the thermal capacitance of the mass of water  $M$  in the radiators is taken into account, the energy balance for the heating system would be

$$\dot{Q}_{HS} = \dot{Q}_{required} + Mc_p \frac{d\bar{T}}{dt}$$

The representative temperature of the water  $\bar{T}$  could be approximated by  $T_{rad}$ . As discussed in section 4.1 however, the responsiveness of the heating system is taken into account by the IDEAS building model. It is implicit in the construction of the model that the rate of heat delivered to the building (in particular at the start and end of heating periods) changes smoothly rather than abruptly. The thermal capacitance of the distribution system is therefore ignored in equation (4.5), and  $\dot{Q}_{required}$  is used in place of  $\dot{Q}_{HS}$ .

Key parameters for the radiator model are the heat transfer coefficient from the surface of the radiators,  $h_{rad}$ , and the total area of radiators in the building,  $A_{rad}$ . Methods of sizing radiator systems and the effect this has on performance is discussed in the results section.

No sources were found in the literature on the heat transfer coefficient of domestic central heating radiators. Instead information was collected from a three radiator suppliers (Traderadiators, 2011; Plumbworld, 2011; and Wickes, 2011) on the rated heat output of typical 0.6m tall double panel, finned units. The standard rating conditions under which output is measured are a room temperature of 20°C, an input temperature of 75°C and a return temperature of 65°C. The average heat transfer coefficient was calculated and found to be 63.3 W/m<sup>2</sup>K (using equation (4.1) and assuming that radiator temperature can be

represented as the mean of the input and return temperatures). There was very little variation in the heat transfer coefficients of radiators from different manufacturers

An important aspect of radiator performance is that  $h_{rad}$  decreases at lower mean flow temperatures. Tables of correction factors were also sourced from three manufacturers, which enabled average heat transfer coefficients at lower temperatures to be calculated (AEL Heating Solutions, 2004; Henrad (UK) Ltd, 2005; and Kermi, 2008). Again there was very little variation in the factors given by different manufacturers for particular flow temperatures. A regression analysis tool was used to find equation (4.6):

$$h_{rad} = 0.392(T_{rad} - T_{room})^{1.30} \quad (4.6)$$

Substituting this expression into equation (4.5) results in the inverse distribution system model used in this project to represent conventional radiators:

$$T_{return} = \left( \frac{\dot{Q}_{required}}{0.392 A_{rad}} \right)^{\frac{1}{1.30+1}} + T_{room} - \frac{\dot{Q}_{required}}{2\dot{m}c_p} \quad (4.7)$$

This can be seen most easily by returning to equation (4.1) at the start of this subsection, and substituting (4.6) for  $h_{rad}$  there:

$$\dot{Q}_{required} = [0.392 (T_{rad} - T_{room})^{1.30}] \times A_{rad}(T_{rad} - T_{room}) \quad (w)$$

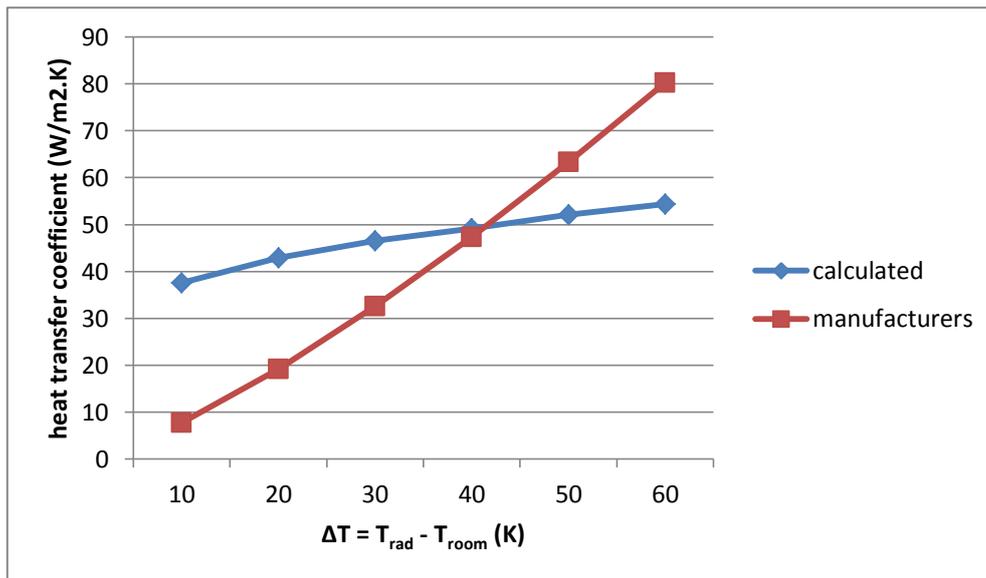
Inverting equation (4.1) then results in

$$T_{rad} = \left( \frac{\dot{Q}_{required}}{0.392 A_{rad}} \right)^{\frac{1}{1.30+1}} + T_{room} \quad (x)$$

The radiator heat transfer coefficients derived from manufacturers' charts decline surprisingly steeply at lower radiator temperature. The methodology employed by manufacturers to generate the correction factor charts is not known, and accuracy is difficult to judge.

Therefore values of  $h_{rad}$  at different flow temperatures were also calculated from the theory of convective and radiative heat transfer, and an equation of the same form as (4.6) was derived. Calculations were based on a typical radiator construction, and are detailed in Annex B. As shown in figure 4.3, there is a substantial difference between the manufacturers' values and those obtained by calculations. The effect of this on heat pump performance is discussed in the results section.

**Figure 4.3: Manufacturers' and calculated radiator heat transfer coefficients**



#### 4.4 ASHP model

The aim in this project is to develop a generalised model of ASHP performance, rather than modelling a specific model. Data on the performance of thirty air source heat pump models was obtained from the Wärmepumpen Testzentrum in Switzerland (WPZ, 2011). Regression models for COP and output are developed from this data.

WPZ tests ASHPs in accordance with the European Heat Pump Association (EHPA) test regulations, which specify eight sets of conditions at which heat pumps should be tested (European Heat Pump Association, 2009a). A sample of the data available relating to a single ASHP model is set out in table 4.2.

The test procedure involves installing the ASHP in a test room which is maintained at a particular temperature and relative humidity according to the test point being used. At test point 1, the ASHP is fed water at 30°C, and the mass flow rate adjusted until the output temperature is 35°C. The output of the ASHP is calculated from the mass flow rate and the difference between the output and return temperatures. The power drawn by the ASHP is also monitored, enabling COP to be calculated.

At the subsequent test points, no return temperature is specified. Instead, the mass flow rate attained at test point 1 is maintained, and the return temperature adjusted until the new output temperature is achieved. The output and COP of the ASHP are calculated as before. Though the return temperatures used in tests 2 to 8 are not given in test results, they can be easily calculated (as has been done in table 4.2) by first calculating the mass flow rate required in the first test.

**Table 4.2: Sample of WPZ test data**

Test point	External air T (°C)	Relative hum. (%)	ASHP output T (°C)	ASHP return T (°C)	heat output kW	COP
1	7	89	35	30	6.3	4.7
2	2	84	35	30.8	5.2	3.9
3	-7	75	35	31.8	4.0	3.1
4	-15	-	35	32.5	3.1	2.6
5	7	89	45	40.3	5.9	3.6
6	20	50	55	49.1	7.4	3.7
7	7	89	55	50.6	5.5	2.9
8	-7	75	55	52.1	3.6	2.1

(WPZ, 2011)

As discussed previously, the COP and maximum output of a heat pump depend on the conditions it is operating in. These conditions are the external air temperature, the temperature of the return flow from the heating system, and relative humidity (which affects the frequency at which defrost cycles are required). It is not possible to include the dependency on relative humidity in regression models developed from the results of tests carried out under the EHPA regulations, as the level of relative humidity specified for tests at any particular external temperature is the same.

Using the 30 ASHP tests results published by WPZ, the following relationships were developed:

$$\text{COP} = 6.70e^{-0.022\Delta T}$$

where  $\Delta T$  = return temperature – external temperature; and

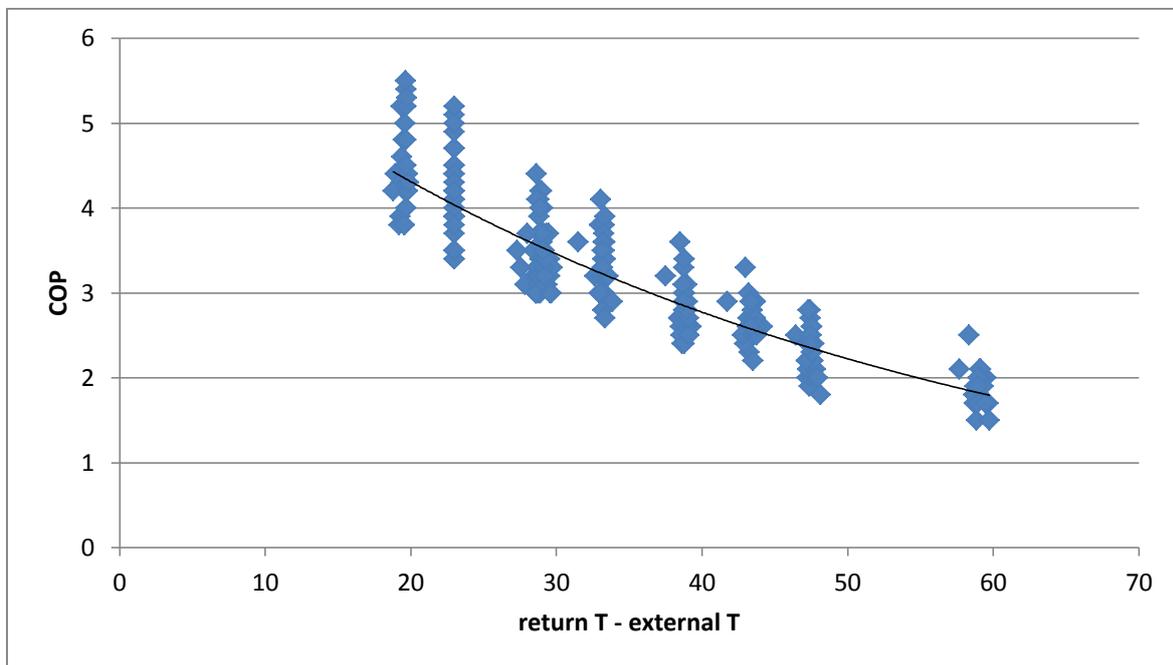
$$\dot{Q}_{\text{factor}} = \frac{\dot{Q}_{\text{max}}}{\dot{Q}_{\text{ref}}} = 0.023T_{\text{ext}} - 0.0031T_{\text{return}} - 4.46$$

$\dot{Q}_{\text{max}}$  is the heat output at each test point, and  $\dot{Q}_{\text{ref}}$  is the heat output at test point 1.  $\dot{Q}_{\text{factor}}$  thus relates the output of an ASHP at any particular set of conditions to its output at test point 1.

This allows the size of the heat pump being modelled to be easily changed. Additionally, output at test point 1 is often quoted as a heat pump's nominal output.

