Theoretical Basis of Primitive Part Modeling

Tin-Tai Chow, Ph.D., C.Eng.  
Member ASHRAE  
Joseph A. Clarke, Ph.D., C.Eng.

ABSTRACT
At the present time, the simulation of buildings and their environmental control systems is enjoying growing acceptance by practitioners worldwide. At the same time, these same programs possess internal structures and modeling approaches that will restrict future extensibility in response to growing user expectations. This situation is particularly problematic in relation to the modeling of HVAC systems, where inflexible component algorithms will restrict applications to standard and conventional system configurations. One reaction to this issue is to move toward a process-oriented representation of the HVAC component models. In this approach, a library of "primitive parts" (PP) is established where each PP describes (mathematically) an elementary heat, mass, or mixed transfer process. HVAC system component models may then be synthesized at simulation time from appropriate PP combinations. This paper explains the PP approach and illustrates its application to practical problems.

INTRODUCTION
The consequence of mechanically supplying or removing heat from a space depends on several factors, such as the thermal response of the fabric and contents, occupant behavior, air movement regimes, HVAC system capabilities, and control system behavior. Taken together, these factors give rise to a system that is nonlinear, time varying, and systemic (in that there are many causal interactions). Such a system requires special mathematical modeling procedures and numerical integration methods (e.g., Clarke 1985). Figure 1 illustrates this complexity as typically found within HVAC systems.

Where applicable, simulation provides a most powerful tool for the appraisal of HVAC systems in the dynamic (i.e., realistic) domain. The technique can be applied to assist in equipment sizing and selection, to support design optimization, and to compare alternative approaches. Unfortunately, simulation programs are not unconditionally applicable because of inherent assumptions and inflexible component models. They are, therefore, unable to deal with the range of problems likely to be encountered in practice (Sowell 1991).

At the present time, simulation is evolving toward an approach that offers full building and HVAC system integration, based on a fundamental mathematical approach. When a model is built up from the mathematical descriptions of basic energy and mass transfer phenomena, the need for experiments is not apparent. Models can be constructed to the point where all but a few key relations or parameters are known, then valuable experiments can be carried out to determine these unknown quantities (Mitchell 1997). A complementary goal is to achieve an approach to HVAC component modeling that offers program interoperability and helps to converge the disparate theories. This goal is apparent within several contemporary developments, such as the neutral model format (NMF) project (Bring et al. 1992; Sahlin et al. 1995), the input-output independent models of the SPARK system (Buhl et al. 1993; Sowell and Mosher 1995), and the primitive parts (PP) approach within the ESP-r system (Clarke 1985; Chow 1993, 1995).

PP and NMF are basically different in that PP approaches are elemental mathematical models whereas NMF is a model description method. PP modeling is an innovation that allows modelers, in the short term, to construct component models from a finite set of elemental process models and, in the longer term, for this to be achieved automatically at run-time as the need arises. The principal advantage of the PP approach is its flexibility in generating component models for the multiplicity of possible plant systems and modeling abstractions. Alternative input-output relationships are represented by different...
Figure 1  Energy and mass flow paths in air-conditioning systems.

PP groupings arranged automatically at the point of component model selection (Chow et al. 1997).

The developmental work to date has been based on the finite volume conservation method, by which a dynamic model may be constructed that conserves system integrity by numerically processing all elements of the problem as a state-connected system. The ultimate aim is to support the automatic formulation of component mathematical models on the basis of explicit descriptions of components as distinct from an approach in which preconstructed models are links to define a system. The former approach is deemed to be more flexible and so able to accommodate a wide range of component types.

FINITE VOLUME CONSERVATION METHOD

In the context of the ESP-r system, the finite volume conservation method was first applied to dynamic building/plant modeling in the late 1970s, continuing to the present day (Clarke 1977; McLean 1982; Tang 1985; Clarke 1986; Clarke and Mac Randal 1993; Aasem 1993). In the approach, plant components are physically decomposed into a finite number of subparts, or control volumes (CV). A CV can be homogeneous, and, therefore, possess uniform hygrothermal properties, or heterogeneous, and, therefore, possess a set of hygrothermal properties. In both cases, the region represented by the CV has a single set of state variables (enthalpy, pressure, voltage, mass, etc.).

For each CV in turn, and in terms of all other CVs in potential thermal, electrical, or flow contact, conservation equations are developed in relation to the transport properties of interest (heat energy and fluid exchange, for example). The arbitrariness of CV discretization allows the technique to be applied to objects with irregular shapes and subject to multidimensional, mixed-mode heat and mass transfer.