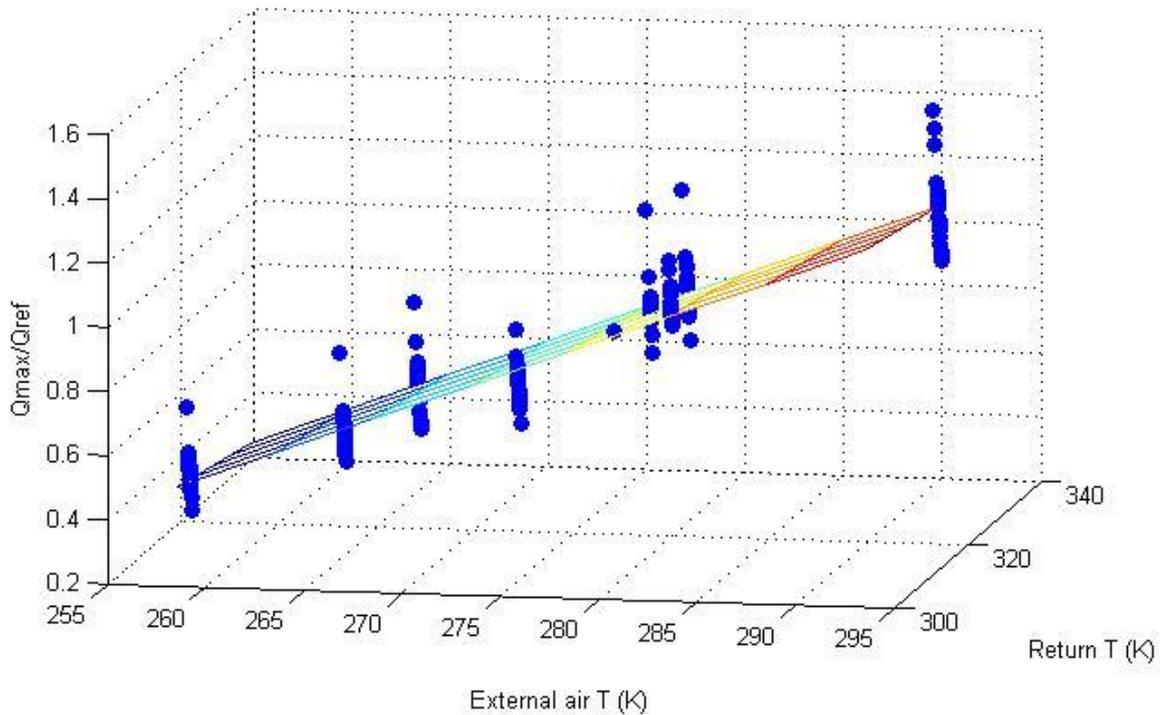
These relationships are simple regression models found using Matlab's generalised linear model regression tool. The coefficient of determination ( $R^2$  value) for the COP regression model is 0.85, and that for the  $\dot{Q}_{\text{factor}}$  regression model is 0.89. This indicates that 85% of the variance in COP is explained by the model, and 89% of the variance in  $\dot{Q}_{\text{factor}}$ . In both cases external temperature, return temperature and linear combinations of the two were trialled as independent predictors<sup>6</sup>, however no improvements on the above models were achieved. Coefficients of determination much closer to 1 are unlikely in any case given that there are 30 values of COP and  $\dot{Q}_{\text{factor}}$  for every test point. The performance data and regression models are represented graphically in figures 4.4 and 4.5.

**Figure 4.4: COP regression model**



<sup>6</sup> Relationships were also sought in the form of polynomials in  $T_{\text{ext}}$  and  $T_{\text{return}}$ , as have been used by others (Kelly and Cockroft, 2008 and Jenkins et al, 2008). However the results were less satisfactory in terms of the coefficients of determination achieved. This should not be the case in theory, but may be caused by a combination of the method of estimation employed by the Matlab regression analysis function, and the large size of powers of  $T_{\text{ext}}$  and  $T_{\text{return}}$  (in Kelvin) compared to COP and  $\dot{Q}_{\text{factor}}$ .

**Figure 4.5:  $\dot{Q}_{\text{factor}}$  regression model**



#### 4.5 Climate Data and Internal Gains Calculation

Before simulations can be run, input data on external temperature, solar gains and internal gains must be provided. Hourly climate data files for a number of UK and European locations were obtained from the website of the US Office of Energy Efficiency and Renewable Energy (EERE, 2011).

External temperature profiles were taken directly from the files. Solar gain profiles relating to the building model were calculated from solar radiation profiles in the climate data using the process described in Annex C. The method used combines equations for calculating the incident solar flux on a surface taken from course material with the process specified in SAP 2009 for the estimation of monthly average solar gains. Values for the transmissivity of the building windows and parameters such as ‘frame factor’ were taken from SAP tables.

SAP 2009 was also used to calculate monthly average internal gains. The process used by SAP is based around an assumed number of occupants calculated from the floor area of the building. Default values for parameters were used where possible (for example, SAP

stipulates that where the fraction of low energy light fittings in building is unknown, 0.3 should be assumed). In order to allow for the dynamic nature of internal gains, the monthly averages were used to scale typical electricity use profiles developed for an International Energy Agency-lead research project (IEA, 2006)

#### **4.6 Heating system sizing methods**

In order to ensure simulations results are realistic, it is important to consider what capacity of heat pump and total area of radiators would be likely to be installed in the building modelled. It appears that at present, there is not a single, widely-accepted approach to the sizing of heat pump systems. Therefore the basic approach used for sizing conventional central heating systems is adapted for this analysis, along with reported approaches which have been used to determine radiator area in heat pump systems.

##### **4.6.1 Sizing of conventional heating systems**

The standard approach to assessing the heating system requirements of a building is based around the design heat loss. Design heat loss is the steady state rate of heat loss at particular internal and external temperatures, and is intended to represent the peak heating load of the building. In the UK, generally 21°C and -1°C are used (British Standards Institution, 2003).

The thermal capacity of conventional boilers is generally taken to be fixed across the range of operating conditions likely to be encountered. Therefore once the design heat loss is known it is relatively straightforward to select the required capacity. The assessment of radiator requirements must also take into account the optimum working temperature of the boiler, which for a gas condensing boiler may be 70°C. Radiators are selected so that their rated output matches the design heat loss when the boiler is working at optimum temperature. It is useful at this stage to define

$$\Delta T(K) = \text{radiator temperature} - \text{room temperature}$$

So for a condensing gas boiler, radiators would often be sized so that their output matches the design heat loss when  $\Delta T = 50K$ . Sometimes an additional margin of 10 or 20% in both boiler capacity and radiator size is allowed for, with the aim of ensuring the heating system is capable of achieving the desired room temperature on particularly cold days.

The design heat loss of building model 1 is 5010W (calculated by adapting the Heat Loss Parameter calculation given in SAP 2009). Were a condensing gas boiler to be installed, the method outlined above would suggest a boiler capacity of 5500W and a total radiator area of 1.75m<sup>2</sup> (allowing a 10% margin, and using the average of the radiator rated output values obtained from manufacturers). This disregards any allowance for domestic hot water provision.

#### **4.6.2 Sizing of heat pump systems**

Two methods of sizing radiators for heat pump systems are mentioned in the literature. In one, it is reported that radiators were oversized by 30% compared to conventional practice for gas systems (Boait et al, 2011). In the other, radiators are sized for a  $\Delta T$  of 35K, most likely chosen as it coincides with the heat pump's maximum output temperature (Kelly and Cockroft, 2008). An important feature of heat pump performance is that to prevent excessive pressure at the compressor inlet, a protective mechanism is normally included which switches the heat pump off if heating system return temperature rises too high.

The former of these methods would suggest a radiator area of 2.25m<sup>2</sup>. The latter would suggest a radiator area of 4m<sup>2</sup> (using manufacturer's radiator output values, adjusted using published correction factors, and assuming a 10% margin.) It is worth noting that this is more than double the area of radiators that would be specified for a conventional boiler. This is due to the steep decline in radiator heat transfer coefficient at smaller  $\Delta T$  implied in manufacturers' correction factor charts.

Selecting the appropriate heat pump capacity is more complex than selecting conventional heating system capacity. The maximum output of a heat pump depends strongly on operating conditions – namely external temperature and return flow temperature. Generally heat pump output at an external temperature of 7°C and a return temperature of 30°C is used as the nominal output figure. These are relatively favourable conditions for heat pump performance. If external temperature is closer to that to which the design heat loss relates, heat pump output will be reduced. Therefore an installer would likely consult tables of heat pump output at a range of conditions, and select one which could meet the design heat load of the building during cold weather.

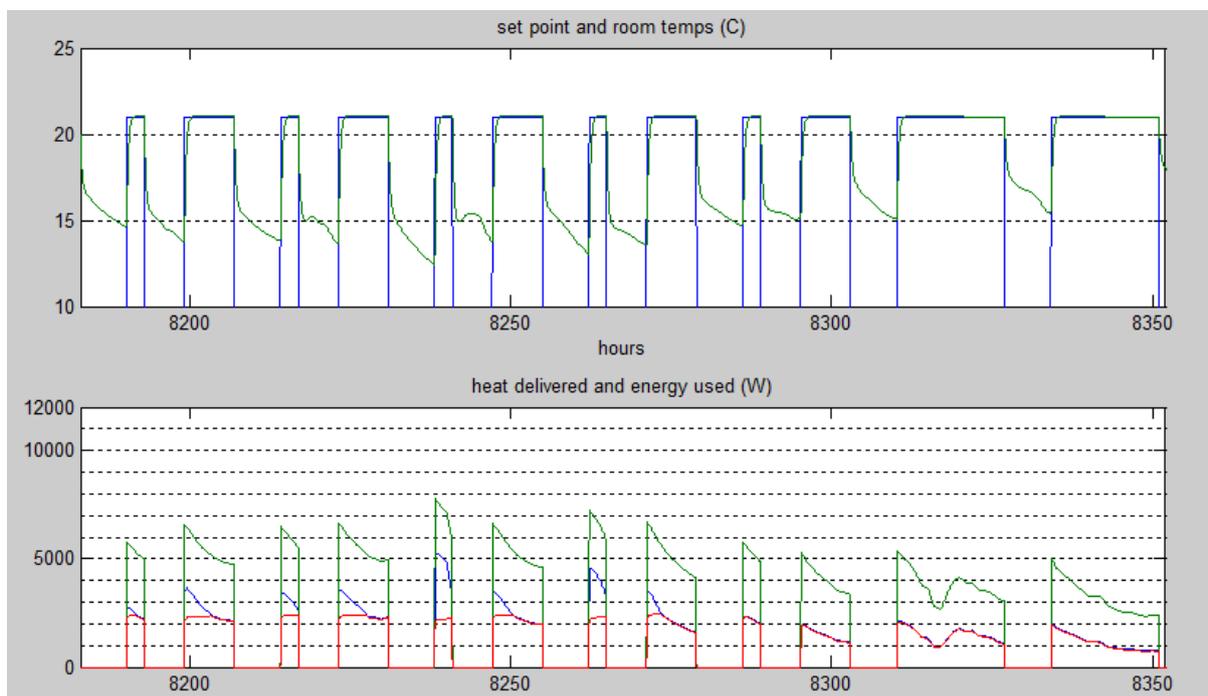
The result of this is that heat pumps will be substantially oversized compared to heat requirements during warmer periods. Where a fixed capacity heat pump is to be installed at least, greater consideration must be given to the minimisation of cycling behaviour. For this reason, heat pumps are sometimes sized to meet 75 to 95% of the design heat loss, with the remainder being met by supplementary heating.

As this project is concerned with generalised heat pump performance, the regression model for heat pump output (section 4.4) is used to determine what nominal capacity of heat pump is likely to be installed. It is assumed that heat pump output at an external temperature of  $-1^{\circ}\text{C}$  and a return temperature of  $55^{\circ}\text{C}$  is used for sizing. If the heat pump is intended to meet 95 % of the design heat load (to reduce cycling), a nominal capacity of approximately 6.3kW is required. This is also the capacity of the smallest heat pump of the thirty tested by WPZ.

## 5. Results

Figure 5.0 depicts the output of a simulation using the ASHP model with separate supplementary heating subject to an intermittent heating pattern. The time period displayed is one week. As seen in the upper graph, the IDEAS method's inverse dynamics temperature controller ensures the set point temperature is reached 15 minutes after the start of a heating period. In the lower graph, the green line depicts heat delivered by the heating system. The blue line is total electricity use, and the red line represents the proportion that is used by the ASHP. Gaps between the blue and red lines indicate that supplementary heating is required in order to maintain the set point.

**Figure 5.0: Demonstration of model outputs**



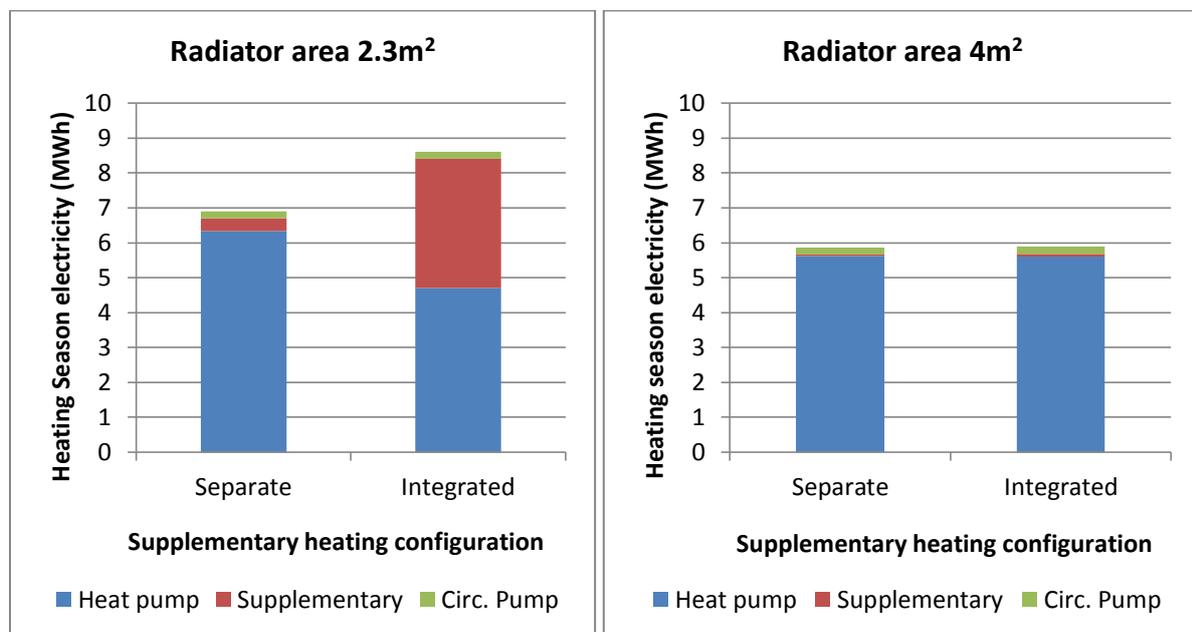
### 5.1 Supplementary heating configuration

The heat pump model is first used to explore the difference in performance between a heat pump system with integrated supplementary heating, and one with separate supplementary heating. In both cases, heat pump nominal capacity is set 6.3kW, and the heating systems are operated continuously (as appears to be recommended practice) with a set point of 21°C. Results of simulations show the set point is maintained year round other than during periods

of overheating. The overheating is caused solely by warm external temperatures and high solar gain: the heat pumps do not deliver heat when the comfort temperature is above 21°C.

Climate data for Aberdeen is used as, in terms of external temperature, this is the most challenging of the UK locations for which climate profiles were obtained (see section 5.3). In each simulation, the heat pump and supplementary systems delivered 15720 kWh of heat over the heating season<sup>7</sup>, exactly matching the requirement determined by the temperature controller. Graphs of the electricity used during the heating season are presented in figure 5.1, and more detailed results are given in table 5.1.

**Figure 5.1: Heating system electricity use – ASHP capacity 6.3kW, continuous heating**



<sup>7</sup> The heating season is assumed to run from 1 September to 31 May.

**Table 5.1: Results of simulations of separate and integrated supplementary heating**

Radiator Area (m <sup>2</sup> )	2.3		4	
	Separate	Integrated	Separate	Integrated
Supp. Heater config.	Separate	Integrated	Separate	Integrated
Heat delivered (kWh)	15720	15720	15720	15720
Total elec (kWh)	6906	8612	5868	5889
Heat pump elec (kWh)	6330	4707	5615	5609
Suppl. Elec (kWh)	369	3698	46	73
Circ. pump elec (kWh)	207	207	207	207
SE - heating season	2.3	1.8	2.7	2.7
SE - Autumn	2.5	2.3	2.9	2.9
SE - Winter	2.1	1.5	2.5	2.5
SE - Spring	2.3	1.9	2.7	2.7
Mean part load factor	0.41	0.31	0.41	0.41

It can be seen that where radiators are sized to meet the design heat loss at a radiator temperature of 55°C (giving a total area of 4m<sup>2</sup>), there is little difference between the two system configurations. The heating season system efficiency in both cases is 2.7, closely matching that found in simulations run by others (Kelly and Cockroft, 2008) and towards the top of the range found in the Energy Saving Trust field trial.

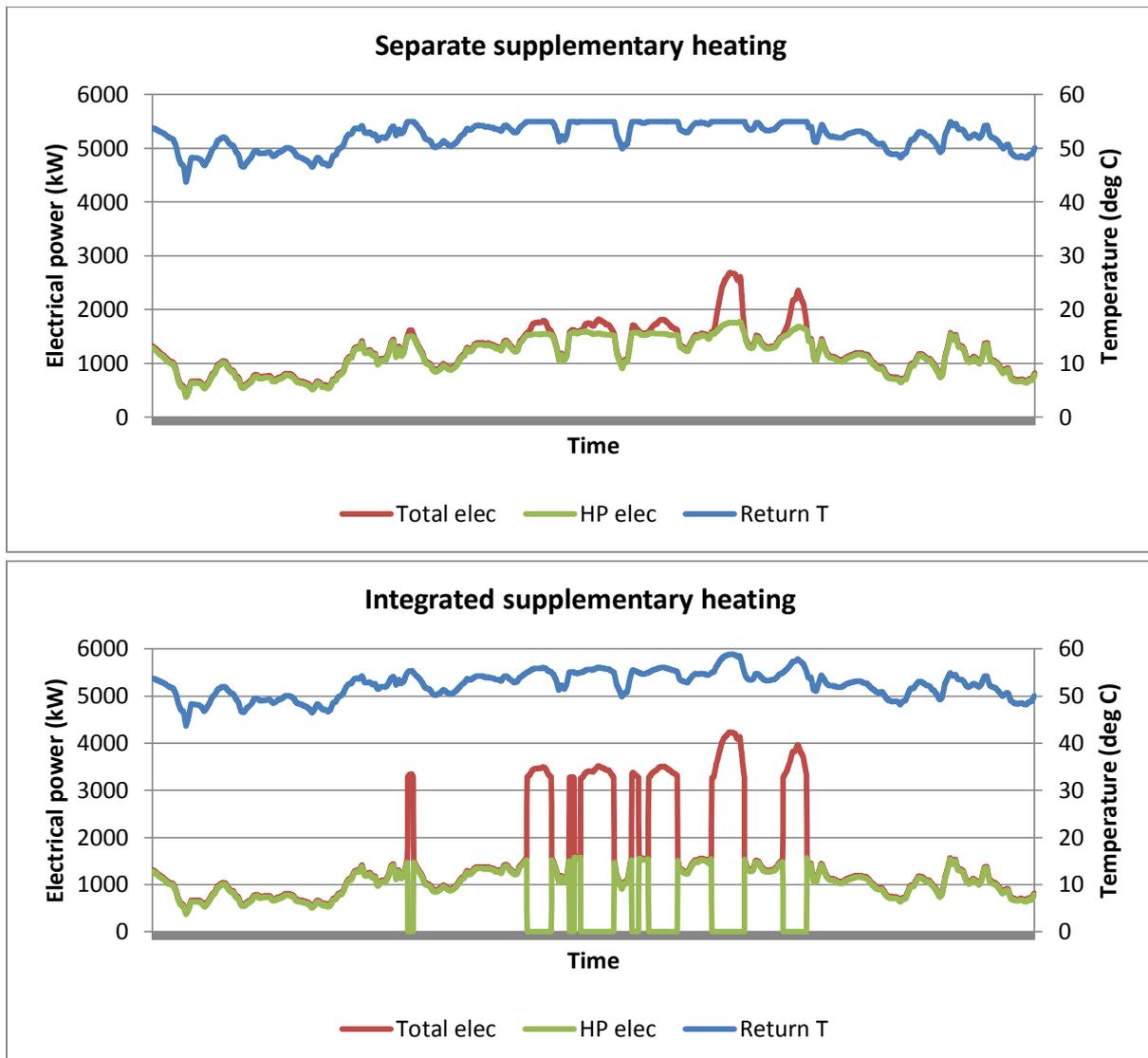
However where a total radiator area of 2.3m<sup>2</sup> is specified, corresponding to approximately 30% oversizing compared to standard practice for gas boilers, a large difference between the two configurations emerges: system efficiency drops to 2.3 in the case of separate supplementary heating, and 1.8 in the case of integrated supplementary heating.

The reasons for this can be seen in figure 5.2. This gives graphs of total system power requirement, heat pump power requirement, and heating system return temperature for the first two weeks of December. As discussed in section 2.5, to avoid excessively high pressure at the compressor outlet, heat pumps must stop operating when the return temperature exceeds a certain level – often 55°C, as is assumed here. Where supplementary heating is delivered separately to the heat pump and radiator system, return temperature can be held within the heat pump’s assumed maximum operating range whilst still ensuring the desired set point temperature is reached.

However where supplementary heating is integrated with the heat pump system, return temperature cannot be limited to the heat pump’s operating range without the risk that comfort temperature in the building may fall below the desired set point. The result of this is

that, when a return temperature of greater than 55°C is required in order to maintain the set point, the heat pump cuts out and all heat is provided by the supplementary heater. The effect of this on power consumption can be seen in the second of the two graphs in figure 5.2.

**Figure 5.2: Heating system electrical power consumption and return temperature over the first two weeks of December.**



From these results it is concluded that, where it is not possible to size the radiator system so that peak heat demand can be met without return temperatures exceeding a heat pump's operating range, systems with separate supplementary heating have the potential to provide the best performance. Inherent in the approach taken in this project is the assumption that the heat pump system is perfectly controlled. The control method used in a real heat pump system with separate supplementary heating may not be sophisticated enough to take full

advantage of the benefit offered by this configuration. However these results show that, where total radiator area is limited, systems with integrated supplementary heating are subject to constraints to which those with separate supplementary heating are not. This conclusion applies whether the heat pump itself is oversized or undersized, though with substantially undersized units the potential benefit is lessened.

## 5.2 Intermittent versus continuous heating

Whereas conventional heating systems are often operated to intermittent heating patterns, it appears to be common practice for suppliers and manufacturers to recommend that heat pump are operated continuously. The heat pump model can be used to examine whether the assumption that this leads to improved performance is always correct.

Simulations were again run using a 6.3kW nominal heat pump capacity and climate data for Aberdeen. A separate supplementary heating system is assumed. The SAP standard intermittent heating pattern was used, which assumes the building is heated to a set point of 21°C for 16 hours each weekend day (0700 to 2300), and 9 hours each weekday (0700 to 0900 and 1600 to 2300).

**Figure 5.3: Electricity use under intermittent and continuous heating patterns**

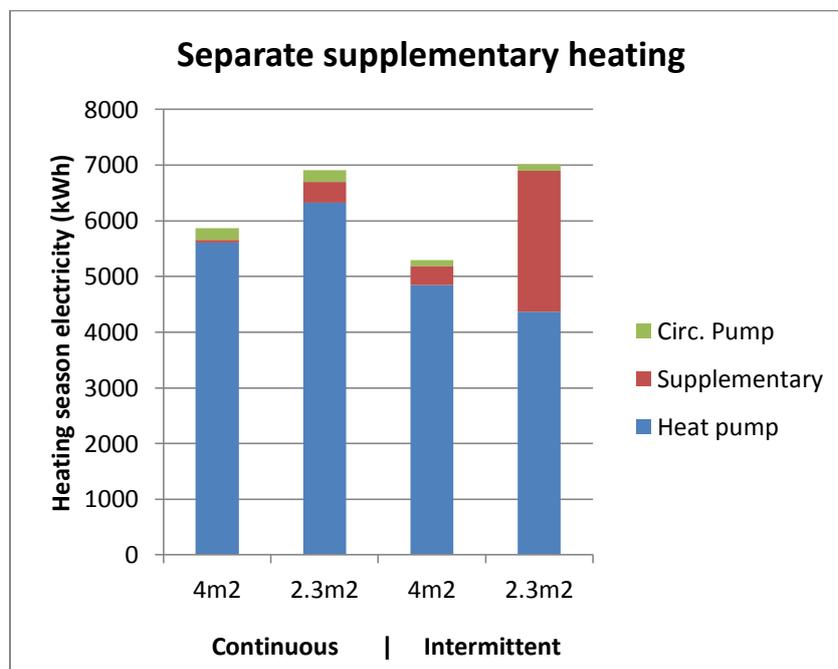


Figure 5.3 compares the electricity used during the heating season under the intermittent heating regime to the results obtained in the previous subsection for continuous heating. For a total radiator area of  $4\text{m}^2$ , the intermittent heating pattern appears to offer a clear energy saving. When a total radiator area of  $2.3\text{m}^2$  is assumed however, the results are less clear: in terms of kWhs of electricity used, a continuous heating pattern appears to have a slight edge. This is because the peak heating loads under an intermittent heating regime are greater, leading to higher return temperatures and poorer system efficiency.

However for many households, electricity bills rather than kWhs used are the primary measure of performance. For the results relating to the simulation with total radiator area of  $4\text{m}^2$ , the annual electricity costs that could be experienced under standard and 'economy 10' offpeak electricity tariffs have been calculated and are given in table 5.2.

Economy 10 offers three periods of off peak electricity daily. Matlab was used to bin electrical load data generated by simulations according to time of day.

The times of the off peak periods in Scotland are different to those in the rest of the UK. The off peak periods in Scotland run from: 0430 to 0730; 1330 to 1430; and 2030 to 0030. The off peak periods in the rest of the UK run from: 0000 to 0500; 1300 to 1600; and 2000 to 2200.