Consider an arbitrary CV, i, representing a single-phase fluid as shown in Figure 2. For this case, the energy and mass balance equations are given by

\[ m_{i-1}C_p\theta_{i-1} - m_iC_p\theta_i + Q = \frac{\partial}{\partial t}(M_iC_p\theta_i) \]  
(1)

and

\[ m_{i-1} - m_i = \frac{\partial}{\partial t}(M_i) \]  
(2)

where \( m \) is the fluid flow rate \((\text{kg/s})\), \( C_p \) is the fluid specific heat \((\text{J/kg K})\), \( \theta \) is temperature \((\text{°C})\), \( M \) is the mass \((\text{kg})\), and \( Q \) is the rate of heat addition \((\text{W})\); \( i-1 \) relates to the upstream CV connected to \( i \).

The time-variant \( \theta \) and \( C_p \) in Equation 1 are the spatially averaged values within a CV. For solid CVs, Equation 2 is
omitted, while a multiphase flow problem (e.g., the dry air and vapor phases within an air-conditioning system) will require a mass balance equation for each phase. Within ESP-r, the CV conservation equations are made time discrete by application of finite differencing (Clarke 1985). For a small time interval $\delta t$, Equation 1 can be expressed in the explicit, implicit, or Crank-Nicolson formulations, respectively, as follows:

$$m^+_{i-1} C_{p}^+ \cdot \theta^+_{i-1} - m^*_i C_{p}^* \cdot \theta^*_i + Q^* =
M_i C_{p}^+ \left( \frac{\theta^+_{i-1} - \theta^+_i}{\delta t} \right)$$

(3a)

$$m^*_{i-1} C_{p}^+ \theta^+_{i-1} - m^*_i C_{p}^* \theta^*_i + Q = M_i C_{p}^+ \left( \frac{\theta^+_{i-1} - \theta^+_i}{\delta t} \right)$$

(3b)

$$\left( \frac{m^*_{i-1} C_{p}^+ \theta^+_{i-1} + m^*_i C_{p}^* \theta^*_i}{2} \right) + \left( \frac{Q + Q^*}{2} \right) =
M_i C_{p}^+ \left( \frac{\theta^+_{i-1} - \theta^+_i}{\delta t} \right)$$

(3c)

where $^*$ signifies quantity evaluation at the present (known) time-row of some arbitrary time-step (all other quantities relate to the future, unknown time-row). The above three equations can also be represented in a generalized form, as shown below, by a weighting factor $\alpha$, which defines the degree of “implicityness” of the equation: 0, 1, and 0.5 corresponding to the explicit, implicit, and Crank-Nicolson weighted average cases, respectively, taking the CV thermal mass as time invariant.

$$\left[ -\alpha m^*_{i-1} \frac{M_i}{\delta t} C_{p}^+ \theta^+_i + \alpha m^*_{i-1} C_{p}^+ \theta^+_{i-1} \right] =
\left[ (1-\alpha)m^*_i \frac{M_i}{\delta t} C_{p}^* \theta^*_i + (1-\alpha)m^*_i C_{p}^* \theta^*_{i-1} + \right.$$

$$\left. \alpha Q + (1-\alpha)Q^* \right]$$

(3d)

**PLANT MATRIX EQUATION**

Any HVAC system can, therefore, be represented as a network of nodes (the CVs) connected by arcs (the conservation equations linking CVs). In an air-conditioning system, for example, each CV will therefore be described by three conservation equations corresponding to energy and two-phase mass balance. The collection of such equations for the nodes (CVs) composing a single component gives rise to a generic mathematical model for that component type. It is the job of the run-time simulator to synthesize the overall plant matrix equation from the individual component models composing the overall network. This requires knowledge of the factors influencing the CVs and a means to evaluate the equation coefficients at each time-step.

Control equations are then added to the matrix equation to prescribe or limit parameter values before the entire equation set is passed to the solver for time-step integration. For an integrated building and plant problem, the solution procedure utilizes several solvers operating in tandem, each one customized to a particular subsystem, e.g., piecewise, direct inversion of linearized equations describing energy and mass balance and iterative solvers for fluid flow (Clarke 1985). Where possible, nonlinear equations are linearized by absorbing the nonlinear elements into the variable coefficients and employing iteration.