Electricity rates from SAP 2009 have been used: these are three-year averages taken from a range of suppliers. They are therefore slightly out of date, however as they are average figures they give a better idea of the typical cost difference between standard and off-peak tariffs. The rates are -

- Standard tariff rate: 11.46 p/kWh
- Economy 10 off peak rate: 6.17 p/kWh
- Economy 10 peak rate: 11.83 p/kWh
- Economy 10 additional standing charge: £18/year

**Table 5.2: Annual electricity costs under different tariffs and heating patterns – 4m<sup>2</sup> radiators**

Heating pattern	Annual electricity (KWh)	Tariff	off-peak fraction	Total cost (£)
Continuous	6640	Standard	-	761
		Economy 10 (Scotland)	0.42	646
		Economy 10 (rest of UK)	0.45	633
Intermittent	5911	Standard	-	677
		Economy 10 (Scotland)	0.41	580
		Economy 10 (rest of UK)	0.25	634

It can be seen that for both an intermittent and continuous heating pattern, and for locations anywhere in the UK, Economy 10 tariffs offer a saving over the standard tariff. For locations in Scotland, economy 10 off-peak times coincide well with the intermittent heating pattern (most likely because of the long off-peak period in the evening), and this offers the lowest costs. Off peak periods in the rest of the UK coincide less well with the intermittent heating pattern, and there is little difference in cost between the continuous and intermittent heating patterns. The main conclusion that can be drawn here is that, where cost is the main measure of performance, tariff structure as well as absolute electricity use should be taken into account when selecting a heating pattern.

Assuming a separate supplementary system or a large total radiator area, it can be concluded that heat pump installations in Scotland offer lower costs when operated on the SAP intermittent heating pattern whether Economy 10 or a standard tariff is used. For the rest of the UK, heating pattern makes little difference assuming Economy 10 is used: the intermittent pattern offers reduced costs if a standard tariff is used. Households of course use electricity for purposes other than heating, and the times at which most electricity is used would need to be considered on a case by case basis to ensure the most beneficial tariff is selected.

For comparison, the cost of heating the building using gas has been estimated using data on boiler efficiency and gas costs from SAP 2009 tables. The calculation was made using heat energy requirement output by the model when simulations are run with the intermittent heating pattern. The annual cost was found to be £658: this is lower than the results in table 5.2 for the heat pumps operated on standard tariffs, but higher than heat pump costs when operated on Economy 10.

Efficiency figures for a standard condensing gas boiler with automatic ignition are used: these are 84% in winter and 74% in summer. Given perfect control of the heat pump is assumed, the +3% adjustment SAP stipulates where weather compensation is used is applied. SAP gives the average cost of mains gas as 3.10p/kWh with a £106 annual standing charge.

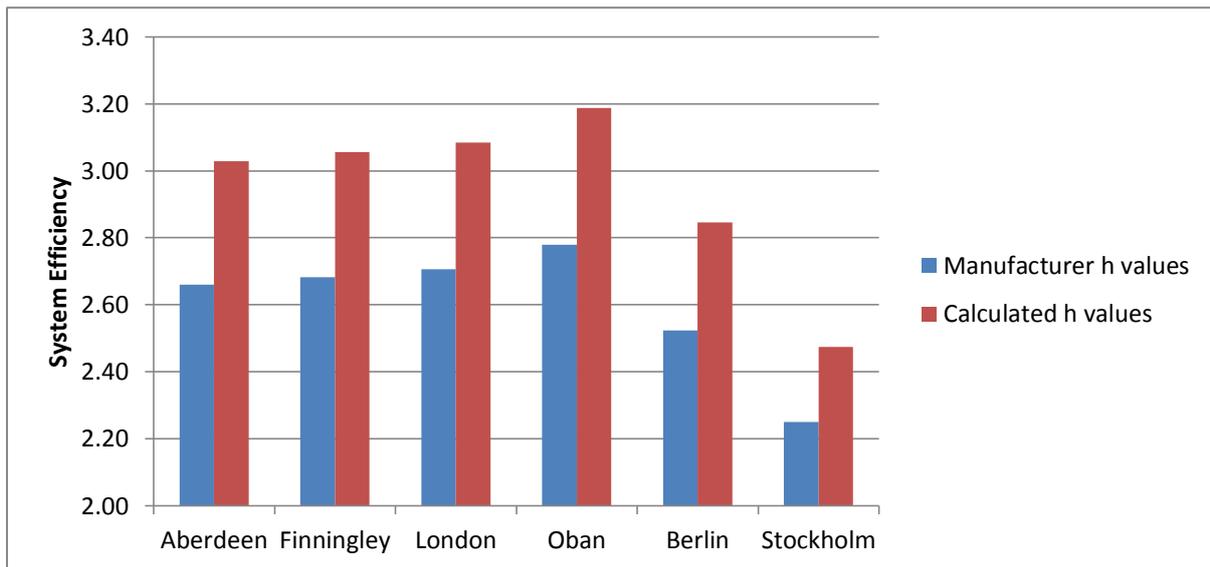
Factors from SAP 2009 were also used to estimate the CO<sub>2</sub> emissions which would result from the generation of the electricity used by a heat pump installation with separate supplementary heating and 4m<sup>2</sup> of radiators. Under the intermittent heating pattern, 3.06 tonnes would be emitted; if operated continuously, 3.43 tonnes would be emitted. This compares to an estimate of 3.53 tonnes for gas central heating with the efficiency described above.

SAP 2009 emissions factors are 0.198kg CO<sub>2</sub> per kWh of mains gas used, and 0.517 kg CO<sub>2</sub> per kWh of electricity.

### **5.3 Comparison of methods of deriving radiator heat transfer coefficients**

Pairs of simulations were run using climate data for 6 different locations. The first of each pair used the relationship between radiator heat transfer coefficient and  $\Delta T$  derived from manufacturer's data. The second used the relationship derived from calculations using the theory of convective and radiative heat transfer described in Annex B. Heat pump nominal capacity was 10kW, and in both cases radiators were sized to meet the design heat loss plus 10% at a radiator temperature of 55°C. Heat pumps were operated to a continuous heating pattern. This meant that supplementary heating was not required. The seasonal efficiencies obtained are depicted in figure 5.4. Finningley, a village in South Yorkshire, represents a fairly central UK location.

**Figure 5.4: Comparison of methods of deriving radiator heat transfer coefficients**



Despite the same sizing method being used, the fact that radiator heat transfer coefficients derived from calculations decline at a slower rate as radiator temperature drops leads to markedly higher system efficiency. It seems unlikely that manufacturers' correction charts are significantly inaccurate. However it may be that standard central heating radiators are optimised for high temperature performance, and alternatives could be designed which are better suited to heat pump applications and cheaper and easier to integrate into existing central heating systems than fan convector units.

As expected, figure 5.4 shows that when external temperature alone is taken into account, the UK offers more favourable conditions for ASHP performance than Northern European locations where they are much more popular. However, without taking account of relative humidity, it is difficult to make a firm comparison.

## 6. Conclusions

The main aim of this project is to develop a model of air source heat pump performance that integrates with the IDEAS method, and to use this to investigate the constraints placed on ASHP performance by the configuration of the supplementary heating system.

The model produced is able to simulate the power required by a heat pump heating system when subject to the perfect control provided by the IDEAS model's inverse dynamics-based temperature controller. Though not considered in this project, it is also able to assess the effect on comfort temperature of an undersized heat pump system without supplementary heating.

A number of simplifications have been required due to the scarcity of extensive ASHP test data on which to base the performance model. The effect of relative humidity on heat pump performance may be an important factor in the UK, however it has not been possible to incorporate this in the model at this stage. It has not been possible to include either the effect of cycling behaviour on ASHP performance, or the effect of part load operation on the efficiency of variable capacity heat pumps. Additionally, the effect of the thermal capacitance of the heating system is only partially accounted for.

Never the less, the system efficiencies achieved by the model correspond well to those reported in field trials and the results of other simulation exercises. It is the author's view that the model is sufficiently realistic to offer an insight into the impact of supplementary heating system configuration on system performance.

The results obtained show that, when supplementary heating is integrated into a heat pump system with less than the 'ideal' total area of radiators, performance is constrained by the operating range of the heat pump. This configuration appears to be common; it has advantages in terms of the provision of heat during defrost cycles, reduced complexity and fewer heat emitters. However where the space available for radiators is limited, it can be harmful to performance in cold spells as the heat pump may be required to cut out.

Based on these findings, integrated supplementary heating may be a contributing factor to some of the particularly poor system efficiencies found in the Energy Saving Trust field trial

(eight of the 29 ASHP installations had system efficiencies of 1.6 or lower). A potential low-cost solution for existing installations experiencing this problem may be to disable the supplementary heater, and use portable direct electric units to supplement heat pump output. The effect of this on thermostatic control systems is uncertain however. A more invasive solution would be to replace radiators with fan convactor units, which have effective heat transfer coefficients approximately double that of a conventional radiator at a  $\Delta T$  of  $50^{\circ}\text{C}$ , and more than double at lower temperatures. Though the additional investment required may be unwelcome, the potential performance improvement could result in a payback period of just a few years.

The model has also been used to explore the relative merits in terms of energy use and costs of different heating patterns and tariff structures. Where it is possible to size radiators so the the design heat loss of a building can be met without exceeding the heat pump operating conditions, it appears that an intermittent heating pattern results in energy savings. However where there are constraints on the total area of radiators which can be installed, intermittent heating may offer little benefit as system efficiency is reduced. Further work would be required to establish whether these results also apply to better insulated buildings. It has also been shown that tariff structure as well as absolute electricity use is relevant to the choice of heating pattern.

Costs and  $\text{CO}_2$  emissions have been compared to those estimated for a gas boiler, as this represents the most advantageous of the conventional heating options in these respects. Again where it is possible to size radiators so the the design heat loss of a building can be met without exceeding the heat pump operating conditions, Emissions savings of 3 to 13% were found. Whether or not cost savings are achieved was found to depend on the choice of electricity tariff. Differences in Cost savings are small however, and would be sensitive to relative changes in gas and electricity prices.

## **7. Further work**

Further work is required to better establish radiator performance at low flow temperatures. The steep decline in heat transfer coefficients derived from manufacturers' correction charts contrasted strongly with the slower decline found when heat transfer coefficients were calculated from heat transfer theory, and certainly inhibited heat pump performance. The effect of radiators on heat pump performance is a key factor for retrofit installations.

A topic that would be interesting to investigate with this model is the effect of relative humidity on ASHP performance in locations around the UK. The UK's oceanic climate means that relative humidity is often higher than in other European countries where ASHPs are more established as a source of domestic heating. A lack of detailed test data meant it has not been possible to assess this aspect of ASHP performance here.

Better integration of the radiator system model with the IDEAS methodology may enable heating system thermal capacity to be more fully taken into account. This would assist in assessing the effect of cycling behaviour on heat pump performance, which in turn would enable a fuller examination of the benefits of different heating system control methods. It would also be interesting to investigate the controllability of ASHP heating systems with respect to the varying emissions intensity of grid electricity.

## **Annex A: Description of the Heat pump model's implementation in Simulink**

Figure A.1 depicts the top-level Simulink block diagram of the model used in this project to simulate a heat pump with separate supplementary heating. Simulink is an extension of the Matlab software package which provides a graphical environment for modelling. The portion of the model above the blue dashed line is the Matlab implementation of the IDEAS method, created by Murphy (Murphy et al, 2011); the portion below the dashed line has been added during this project.

Simulink models consist of blocks representing mathematical or logical operations. These act on the input signal to the block (denoted by arrows) at each timestep of a simulation. Signals begin at source blocks, highlighted in blue in the diagram. These contain files of input values for each timestep. Three source blocks are required for the model, containing the external temperature profile; the internal and solar gains profile (labelled free heats); and the heating set point profile. The sources for the profiles used in this project are described in section 4.5.

Signals end at sink blocks, which represent the model's outputs. These are the numbered ovals on the right hand side of the diagram.

Larger blocks contain subsystems, which are essentially a way of compartmentalising the model for easier viewing. The IDEAS model consists of two subsystems, one containing equations representing the thermal performance of the building and another containing the inverse dynamics-based temperature controller.

Two further subsystems have been added during this project. The equations discussed in section 4.3, which represent a radiator system and heat pump performance, are implemented in these subsystems. There are two versions of each subsystem, in order that both integrated and separate supplementary heating systems can be simulated. The versions relating to separate supplementary heating can be seen in figures A.2 and A.3 below. The versions relating to integrated supplementary heating can be seen in figures A.4 and A.5.

Two versions of the top-level block diagram are also required, however as they are very similar only the one relating to the separate supplementary heating configuration is depicted here. The main difference is that three input signals are required to link the radiator and heat

pump subsystems together, whereas the integrated supplementary heating version only requires one.

In both versions of the top level block diagram, three key signals carry information from the IDEAS subsystems to those representing the radiator system and heat pump model. These are highlighted in green in figure A.1; they are the heat requirement determined by the temperature controller; the comfort temperature in the building; and external temperature. The return signal to the IDEAS model, the rate of heat output, is highlighted in red.

**Figure A.1: Heat pump model integrated into IDEAS - Simulink block diagram**

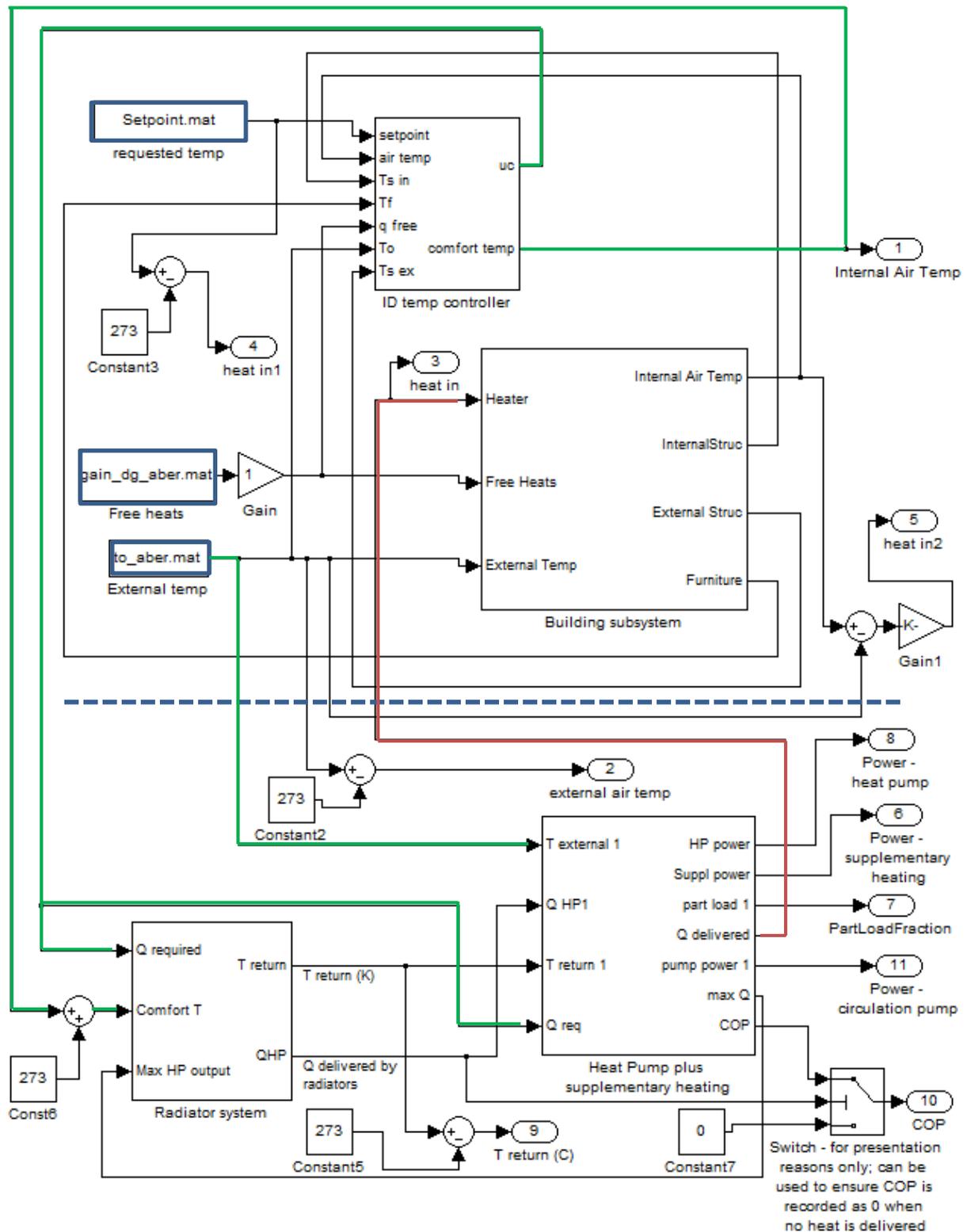
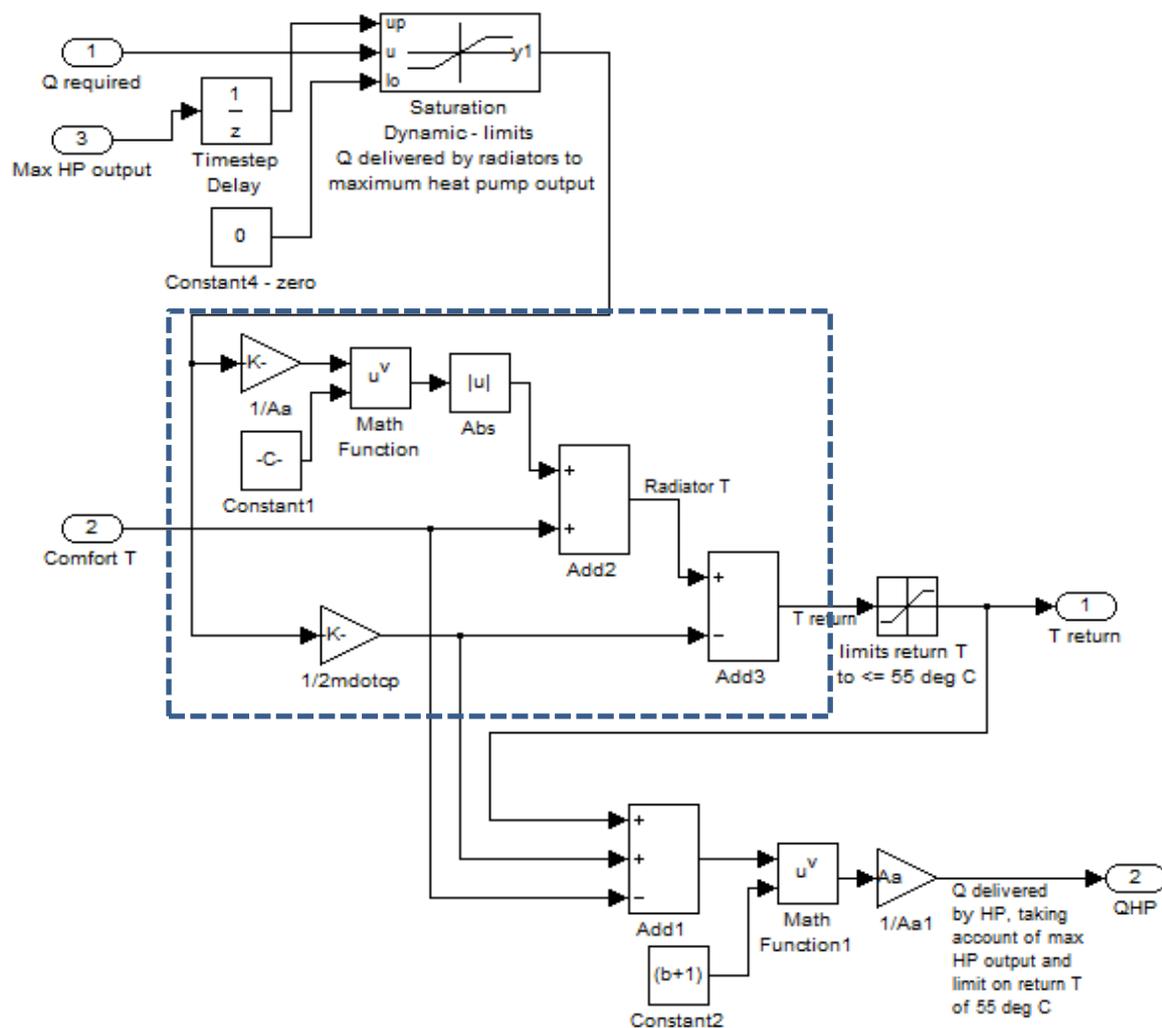


Figure A.2 depicts the version of the radiator subsystem relating to the case where supplementary heating is supplied separately from the heat pump system. The input signals from the top level of the model arrive at the oval blocks on the left.

The heat pump's maximum output is determined by the heat pump subsystem based on the return and external temperatures. The rate of heat delivery requested by the temperature controller is limited to the maximum output of the heat pump at the previous timestep. The timestep delay is necessary to avoid an 'algebraic loop', but also reflects reality in that there would be a few minutes' delay after a change in heat pump output before return temperature is affected.

**Fig A.2: Radiator subsystem block diagram – separate supplementary heating**



The portion of the model highlighted by the blue rectangle implements equation (4.7). This calculates the radiator system return temperature which would result if heat is delivered at the requested rate. As discussed in section 2.5, the heat pump cannot operate if the return temperature exceeds 55°C. It is assumed the heat pump is perfectly controlled, and does not output heat at a rate which would cause the return temperature to exceed 55°C. Using the inverse of equation (4.7), the remaining blocks calculate the rate at which the heat pump should output heat in order to meet as much of the heat requirement as is possible without exceeding this limit.

**Fig A.3: Heat pump subsystem block diagram - separate supplementary heating**

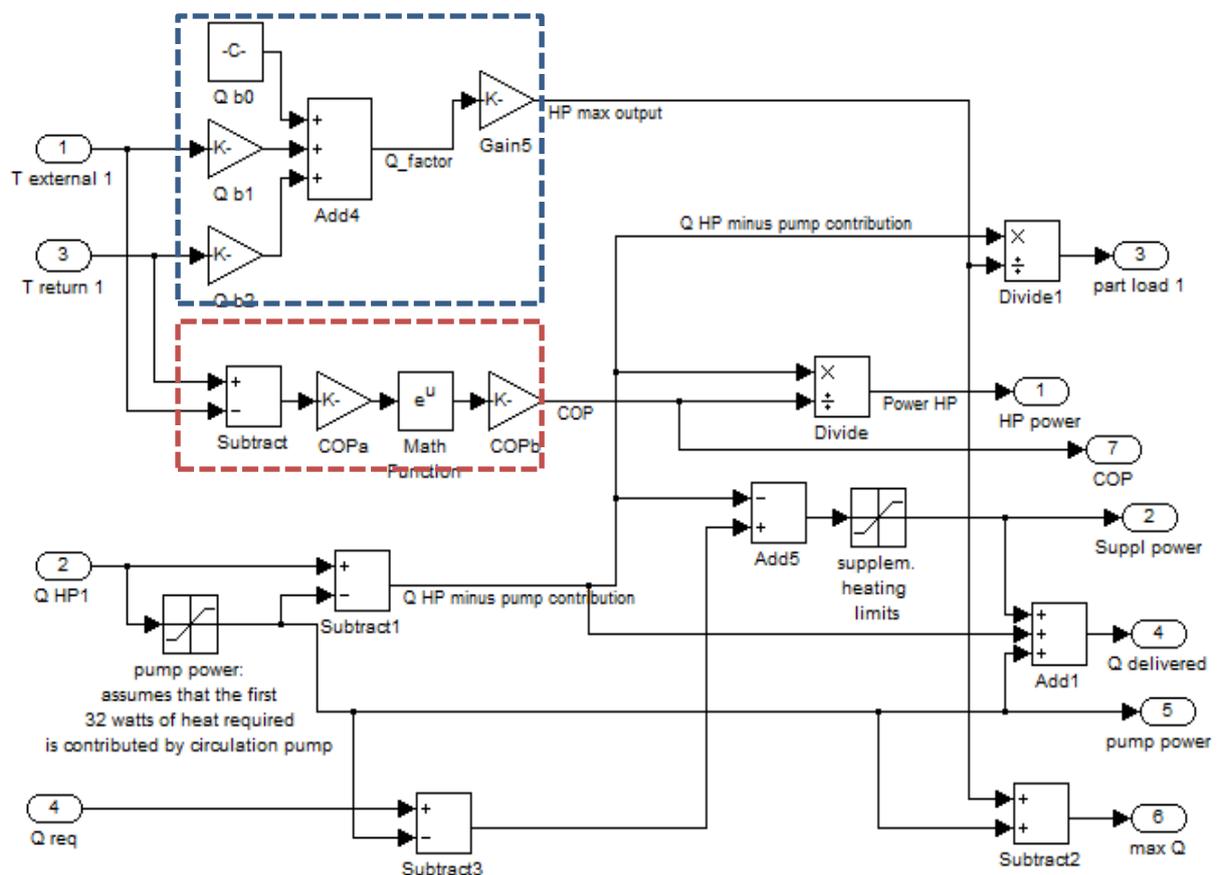


Figure A.3 depicts the heat pump subsystem for the case where supplementary heating is provided by a separate system. As the rate of heat output of the heat pump is already determined by the radiator subsystem, this serves two main functions: calculation of the heat pump's maximum output for the next timestep; and calculation of the power drawn by the heat pump.

The regression model representing heat pump maximum output is implemented by the blocks highlighted by the blue rectangle. The maximum output is linked back to the radiator subsystem. The regression model of COP is implemented by the blocks highlighted by the red rectangle. Heat pump power is calculated by dividing heat pump output by COP.