The essential consequence of linearization on the plant side is that the principle of superposition can then be applied. It is the application of this principle that gives rise to the theory of primitive parts.

**THEORY OF PRIMITIVE PARTS**

Consider Figure 3, which shows a surface, $S$, in conductive contact with a region $X$, in convective contact with a region $Y$, and in radiative contact with a region $Z$. The energy balance equation for $S$ is given by

$$C_{sx} (\theta_s - \theta_x) + C_{sy} (\theta_s - \theta_y) + C_{sz} (\theta_s - \theta_z) = M_s C_s \frac{d\theta_s}{dt}$$

(4)

where $C_{sx} = k_{sx} A_{sx}/l_{sx}$, $C_{sy} = h_{sy} A_{sy}$, and $C_{sz} = h_{sz} A_{sz}$. Applying a finite difference approximation to the time derivative gives

![remoted surface](image)

![solid surface](image)

**Figure 3** Discretization scheme for surface node “S.”
\[
\begin{align*}
\left[\alpha(C_{ss} + C_{s1} + C_{s2}) - \frac{M_C}{\delta t} \right] \theta_{s} + \alpha C_{s1} \theta_{1} + \\
\alpha C_{s2} \theta_{2} &= (1 - \alpha)C_{s1}^* \theta_{1}^* - (1 - \alpha)C_{s2}^* \theta_{2}^* \\
&\quad - (1 - \alpha)C_{s1}^* \theta_{1}^* - (1 - \alpha)C_{s2}^* \theta_{2}^* \tag{5}
\end{align*}
\]

where \(^*\) denotes the present time row (\(M_f = M_{f1}^*\) for most practical applications).

Equation 5 can be rewritten in its matrix equation form:

\[
[C(1) C(2) C(3) C(4)] \begin{bmatrix}
\theta_{s} \\
\theta_{1} \\
\theta_{2} \\
\end{bmatrix} = [C(5)].
\] \tag{6}

In ESP-r’s conventional, component-centered approach, pre-constructed routines of various plant components exist to generate the numerical values of a component’s equation coefficients at each time-step, i.e., \(C(1)\) through \(C(5)\) in Equation 6, for transmission to a central matrix equation construction and solution controller (i.e., the simulation engine). However, by the principle of superposition, it is possible to synthesis component models from a common set of primitive parts so that the need for pre-formed component models is eliminated. This significantly increases program applicability and reduces complexity.

Table 1 lists the primitive parts (PPs) now available for use by ESP-r and gives an example application in each case. These PPs are grouped under ten categories and numbered accordingly. For the current component, three PPs are involved:

<table>
<thead>
<tr>
<th>PP</th>
<th>Description</th>
<th>Nodes Involved</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Thermal conduction: solid to solid</td>
<td>S, X</td>
</tr>
<tr>
<td>2.1</td>
<td>Surface convection: with moist air</td>
<td>S, Y</td>
</tr>
<tr>
<td>3.1</td>
<td>Surface radiation: with local surface</td>
<td>S, Z</td>
</tr>
</tbody>
</table>

While the matrix equations underlying the above PPs, and another six as employed in this paper, are given in the Appendix, a full description of the entire family of 27 is given elsewhere (Chow 1995b). Within a PP matrix equation, each \(A(i,j)\) represents a future time-row coefficient of a nonhomogeneous matrix of the nodal temperature/mass flow rates; \(B(i,j)\) represents a present time-row coefficient of the corresponding matrix, absorbed with the known boundary excitations relating to both the present and future time-rows.

The rule of superposition (Ogata 1992) may then be applied by which each matrix equation coefficient is determined as the sum of constituent PP equation coefficients. Hence, in the current problem:

\[
\begin{align*}
C(1) &= A(11,1) + A(21,1) + A(31,1) = -\alpha (C_{ss} + C_{s1} + C_{s2}) \\
&\quad - \frac{M_C}{\delta t} \tag{5} \\
C(2) &= A(11,2) = \alpha C_{ss} \\
C(3) &= A(21,2) = \alpha C_{s1} \\
C(4) &= A(31,2) = \alpha C_{s2} \\
C(5) &= B(11,1) + B(21,1) + B(31,1) = (1 - \alpha)(C_{s1}^* + C_{s2}^* + C_{ss}^* - M_{f1}^* C_{s1}^*/\delta t) \theta_{s} + (1 - \alpha) C_{s1}^* \theta_{1}^* - (1 - \alpha) C_{s2}^* \theta_{2}^*.
\end{align*}
\]

These relationships are identical to those that would result from first principle considerations applied to Equation 5. A key parameter in PP modeling is \(N_i\), defined as the “primitive part connection index” (hereinafter, the “PP index”) of a particular node. This index indicates the number of PPs used in describing the energy/mass flow paths involving the thermal mass of the particular node. In this case, the number of participating PPs for the node S is 3; hence, \(N_i\) is equal to 3. The specific role of \(N_i\) is to maintain the thermal capacitance of \(S\) equal to \(M_f C_s\) and, therefore, make the capacitance term independent of the number of PPs participating in the addition process.