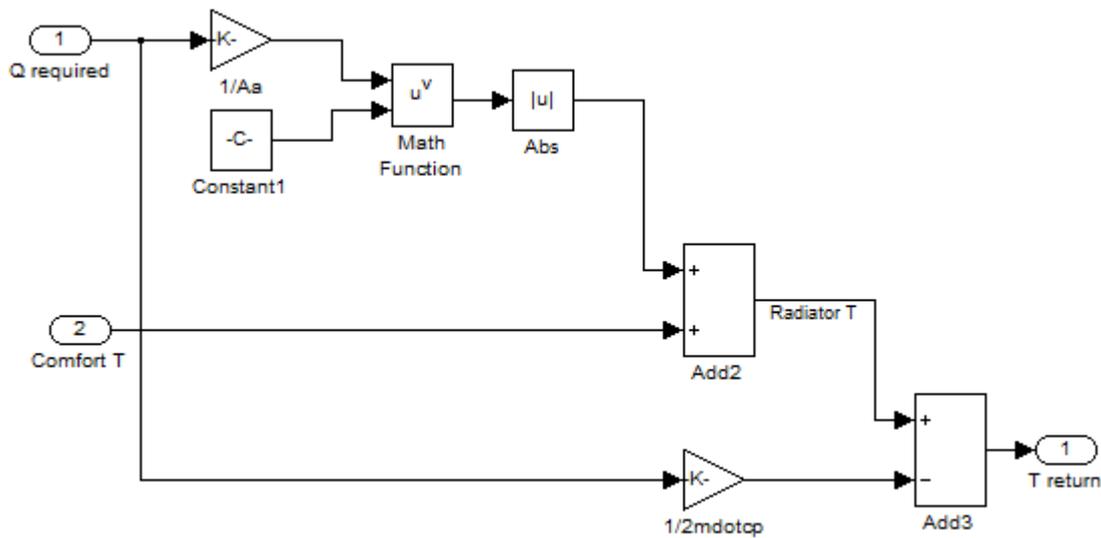
It is assumed that the first 32.1 Watts of heat delivered by the heat pump system are contributed by the circulation pump, but that the pump consumes no power when heat is not required. This is achieved by taking a branch of the heat pump heat output signal from radiator model (which arrives at the source block labelled Q HP1), and limiting its value to 32.1.

The supplementary heating requirement is calculated by subtracting the heat pump output from the rate of heat delivery required by the temperature controller. It is assumed in simulations that the supplementary heating system is 100% efficient.

The rate of heat delivered to the building subsystem is calculated by adding the heat pump, circulation pump and supplementary heating contributions. It is possible to limit or even turn off the supplementary heating. If this is done, the rate of heat delivered could be less than that requested by the temperature controller. However as the result would be that the set point temperature is not maintained, it is not done in this project.

The version of the radiator subsystem model for the case where supplementary heating is integrated in to the heat pump system (figure A.4) is straightforward in comparison to that for separate supplementary heating. The output of the radiator system is not constrained by the heat pump's maximum output, as the supplementary heater boosts its output when required. The subsystem therefore just implements equation (4.7), and outputs the radiator system return temperature resulting from the delivery of heat at the rate requested by the temperature controller.

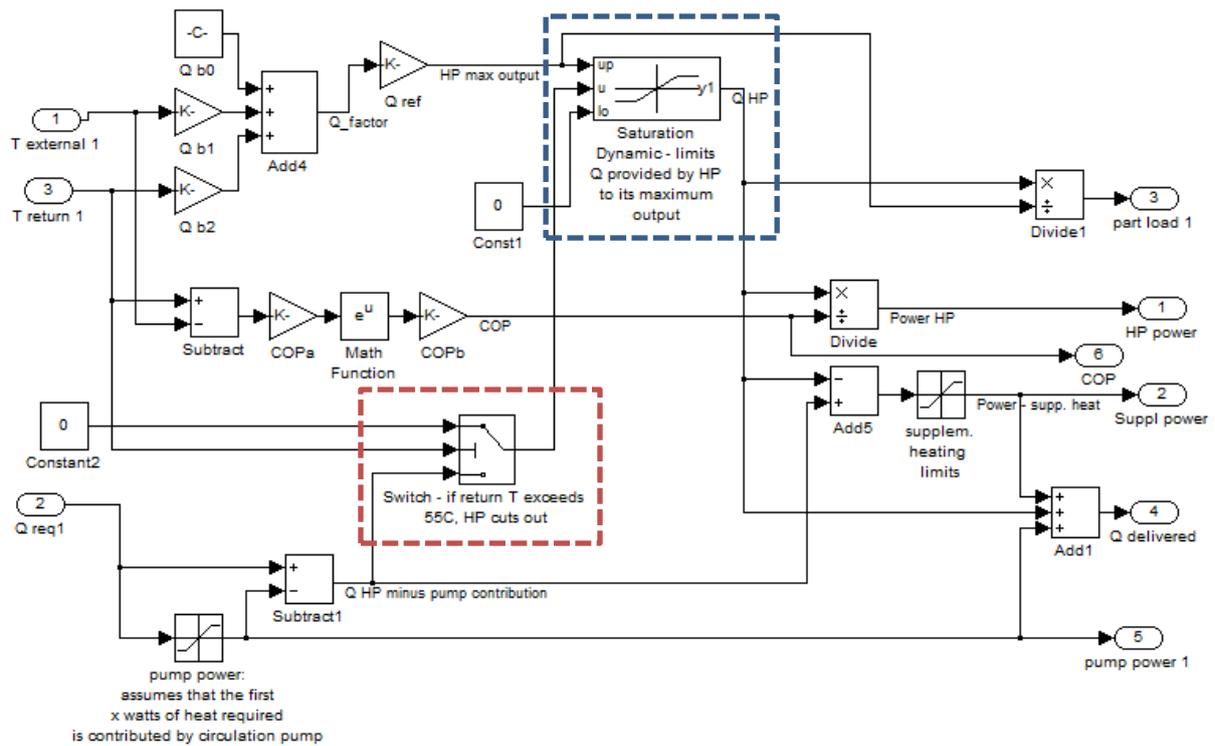
**Figure A.4: Radiator subsystem block diagram – integrated supplementary heating**



The version of the heat pump subsystem for the integrated supplementary heating case is similar to that for the separate supplementary heating case. It is depicted in figure A.5 below. A key difference is that the contribution of the heat pump to the heat output of the radiator system is limited by its maximum output. This is achieved by the block highlighted by a blue rectangle. Any deficit is assumed to be made up by the supplementary heater with 100% efficiency.

As before, it is assumed that the heat pump cannot operate if the return temperature exceeds 55°C. However as the supplementary heater is integrated with the radiator system, the return temperature must now be allowed to exceed 55°C if this is required in order to achieve the heating set point. A switch block is therefore included (highlighted by the red box), which sends the heat pump output to zero whenever 55°C is exceeded .

**Figure A.5: Heat pump subsystem block diagram – integrated supplementary heating**



At the start of simulations, the model is ‘primed’ with parameters held in a Matlab M-file. Three portions of the M-file are included on the following pages. The first part of the file (figure A.6) contains parameters relating to the IDEAS building model. This was not developed as part of this project, but is included as building parameters are relevant to the results obtained. The authors are credited in the text. The second part of the M-file (figure A.7) contains parameters required for the ASHP system model, and was developed as part of this project. A third part of the M-file (figure A.8) contains code to collate and analyse the data output by the model, again developed as part of this project.

Figure A.6: IDEAS building model parameters

```

%PARAMETERS Parameters for single zone building model
%M-File Initially Created by Joseph Brindley 05 April 11,
%equations from IDEAS model by Gavin Murphy.
%Updated by G.B.MURPHY
%FOURTH ORDER MODEL
%This version is a test case example for a Poorly Insulated Dwelling
%Excel Information: 20110606 - SAP_20110606_Us_2.1.m - very good
%match.xlsx
% DATE: 2011/06/07
%*****
clc                                %Clear Command Window
close all                          %Close Graphs from Previous iterations

Mv = 0.039662493237701; %FROM BREDEM      %(Kg/s)      Mass of the dwelling
air
%REF: = (('Semi-Detached'!AA41*'Semi-Detached'!AC6)/3600)*E11
%Where AA41 = Annual Effective Air Change Rate / AC6 = House Volume (m2) /
%E11 = Pa      = 1.205;          %STANDARD VALUE %kg/m3      Density of Air

Ma = 267.51;          %Va (Volume of Air from FROM BREDEM CELL AC6)* Pa
(Density of Air STANDARD VALUE)
%kg Mass of the air

Ca = 1005;           %STANDARD VALUE      %J/(kgK)      Specific heat capacity
of air
%REF:               http://www.engineeringtoolbox.com/air-properties-
d_156.html

Usap = 1;           %FROM BREDEM CELL AB47      %(W/m²K)      SAP Heat transfer
coeff. of the structure

As = 81.8;          %FROM BREDEM CELL AA47      %m²          Surface area of
structure

Ar = 44.4;          %FROM BREDEM CELL AA49      %(m²)        Area of Roof

Ur = 1;            %FROM BREDEM CELL AB53      %(W/m²K)      Heat Transfer Co-
Efficient of the Roof

Uw = 1.852;        %FROM BREDEM CELL AB53      %(W/m²K)      Heat Transfer Co-
Efficient of the Windows

Aw = 16.9;          %FROM BREDEM CELL AA53      %(m²)        Area of the
Windows

Pa = 1.205;        %STANDARD VALUE %kg/m3      Density of Air
%REF:              %http://www.engineeringtoolbox.com/air-properties-
d_156.html

Va = 222;          %FROM BREDEM CELL AC6      %m³          Volume of Air

Uf = 0.7;          %FROM BREDEM CELL AB48      %(W/m²K)      Heat Transfer Co-
Efficient of the Floor

Af = 44.4;          %FROM BREDEM CELL AA48      %(m²)        Area of the Floor

```

```

Aft = 120.7          %FROM BREDEM CELL AA75 + AA76 (Internal Mass)   %m2
Area of Internal Mass in a Dwelling, 120.7 is figure from BREDEM

Kwall = 0.170;      %STANDARD VALUE          %W/(m.K)      Thermal conductivity of
internal wall structure: k value
Twall = 0.1411;     %STANDARD VALUE          %m            Wall thickness (9inch
brick in this example)
%REF:               20110820 - Freundorfer House - PHPP2007_English - Used
to give K value for U Value of 1.xlsx

Mft = 5193.412;     %FROM BREDEM CELL AJ74          %kg           Mass of the
Furniture

Msi = 16062.12/2;   %FROM BREDEM CELL AM74          %kg           Mass of
Structure Internal

Mse = 16062.12/2;   %FROM BREDEM CELL AM74          %kg           Mass of
Structure External

Cs = 1700;          %J/(kg.K)      Specific Heat Capacity of Structure &
Internal Mass
%REF:               %http://www.engineeringtoolbox.com/specific-heat-
capacity-d_391.html

Cft = 1700;          %J/(kg.K)      Specific Heat Capacity of Furniture &
Internal Mass
%REF:               %%http://www.engineeringtoolbox.com/specific-heat-
capacity-d_391.html

Uft = 1.2;          % (W/m²K)      Heat Transfer Co-Efficient of the Furniture
& Internal Mass

%LEGACY:            Tfur = 0.5;          %m            Thickness of
Furniture
%
                    Available From:
"www.energysavingtrust.org.uk/Publication-Download/?p=1&pid=877"
%LEGACY:            Kt = Twall/Kwall;      % was Kwall/Twall; Thermal
Resistance of the Brick - Check units (unit for R Value = m²·K/W)
%LEGACY Ma = 249.795; %FROM BREDEM          %kg           Mass of the air

g = 1/(3*300);      %controller time constant

```

Figure A.7: ASHP heating system parameters

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Parameters for ASHP heating system model
% Added by M.E. Baster 15/07/11
% version 07/09/11

Arad = 2.3                %combined area of radiators in dwelling

a = 0.3905;              %coefficients of equation which calculates heat
trans coeff of radiators using temp difference between rads and room

b = 1.3001;              % as above

Aa = Arad*a;

CutOut = 1;              %presentation - for recording COP as 0 when no heat is
delivered (W)

Tlim = 55;               %maximum return temperature

pump = 32.1              %power drawn by circulation pump whenever heat is
delivered

Limit = 20000;           %max output of supplementary heater

HPcap = 6300             %heat pump thermal capacity at external air 7C, output
35C and return 30C

mdot = 0.000047847*6300; %mass flow rate of water in heating
system (kg/s)

cp = 4180;               % water specific heat at constant pressure at
approx 40C (J/kg)

mcp = mdot*cp;

COPa = -0.022;           %coefficients of Heat Pump COP regression
model

COPb = 6.7002;           %as above

Qbeta0 = -4.4648;        % coefficients of maximum Heat Pump output
regression model

Qbeta1 = 0.0229;         % as above

Qbeta2 = -0.0031;        % as above
```

Figure A.8: Matlab code for collating data output by ASHP heating system model