The PP approach is further illustrated in the following section, where two practical examples are considered.

**AIR DUCT MODEL**

A section of an insulated air duct is arbitrarily discretized into four finite volumes as shown in Figure 4. Two of these, denoted by nodes S1 and S2, represent the thermal insulation and the sheet metal, respectively; node AM represents the air inside the duct; and nodes A0 and A1 represent the entering and leaving air states and are used to connect to the upstream and downstream components, respectively. This arrangement facilitates the modeling of the transport delay phenomenon where necessary. In the following derivation, no transport delay is first assumed for simplicity.

Assuming that the ambient energy exchange takes place with a known boundary condition, \(\theta_{s}\), the energy conservation equations for the four nodes are as follows:

![Figure 4 Four-node insulated air duct.](image-url)
<table>
<thead>
<tr>
<th>Categories</th>
<th>Application Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Thermal Conduction</strong></td>
</tr>
<tr>
<td>1.1</td>
<td>Solid to Solid</td>
</tr>
<tr>
<td>1.2</td>
<td>With Ambient Solid</td>
</tr>
<tr>
<td>2</td>
<td><strong>Surface Convection</strong></td>
</tr>
<tr>
<td>2.1</td>
<td>With Moist Air</td>
</tr>
<tr>
<td>2.2</td>
<td>With 2-Phase Fluid</td>
</tr>
<tr>
<td>2.3</td>
<td>With 1-Phase Fluid</td>
</tr>
<tr>
<td>2.4</td>
<td>With Ambient</td>
</tr>
<tr>
<td>3</td>
<td><strong>Surface Radiation</strong></td>
</tr>
<tr>
<td>3.1</td>
<td>With Local Surface</td>
</tr>
<tr>
<td>3.2</td>
<td>With Ambient Surface</td>
</tr>
<tr>
<td>4</td>
<td><strong>Flow Upon Surface</strong></td>
</tr>
<tr>
<td>4.1</td>
<td>For Moist Air; 3 Nodes</td>
</tr>
<tr>
<td>4.2</td>
<td>For 2-Phase Fluid; 3 Nodes</td>
</tr>
<tr>
<td>4.3</td>
<td>For 1-Phase Fluid; 3 Nodes</td>
</tr>
<tr>
<td>4.4</td>
<td>For Moist Air; 2 Nodes</td>
</tr>
<tr>
<td>4.5</td>
<td>For 1-Phase Fluid; 2 Nodes</td>
</tr>
<tr>
<td>5</td>
<td><strong>Flow Divider and Inducer</strong></td>
</tr>
<tr>
<td>5.1</td>
<td>Flow Diverger (For All Fluid)</td>
</tr>
<tr>
<td>5.2</td>
<td>Flow Multiplier (For All Fluid)</td>
</tr>
<tr>
<td>5.3</td>
<td>Flow Inducer (For All Fluid)</td>
</tr>
<tr>
<td>6</td>
<td><strong>Flow Converger</strong></td>
</tr>
<tr>
<td>6.1</td>
<td>For Moist Air</td>
</tr>
<tr>
<td>6.2</td>
<td>For 2-Phase Fluid</td>
</tr>
<tr>
<td>6.3</td>
<td>For 1-Phase Fluid</td>
</tr>
<tr>
<td>6.4</td>
<td>For Leak-in Moist Air from Outside</td>
</tr>
<tr>
<td>7</td>
<td><strong>Flow Upon Water Spray</strong></td>
</tr>
<tr>
<td>7.1</td>
<td>For Moist Air</td>
</tr>
<tr>
<td>8</td>
<td><strong>Fluid Injection</strong></td>
</tr>
<tr>
<td>8.1</td>
<td>Water/Steam to Moist Air</td>
</tr>
<tr>
<td>9</td>
<td><strong>Fluid Accumulator</strong></td>
</tr>
<tr>
<td>9.1</td>
<td>For Moist Air</td>
</tr>
<tr>
<td>9.2</td>
<td>For Liquid</td>
</tr>
<tr>
<td>10</td>
<td><strong>Heat Injection</strong></td>
</tr>
<tr>
<td>10.1</td>
<td>To Solid</td>
</tr>
<tr>
<td>10.2</td>
<td>To Vapor-Generating Fluid</td>
</tr>
<tr>
<td>10.3</td>
<td>To Moist Air in Solar Collector</td>
</tr>
</tbody>
</table>

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(i) Thermal insulation:

\[ h_i A_i (\theta_i - \theta_{i1}) + \frac{k_{i1} A_{i1}}{l_{i2}} (\theta_{i2} - \theta_{i1}) = M_{i1} C_{i1} \frac{d\theta_{i1}}{dt} \]  

(7)

(ii) Sheet metal:

\[ \frac{k_{i1} A_{i1}}{l_{i2}} (\theta_{i1} - \theta_{i2}) + h_i A_i (\theta_{am} - \theta_{i2}) = M_{i2} C_{i2} \frac{d\theta_{i2}}{dt} \]  

(8)

(iii) Contained air:

\[ (m_{a0} C_{p0} + m_{r0} C_{p0}) \theta_{a0} - (m_{a1} C_{p1} + m_{r1} C_{p1}) \theta_{a1} + \frac{h_i A_i (\theta_{am} - \theta_{a1})}{M_{am} C_{pam}} \frac{d\theta_{am}}{dt} \]  

(9)

(iv) Leaving air (with no transport delay):

\[ \theta_{a1} = \theta_{am} \]  

(10)

Equation 7, in its finite difference form, becomes

\[ -\left[ \frac{\alpha (C_{12} + C_{e})}{\delta t} \right] \theta_{a1} + \frac{M_{12} C_{12}}{\delta t} \theta_{a1} + \frac{\alpha C_{12} \theta_{a2}}{\delta t} = \left[ (1 - \alpha) C_{12} + C_{e} \right] \theta^{*}_{a1} - \frac{M_{a1} C_{a1}}{\delta t} \theta^{*}_{a1} \]  

(11)

where

\[ C_{e} = h_i A_i \]  

\[ C_{12} = \frac{k_{i1} A_{i1}}{l_{i2}} \]

Similar treatment applied to Equations 8 and 9 gives

\[ \alpha C_{a1} \theta_{a1} - \left[ \frac{\alpha (C_{12} + C_{a})}{\delta t} \right] \theta_{a2} + \frac{M_{a1} C_{a1}}{\delta t} \theta_{a1} + \frac{\alpha C_{am} \theta_{am}}{\delta t} = \left[ (1 - \alpha) C_{12} + C_{a} \right] \theta^{*}_{a2} - \frac{M_{a1} C_{a1}}{\delta t} \theta^{*}_{a1} \]  

(12)

and

\[ \alpha C_{a1} \theta_{a2} - \left[ \frac{\alpha (M_{am} C_{pa0})}{\delta t} \right] \theta_{am} - \frac{M_{a1} C_{a1}}{\delta t} \theta_{a1} + \frac{\alpha C_{am} \theta_{am}}{\delta t} = \left[ (1 - \alpha) C_{12} + C_{a} \right] \theta^{*}_{a1} 
- \left[ \frac{M_{a1} C_{a1} C_{pa0}}{\delta t} \right] \theta^{*}_{a0} \]  

(13)

where

\[ C_{as} = h_i A_i \]  

\[ C_{a1} = m_{a1} C_{pa1} + m_{r1} C_{pa1} \]  

\[ C_{a0} = m_{a0} C_{pa0} + m_{r0} C_{pa0} \]

The energy conservation matrix equation for the air duct is then given by

\[ \begin{bmatrix} C(1) & C(2) & 0 & 0 & 0 \\ C(3) & C(4) & C(5) & 0 & 0 \\ 0 & C(6) & C(7) & C(8) & C(11) \\ 0 & 0 & C(9) & C(10) & 0 \end{bmatrix} \begin{bmatrix} \theta_{a1} \\ \theta_{a2} \\ \theta_{am} \\ \theta_{a1}^{*} \\ \theta_{a0}^{*} \end{bmatrix} = \begin{bmatrix} C(12) \\ C(13) \\ C(14) \\ C(15) \end{bmatrix} \]  