```
% Heat energy delivered

year_energy =
jan_energy+feb_energy+mar_energy+apr_energy+may_energy+jun_energy+jul_energ
y+aug_energy+sep_energy+oct_energy+nov_energy+dec_energy

heat_seas_energy = year_energy - jun_energy - jul_energy - aug_energy

% Electricity used by Supplementary heating

janee = y6(1:44640);
febee = y6(44641:84961);
maree = y6(84962:129602);
apree = y6(129602:172803);
mayee = y6(172804:217444);
junee = y6(217445:260644);
julee = y6(260645:305285);
augee = y6(305286:349926);
sepee = y6(349927:393127);
octee = y6(393128:437768);
novee = y6(437769:480969);
decee = y6(480970:525600);

jan_elec = (sum(janee)/(60*1000));
feb_elec = (sum(febee)/(60*1000));
mar_elec = (sum(maree)/(60*1000));
apr_elec = (sum(apree)/(60*1000));
may_elec = (sum(mayee)/(60*1000));
jun_elec = (sum(junee)/(60*1000));
jul_elec = (sum(julee)/(60*1000));
aug_elec = (sum(augee)/(60*1000));
sep_elec = (sum(sepee)/(60*1000));
oct_elec = (sum(octee)/(60*1000));
nov_elec = (sum(novee)/(60*1000));
dec_elec = (sum(decee)/(60*1000));

% Heat pump electricity

janHPee = y8(1:44640);
febHPee = y8(44641:84961);
marHPee = y8(84962:129602);
aprHPee = y8(129602:172803);
mayHPee = y8(172804:217444);
junHPee = y8(217445:260644);
julHPee = y8(260645:305285);
augHPee = y8(305286:349926);
sepHPee = y8(349927:393127);
octHPee = y8(393128:437768);
novHPee = y8(437769:480969);
decHPee = y8(480970:525600);

jan_HPelec = (sum(janHPee)/(60*1000));
feb_HPelec = (sum(febHPee)/(60*1000));
mar_HPelec = (sum(marHPee)/(60*1000));
apr_HPelec = (sum(aprHPee)/(60*1000));
```

```

may_HPelec = (sum(mayHPee)/(60*1000));
jun_HPelec = (sum(junHPee)/(60*1000));
jul_HPelec = (sum(julHPee)/(60*1000));
aug_HPelec = (sum(augHPee)/(60*1000));
sep_HPelec = (sum(sepHPee)/(60*1000));
oct_HPelec = (sum(octHPee)/(60*1000));
nov_HPelec = (sum(novHPee)/(60*1000));
dec_HPelec = (sum(decHPee)/(60*1000));

% Circulation pump electricity

janCPee = y11(1:44640);
febCPee = y11(44641:84961);
marCPee = y11(84962:129602);
aprCPee = y11(129602:172803);
mayCPee = y11(172804:217444);
junCPee = y11(217445:260644);
julCPee = y11(260645:305285);
augCPee = y11(305286:349926);
sepCPee = y11(349927:393127);
octCPee = y11(393128:437768);
novCPee = y11(437769:480969);
decCPee = y11(480970:525600);

jan_CPelec = (sum(janCPee)/(60*1000));
feb_CPelec = (sum(febCPee)/(60*1000));
mar_CPelec = (sum(marCPee)/(60*1000));
apr_CPelec = (sum(aprCPee)/(60*1000));
may_CPelec = (sum(mayCPee)/(60*1000));
jun_CPelec = (sum(junCPee)/(60*1000));
jul_CPelec = (sum(julCPee)/(60*1000));
aug_CPelec = (sum(augCPee)/(60*1000));
sep_CPelec = (sum(sepCPee)/(60*1000));
oct_CPelec = (sum(octCPee)/(60*1000));
nov_CPelec = (sum(novCPee)/(60*1000));
dec_CPelec = (sum(decCPee)/(60*1000));

% yearly electricity totals

year_HP_elec =
jan_HPelec+feb_HPelec+mar_HPelec+apr_HPelec+may_HPelec+jun_HPelec+jul_HPelec
+aug_HPelec+sep_HPelec+oct_HPelec+nov_HPelec+dec_HPelec

year_sup_elec =
jan_elec+feb_elec+mar_elec+apr_elec+may_elec+jun_elec+jul_elec+aug_elec+sep
_elec+oct_elec+nov_elec+dec_elec

year_pump_elec =
jan_CPelec+feb_CPelec+mar_CPelec+apr_CPelec+may_CPelec+jun_CPelec+jul_CPelec
+aug_CPelec+sep_CPelec+oct_CPelec+nov_CPelec+dec_CPelec

year_tot_elec = year_HP_elec + year_pump_elec + year_sup_elec

% Heating season electricity totals

heat_seas_HP_elec = year_HP_elec-(jun_HPelec+jul_HPelec+aug_HPelec)

heat_seas_sup_elec = year_sup_elec-(jun_elec+jul_elec+aug_elec)

```

```

heat_seas_pump_elec = year_pump_elec-(jun_CPelec+jul_CPelec+aug_CPelec)

heat_seas_tot_elec = year_tot_elec -
(jun_elec+jul_elec+aug_elec+jun_HPelec+jul_HPelec+aug_HPelec+jun_CPelec+jul
_CPelec+aug_CPelec)

```

```

% Monthly total electricity

```

```

jan_tot_elec = jan_HPelec + jan_CPelec + jan_elec;
feb_tot_elec = feb_HPelec + feb_CPelec + feb_elec;
mar_tot_elec = mar_HPelec + mar_CPelec + mar_elec;
apr_tot_elec = apr_HPelec + apr_CPelec + apr_elec;
may_tot_elec = may_HPelec + may_CPelec + may_elec;
jun_tot_elec = jun_HPelec + jun_CPelec + jun_elec;
jul_tot_elec = jul_HPelec + jul_CPelec + jul_elec;
aug_tot_elec = aug_HPelec + aug_CPelec + aug_elec;
sep_tot_elec = sep_HPelec + sep_CPelec + sep_elec;
oct_tot_elec = oct_HPelec + oct_CPelec + oct_elec;
nov_tot_elec = nov_HPelec + nov_CPelec + nov_elec;
dec_tot_elec = dec_HPelec + dec_CPelec + dec_elec;

```

```

% For loops - these bin electricity used at low rate tariffs

```

```

e=y6+y8+y11;

```

```

EW = zeros(453,1);
S = zeros(604,1);

```

```

sev23 = zeros(302,1);
sev00 = zeros(151,1);
sev01 = zeros(151,1);

```

```

for i = 0:150

```

```

    EW1=sum(e(1+(i*1440):300+(i*1440)));
    EW2=sum(e(781+(i*1440):960+(i*1440)));
    EW3=sum(e(1201+(i*1440):1320+(i*1440)));

```

```

    EW((3*i)+1)=EW1;
    EW((3*i)+2)=EW2;
    EW((3*i)+3)=EW3;

```

```

    S1=sum(e(1+(i*1440):30+(i*1440)));
    S2=sum(e(271+(i*1440):420+(i*1440)));
    S3=sum(e(811+(i*1440):990+(i*1440)));
    S4=sum(e(1231+(i*1440):1440+(i*1440)));

```

```

    S((4*i)+1)=S1;
    S((4*i)+2)=S2;
    S((4*i)+3)=S3;
    S((4*i)+4)=S4;

```

```

    seven23_1=sum(e(1+(i*1440):360+(i*1440)));
    seven23_2=sum(e(1381+(i*1440):1440+(i*1440)));

```

```

    sev23((2*i)+1)=seven23_1;
    sev23((2*i)+2)=seven23_2;

```

```

seven00=sum(e(1+(i*1440):420+(i*1440)));

sev00(i+1)=seven00;

seven01=sum(e(61+(i*1440):480+(i*1440)));

sev01(i+1)=seven01;

end

EW2 = zeros(366,1);
S2 = zeros(488,1);

sev23_2 = zeros(244,1);
sev00_2 = zeros(122,1);
sev01_2 = zeros(122,1);

for i = 243:364

    EW21=sum(e(1+(i*1440):300+(i*1440)));
    EW22=sum(e(781+(i*1440):960+(i*1440)));
    EW23=sum(e(1201+(i*1440):1320+(i*1440)));

    EW2((3*i)+1)=EW21;
    EW2((3*i)+2)=EW22;
    EW2((3*i)+3)=EW23;

    S21=sum(e(1+(i*1440):30+(i*1440)));
    S22=sum(e(271+(i*1440):420+(i*1440)));
    S23=sum(e(811+(i*1440):990+(i*1440)));
    S24=sum(e(1231+(i*1440):1440+(i*1440)));

    S2((4*i)+1)=S21;
    S2((4*i)+2)=S22;
    S2((4*i)+3)=S23;
    S2((4*i)+4)=S24;

    seven23_1_2=sum(e(1+(i*1440):360+(i*1440)));
    seven23_2_2=sum(e(1381+(i*1440):1440+(i*1440)));

    sev23_2((2*i)+1)=seven23_1_2;
    sev23_2((2*i)+2)=seven23_2_2;

    seven00_2=sum(e(1+(i*1440):420+(i*1440)));

    sev00_2(i+1)=seven00_2;

    seven01_2=sum(e(61+(i*1440):480+(i*1440)));

    sev01_2(i+1)=seven01_2;

end

EWlow_heatseas=(sum(EW)+sum(EW2))/(60*1000);

EWfraction=EWlow_heatseas/heat_seas_tot_elec %fraction of total electricity
used at off-peak times - Economy 10, rest of UK

```

```

Slow_heatseas=(sum(S)+sum(S2))/(60*1000);

Sfraction=Slow_heatseas/heat_seas_tot_elec %fraction of total electricity
used at off-peak times - Economy 10, Scotland

Sev_23fraction=(sum(sev23)+sum(sev23_2))/(60*1000*heat_seas_tot_elec) %
fraction of total electricity used at off-peak times Economy 7, 11-6am

Sev_00fraction=(sum(sev00)+sum(sev00_2))/(60*1000*heat_seas_tot_elec) %
fraction of total electricity used at off-peak times Economy 7, 12-7am

Sev_01fraction=(sum(sev01)+sum(sev01_2))/(60*1000*heat_seas_tot_elec) %
fraction of total electricity used at off-peak times Economy 7, 01-8am

% part load fraction (heating season)

plf1=sum(y7(1:217440));
plf2=sum(y7(349921:525600));
heat_seas_PLF=(plf1+plf2)/(217440+525600-349921)

% Calculation of system efficiency (HP + Supplementary + pump)

winSE = (dec_energy + jan_energy + feb_energy)/(dec_tot_elec + jan_tot_elec
+ feb_tot_elec)
sprSE = (mar_energy + apr_energy + may_energy)/(mar_tot_elec + apr_tot_elec
+ may_tot_elec)
sumSE = (jun_energy + jul_energy + aug_energy)/(jun_tot_elec + jul_tot_elec
+ aug_tot_elec)
autSE = (sep_energy + oct_energy + nov_energy)/(sep_tot_elec + oct_tot_elec
+ nov_tot_elec)

yearSE = year_energy/year_tot_elec

heat_seas_SE = heat_seas_energy/heat_seas_tot_elec

% Calculation of Seasonal performance factor

winSPF = (dec_energy + jan_energy + feb_energy - dec_CPelec - jan_CPelec -
feb_CPelec - dec_elec - jan_elec - feb_elec)/(dec_HPelec + jan_HPelec +
feb_HPelec)
sprSPF = (mar_energy + apr_energy + may_energy - mar_CPelec - apr_CPelec -
may_CPelec - mar_elec - apr_elec - may_elec)/(mar_HPelec + apr_HPelec +
may_HPelec)
sumSPF = (jun_energy + jul_energy + aug_energy - jun_CPelec - jul_CPelec -
aug_CPelec - jun_elec - jul_elec - aug_elec)/(jun_HPelec + jul_HPelec +
aug_HPelec)
autSPF = (sep_energy + oct_energy + nov_energy - sep_CPelec - oct_CPelec -
nov_CPelec - sep_elec - oct_elec - nov_elec)/(sep_HPelec + oct_HPelec +
nov_HPelec)

yearSPF = (year_energy - year_pump_elec - year_sup_elec)/year_HP_elec

% maximum and mean (heating season) return temperature

return_T = y9(1440:525600);

```

```

max_return_T = max (return_T)
T1 = y9(1440:217400);
T2 = y9(349921:525600);
T3 = [T1
      T2];
mean_heat_seas_return_T = mean(T3);

% maximum rate of heat required
qreq = y3(1440:525600);