(14)

If the airflow velocity is low, for a given simulation time step the distance traveled by the air can be less than the length of the duct section. Then the error introduced by Equation 10 can be significant because it merely equates the leaving air temperature to the mean (well-mixed) air temperature within the duct. For such a case \( \theta_{am} \) and \( \theta_{a1} \) are first evaluated separately via a transport delay subprogram, DELAY, and then substituted back into Equation 14. This subprogram determines the array of temperature distribution along the flow conduit at a future time step based on the temperature distribution at the present time step. Details of the numerical technique have been reported elsewhere (Chow 1997). Accordingly, some of the Equation 14 coefficients are modified:

\[ C(6) = 0, \]  

\[ C(7) = 1, \]  

\[ C(8) = 0, \]  

\[ C(9) = 0, \]  

\[ C(11) = 0, \]  

\[ C(14) = \text{DELAY}(\theta_{am}), \]  

\[ C(15) = \text{DELAY}(\theta_{a1}). \]  

Assuming an incompressible fluid, the flow balance for the duct dry air is given by

\[ m_{a1} = m_{am} = m_{a0}. \]  

(15)

Because condensation may occur when the inside duct surface temperature falls below the dew-point temperature of the duct air, a water vapor flow balance is also required as follows:

\[ m_{w1} = m_{wm} = m_{w0} - Cc \]  

(16)

where \( Cc \) is the rate of moisture condensation. Likewise, humidification of the airstream might occur for wetted interior surface on some other occasions.

Because of the complexity of latent heat transfer calculations, the transport delay calculations are not performed when moisture transfer takes place. While this will adversely affect accuracy in the transient response for small time-step simulations, the problem can be resolved by the use of two or more consecutive duct sections, with the initial shorter section dealing with the condensation and the subsequent longer sections the transport delay.

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TABLE 2
Mass Flow Matrix Coefficients of Air Duct Model

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>First Phase</th>
<th>Second Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C(2)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C(3)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C(4)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C(5)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C(6)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C(7)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C(8)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C(9)</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>C(10)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C(11)</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>C(12)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C(13)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C(14)</td>
<td>0</td>
<td>Cc</td>
</tr>
<tr>
<td>C(15)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

For the first- and second-phase flows, the mass conservation matrices have the same topology as that given in Equation 14. The expressions for the equation coefficients are given in Table 2.

The air duct model overall—comprising a multi-node energy and two-phase flow conservation equation set—can be constructed from 3 PPs:

<table>
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<tr>
<th>PP</th>
<th>Description</th>
<th>Nodes Involved</th>
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<td>Thermal conduction: solid to solid</td>
<td>S1, S2</td>
</tr>
<tr>
<td>2.4</td>
<td>Surface convection: with ambient</td>
<td>S1</td>
</tr>
<tr>
<td>4.1</td>
<td>Flow over surface: for moist air (3 nodes)</td>
<td>S2, AM, A1, A0</td>
</tr>
</tbody>
</table>

The expressions for the matrix equation coefficients, in terms of the constituent primitive part coefficients, are given in Table 3. The PP indices are 2, 2, 1, and 1 for nodes S1, S2, AM, and A1 respectively.

FAN MODEL

Airflow dynamics in fan systems are mainly due to air compressibility and fan motor dynamics (Bourdouxhe and Lebrun 1996). The flow performance of a fan is usually characterized by a performance curve that relates the total pressure rise to a polynomial expression of the mass flow rate at a definite fan speed and air density. Such a pressure rise overcomes the friction and dynamic losses at all other fan-duct system components. In a fluid flow network, the flows are related nonlinearly to the pressures at the nodes and thus require the iterative processing of a set of simultaneous nonlinear equations that are subjected to a given set of boundary conditions. The conservation of mass at each internal node is equivalent to the mathematical statement that the sum of the mass flows is equal to zero at that node. The solution technique used in ESP-r is to assign an arbitrary initial pressure to each internal node and then determine the node pressure correction vector based on a simultaneous solution of a Jacobian matrix, which represents the nodal pressure corrections in terms of all branch flow partial derivatives. The pressures are then iteratively corrected and the mass balance at each internal node is reevaluated until some convergence criterion is met. The fluid flow solver, as described, can work together with the building and plant simulation engine to handle the mass flow simulation for the plant and/or building (Clarke and Hensen 1990).

Figure 5 shows a possible finite volume model for a fan with an integral motor. Nodes S4 and S5 are solid nodes representing the motor/impeller body and fan casing, respectively. Note that the impeller and motor, here treated as a single finite volume, can be disaggregated into two or more nodes if necessary. Node A1 represents the air in contact with the motor-impeller body, node A2 represents the air in contact with the casing, node A3 represents the air at the fan exit, node A0 represents the air exiting the upstream component, and the excitation θ represents the ambient air temperature.

TABLE 3
Development of Insulated Air Duct Model from Primitive Part Coefficients

<table>
<thead>
<tr>
<th>Matrix Coefficients</th>
<th>PP 1.1 Thermal Conduction (Solid to Solid) S1, S2</th>
<th>PP 2.4 Surface Convection (with Ambient) S1, E</th>
<th>PP 4.1 Flow Upon Surface (for Moist Air) S2, A1, AM, A0</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>A(11,1)</td>
<td>+ A(24,1)</td>
<td></td>
</tr>
<tr>
<td>C(2)</td>
<td>A(11,2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(3)</td>
<td>A(11,3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(4)</td>
<td>A(11,4)</td>
<td>+ A(41,1)</td>
<td></td>
</tr>
<tr>
<td>C(5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(6)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(7)</td>
<td></td>
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<tr>
<td>C(8)</td>
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<td></td>
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<tr>
<td>C(9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(11)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(12)</td>
<td>B(11,1)</td>
<td>+ B(24,1)</td>
<td></td>
</tr>
<tr>
<td>C(13)</td>
<td>B(11,2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(14)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(15)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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This model comprises five PPs, with one PP, No. 4.4, used twice:

<table>
<thead>
<tr>
<th>PP</th>
<th>Description</th>
<th>Nodes Involved</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>Surface convection: with ambient</td>
<td>S5</td>
</tr>
<tr>
<td>4.4(1)</td>
<td>Flow upon surface: for moist air</td>
<td>A0, A1, S4</td>
</tr>
<tr>
<td>4.4(2)</td>
<td>Flow upon surface: for moist air</td>
<td>A1, A2, S5</td>
</tr>
<tr>
<td>5.3</td>
<td>Flow inducer (for all fluid)</td>
<td>A3</td>
</tr>
<tr>
<td>10.1</td>
<td>Heat injection: to solid</td>
<td>S4</td>
</tr>
<tr>
<td>10.3</td>
<td>Heat injection: to moist air</td>
<td>A1</td>
</tr>
</tbody>
</table>

The energy imparted to the airstream eventually dissipates as friction, resulting in a downstream temperature rise. This is approximated in the model by including an instantaneous energy pickup at the leaving air node given by

\[ Q_{\text{h}} = \eta_w \]

where \( W \) is the corresponding motor power and \( \eta \) is the fan efficiency. Likewise, the motor inefficiency results in heat generation within the motor-impeller body:

\[ Q_{\text{i}} - \eta = (1 - \eta)W. \]

The energy conservation matrix equation for this fan model is given by

\[

\begin{bmatrix}
C(1) & 0 & 0 & 0 \\
0 & C(2) & 0 & 0 \\
C(5) & 0 & C(6) & 0 \\
0 & C(7) & C(8) & C(9) \\
0 & 0 & 0 & C(10) \\
0 & 0 & 0 & C(11) \\
\end{bmatrix}

= 

\begin{bmatrix}
C(13) \\
C(14) \\
C(15) \\
C(16) \\
C(17) \\
\end{bmatrix}

\]