% write results to a csv file

results = [year_energy
          year_tot_elec
          year_HP_elec
          year_sup_elec
          year_pump_elec
          heat_seas_energy
          heat_seas_tot_elec
          heat_seas_HP_elec
          heat_seas_sup_elec
          heat_seas_pump_elec
          EWfraction
          Sfraction
          Sev_23fraction
          Sev_00fraction
          Sev_01fraction
          heat_seas_SE
          winSE
          sprSE
          autSE
          winSPF
          sprSPF
          %sumSPF
          autSPF
          %yearSPF
          heat_seas_PLF
          max_return_T
          mean_heat_seas_return_T
          max(qreq)];

%csvwrite('res_sepSup_consp_manu_fmF_923aber_test.csv',results);

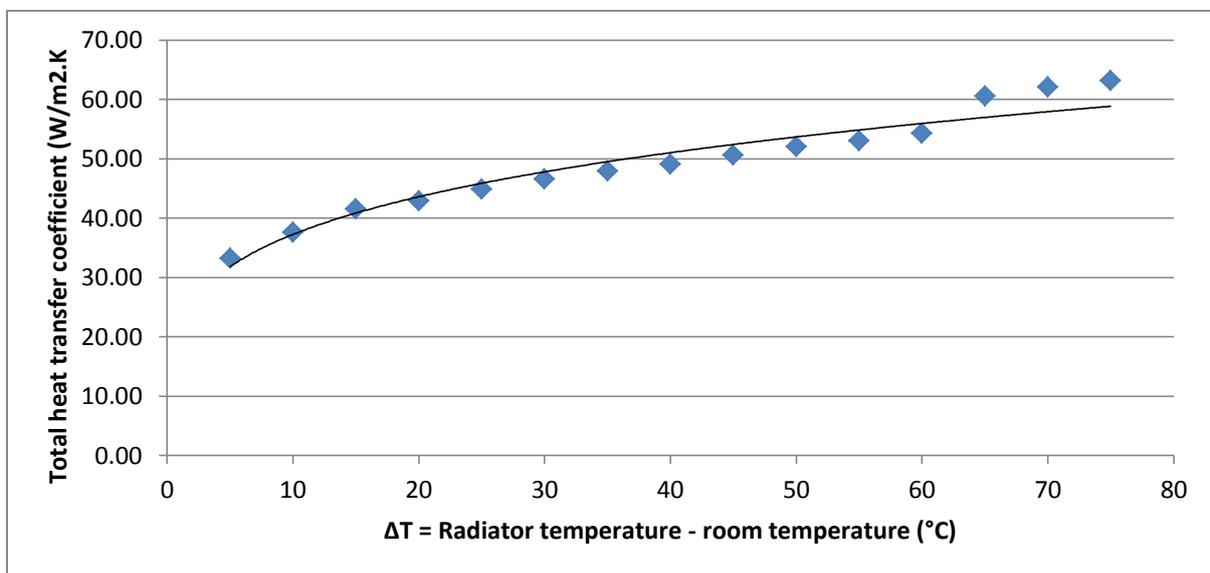
```

## Annex B: Calculation of radiator heat transfer coefficients

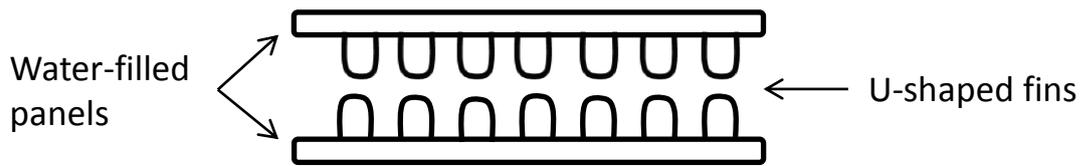
The method used to calculate the heat transfer coefficient of a typical central heating radiator is based on lecture material from the University of Strathclyde's MSc course in Renewable Energy Systems and the Environment. The equations given below were implemented in a spreadsheet and evaluated for a range of radiator surface temperatures.

The results obtained are depicted in Figure B.1. The jump in heat transfer coefficient is because different equations are required to be used to calculate the convective heat transfer coefficient over certain ranges. A schematic of the radiator construction used as the basis for calculations is given in figure B.2.

**Figure B.1: Calculated radiator heat transfer coefficient**



**Figure B.2: Schematic of assumed radiator construction: view from above**



Height of radiator  $l = 0.6\text{m}$  (fins run the full height of the radiator)

Viewing half of each U-shape as an individual fin:

Fin length  $L = 0.03\text{m}$

Fin spacing  $= 0.025\text{m}$

Fin thickness  $= 0.001\text{m}$

Fins and panels are assumed to be constructed of steel with conductivity  $K = 43\text{W/m.K}$

### Convection heat transfer coefficient

The convective heat transfer from the panels of the radiator is calculated using the following approximate relationships for natural convection from vertical flat plates:

$$10^4 < Gr Pr < 10^9, \quad Nu = 0.6 (Gr Pr)^{0.25}$$

$$10^9 < Gr Pr < 10^{12}, \quad Nu = 0.13 (Gr Pr)^{0.33}$$

The Prandl Number  $Pr$  is taken from standard charts of the properties of air, and is evaluated at the mean of the surface temperature and air bulk temperature

The Nusselt number  $Nu$  is given by

$$Nu = \frac{hl}{k}$$

where  $h$  is the heat transfer coefficient,  $l$  is the characteristic dimension (height in this case) and  $k$  is thermal conductivity of air.

The Grashof number  $Gr$  is given by

$$Gr = \frac{\beta g (T_s - T_f) l^3 \rho^2}{\mu^2}$$

where  $g$  is acceleration due to gravity,  $T_s$  is the temperature of the radiator surface,  $T_f$  is air temperature,  $\rho$  is density and  $\mu$  is viscosity.

Coefficient of cubical expansion  $\beta$  is approximated as

$$\beta \sim \frac{1}{T}$$

Density, conductivity and viscosity are taken from standard tables of properties of air and are evaluated at the mean of the surface and air temperatures.

### **Fin heat transfer coefficient**

The temperature distribution along a fin is described by

$$\frac{d^2\Phi}{dx^2} - m^2 = 0$$

where  $\Phi$  is defined as “excess temperature”  $\Phi = T - T_f$ .  $T$  is temperature at some point distance  $x$  from the base of the fin and  $T_f$  is air temperature. The constant  $m$  is defined as

$$m = \sqrt{\frac{hP}{kA}}$$

where  $h$  is assumed to be the same as the convective heat transfer coefficient from the surface of the panels, calculated using the equations above.  $A$  is the cross-sectional area of a fin,  $P$  is its perimeter and  $k$  is the conductivity of the fin material.

General solutions to the differential equation for different pairs of boundary conditions are given in lecture material. As fins are U-shapes of total length 0.6m and attached to the radiator surface at both ends, at the midpoint of the curve of the U there is the boundary condition

$$\frac{d\Phi}{dx} \Big|_{x=0.3} = 0$$

From the solution this gives, the following equation for the heat transferred from a fin follows:

$$Q_{fin} = \sqrt{hPkA} \Phi_b \tanh mL$$

Here  $\Phi_b$  is the “excess temperature” evaluated at the base of the fin, i.e. radiator surface temperature minus air temperature.

Given the assumed radiator height of 0.6m and fin spacing of 0.025m, there are 66.6 fins per unit area of radiator. Using this number and noting the definition of  $\Phi_b$ , it is possible to adapt the equation for  $Q_{fin}$  to give an effective fin heat transfer coefficient relating to the radiator area to which the fins are attached:

$$h_{fin} = 66.6\sqrt{hPkA} \tanh mL$$

### **Radiation heat transfer coefficient**

In calculating the radiative heat transfer coefficient, it is assumed that the simplification that the radiator is a “small convex object in a large enclosure” can be applied.

$$h_R = \sigma \varepsilon (T_1 + T_2)(T_1^2 + T_2^2)$$

$T_1$  is taken to be radiator temperature.  $T_2$  is taken to be 20°C

### **Assumptions**

A key assumption used in these calculations is that air and room surfaces are all at 20°C (including the wall surface behind the radiator). Partly, this allows the straightforward combining of the convective, fin and radiative heat transfer coefficients into a total radiator heat transfer coefficient. Additionally, the total coefficient calculated here is compared elsewhere in this project against radiator rated outputs derived from standard tests. The test process does involve placing the radiators in a room in which air temperature is maintained at 20°C. However, in order to maintain this temperature, the walls of the test room are cooled internally by water. As no standard size appears to be stipulated for test rooms, it is not possible to know the temperature of the wall surfaces in tests.

In order to combine the convective, fin and radiative heat transfer coefficients into an total radiator heat transfer coefficient, it is assumed that: convection takes place at the front and back of each radiator panel (so from a total area 4 times the radiator area); heat is radiated from the front of the front panel and the rear of the rear panel (twice the radiator area); and that fins cover one surface of each of the panels (again twice the radiator area). As all these heat transfers happen independently, the total radiator heat transfer coefficient is assumed to be

$$h_{rad} = 4h_{conv} + 2h_{fin} + 2h_R$$

Additionally, it is assumed in this project that the surface temperature of the radiator is the mean temperature of the water flowing through it.

## Annex C: Calculation of solar gain

The method used to calculate the solar gain profile used an input for the building model is based on lecture material from the University of Strathclyde's MSc course in Renewable Energy Systems and the Environment, combined with standard parameters from SAP 2009.

The total area of windows in the model is 16.9m<sup>2</sup>. For generality, it is assumed the building walls are orientated due North, South, East and West, and that the total window area is divided equally between each wall.

First the total solar flux incident  $S$  on vertical surfaces with the above orientations must be determined. The equations given in the latter part of this annex were implemented in a spreadsheet, and used to calculate hourly incident solar flux for each wall orientation based on solar radiation data from the climate files discussed in section 4.5.

Once this was done, the following formula from SAP 2009 was used to calculate an hourly solar gain profile. In SAP, this would normally be used with monthly average solar flux data, however for dynamic simulation this is unlikely to give accurate results.

$$\text{Solar gain} = 0.9 \times A_W \times S \times g_g \times FF \times Z$$

- $A_W$  = area of windows at a particular orientation
- $S$  = total solar flux for that orientation

SAP reference values (or default values for use when characteristics are unknown) are used:

- Transmittance factor  $g_g = 0.72$ . Corresponds to double glazed windows with a low- $\epsilon$  hard coat.
- Frame factor  $FF$  represents the proportion of the window opening which is glazed. The reference value of 0.7 is used.
- The default value for access factor  $Z$  is 0.77.

The calculation of incident solar flux  $S$  requires the direct and diffuse components to be derived separately.

## Direct solar radiation

The calculation of the direct solar radiation falling on a surface of the collector requires several stages. First declination, the angle of tilt of the planet's axis in relation to the position of the sun, and the parameters B and E are calculated. E is a correction factor allowing for the eccentricity in the earth's orbit

$$\text{Declination } \delta = \delta_0 + \sin \frac{360 \times (284 + n)}{365}$$

where  $\delta_0 = 23.5^\circ$  and n is the day of the year.

$$B = \frac{360 \times (n - 1)}{364}$$

$$E = 9.87 \sin(2B) - 7.35 \cos B - 1.5 \sin B$$

Local solar time  $t_{sol}$  and hour angle h are then calculated:

$$t_{sol} = t_{ref} + \frac{4(L_{ref} - L) + E}{60}$$

where  $t_{ref}$  is GMT and  $L_{ref} = 0$ , the longitude of Greenwich. Hour angle h is given by

$$h = 15 \times (12 - t_{sol}) \quad am$$

$$h = 15 \times (t_{sol} - 12) \quad pm$$

Solar elevation  $\beta$  and azimuth angle  $\gamma_s$  can now be calculated using the above quantities:

$$\beta = \sin^{-1}(\cos l \cos h \cos \delta + \sin l \sin \delta)$$

$$\gamma_s = \cos^{-1}\left(\frac{\sin l \cos h \cos \delta - \cos l \sin \delta}{\cos \beta}\right)$$

Here l is latitude of the site.

Once elevation  $\beta$  and azimuth angle  $\gamma_s$  are obtained, the direct solar radiation falling on a the surface for some angle of tilt  $\Phi$  is calculated using the following formula. For a vertical surface,  $\Phi = 0$ .

$$I_{b\Phi} = I_b (\cos \beta \cos \alpha \cos \Phi + \sin \beta \sin \Phi)$$

Parameter  $\alpha$  is the surface-solar azimuth angle  $|\gamma - \gamma_s|$ .  $I_b$  is the measured direct solar radiation from the climate file.

### Diffuse radiation

The values for diffuse solar radiation (denoted  $I_{dh}$  in the equation below) given in the climate data relate to a horizontal surface. The diffuse radiation falling on a vertical surface,  $I_{d\phi}$ . This is approximated in the spreadsheet using the following formula. For a vertical surface,  $\Phi = 0$ .

$$I_{d\phi} = I_{dh} \left( \frac{1 + \sin\phi}{2} \right)$$

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