where, in the case of energy balance,

\[ C(1) = -\alpha h_{14} A_{14} - M_{a4} C_{a4} / \delta t \]
\[ C(2) = -\alpha h_{14} A_{14} \]
\[ C(3) = -\alpha (h_{25} A_{25} + h_{25} A_{25}) - M_{a5} C_{a5} / \delta t \]
\[ C(4) = \alpha h_{25} A_{25} \]
\[ C(5) = \alpha h_{14} A_{14} \]
\[ C(6) = -\alpha (m_{a1} C_{p_{a1}} + m_{a1} C_{p_{a1}} + h_{14} A_{14} - M_{a1} C_{p_{a1}}) / \delta t \]
\[ C(7) = \alpha h_{14} A_{14} \]
\[ C(8) = -\alpha (m_{a2} C_{p_{a2}} + m_{a2} C_{p_{a2}}) \]
\[ C(9) = -\alpha (m_{a2} C_{p_{a2}} + m_{a2} C_{p_{a2}} + h_{25} A_{25} - M_{a2} C_{p_{a2}}) / \delta t \]
\[ C(10) = -1 \]
\[ C(11) = 1 \]
\[ C(12) = \alpha (m_{a2} C_{p_{a2}} + m_{a2} C_{p_{a2}}) \]
\[ C(13) = [(1-\alpha)h_{14} A_{14} - M_{a4} C_{a4} / \delta t] \theta_{a4} - \]
\[ (1-\alpha)h_{14} A_{14} \theta_{a4} - [\alpha H_{14} C_{a4} + (1-\alpha)H_{14} C_{a4}] - [\alpha Q_{14} + \alpha Q_{14}] \]
\[ C(14) = [(1-\alpha)h_{25} A_{25} + h_{25} A_{25} - M_{a5} C_{a5} / \delta t] \theta_{a5} - \]
\[ (1-\alpha)h_{25} A_{25} \theta_{a5} + [\alpha H_{25} C_{a5} + (1-\alpha)H_{25} C_{a5}] - [\alpha Q_{25} + \alpha Q_{25}] \]
\[ C(15) = [(1-\alpha)(m_{a1} C_{p_{a1}} + m_{a1} C_{p_{a1}}) - M_{a1} C_{p_{a1}} / \delta t] \theta_{a1} - (1-\alpha)h_{14} A_{14} \theta_{a4} - (1-\alpha)(m_{a1} C_{p_{a1}} + m_{a1} C_{p_{a1}}) \theta_{a4} \]
\[ C(16) = [(1-\alpha)(m_{a2} C_{p_{a2}} + m_{a2} C_{p_{a2}}) - M_{a2} C_{p_{a2}} / \delta t] \theta_{a2} - (1-\alpha)h_{25} A_{25} \theta_{a5} - (1-\alpha)(m_{a2} C_{p_{a2}} + m_{a2} C_{p_{a2}}) \theta_{a5} \]
\[ C(17) = 0 \]

By a "mix-and-match" procedure, the expressions for the individual coefficients of this matrix equation are obtained in terms of the constituent PPs. These are given in Table 4. The PP indices are 2, 1, 1, 2, and 2 for nodes A1, A2, A3, S4, and S5, respectively.

Note that in this model the air mass flow rate can be either given as an input or treated as a variable. In the latter case, the flow is solved by ESP-r's flow network solver based on the pressure-resistance relationship at each time-step. Note also that in the PP approach, the air nodes A1, A2, and A3 cannot be combined to form one single node even though this is possible within the discretization scheme. If they were combined, then the matrix equation coefficients, when expressed in terms of the PP coefficients, would be misrepresented. Apart from this, the matrix equation template of a plant model constructed from PPs is identical to one built from a conventional finite volume conservation approach.

**SIMULATION EXAMPLES**

The following examples are included to illustrate the PP approach in use and to demonstrate that it is comparable to, if not better than, the conventional component-based modeling approach. It is reckoned that the conventional plant components in ESP-r have been tested via various means, including comparisons with measured data from IEA Annex (Tang 1985). The PP approach can be seen as a reimplementation of these previously validated finite volume component models.
TABLE 4

Development of Fan Model from Primitive Part Coefficients

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>PP2.4 S5</th>
<th>PP4.4(1) S4 A1, A0</th>
<th>PP4.4(2) S5, A2, A1</th>
<th>PP5.3 A3, A2</th>
<th>PP10.1 S4 A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>A(44,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(2)</td>
<td>A(44,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(3)</td>
<td>A(44,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(4)</td>
<td>A(44,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(5)</td>
<td>A(44,3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(6)</td>
<td>A(44,4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(7)</td>
<td>A(44,3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(8)</td>
<td>A(44,6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(9)</td>
<td>A(44,4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(10)</td>
<td>A(53,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(11)</td>
<td>A(53,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(12)</td>
<td>A(44,6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(13)</td>
<td>B(44,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(14)</td>
<td>B(44,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(15)</td>
<td>B(44,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(16)</td>
<td>+ B(44,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(17)</td>
<td>B(53,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A match of simulation results serves to verify the validity of the PP approach used in the construction of finite volume component models.

Fan-Duct System

Figure 6 details a fan-duct system serving an industrial plant. The system draws in outside air to the plant room at a constant volume flow rate. The required operating states were chosen according to practical design standards. The industrial plant operates on a 24-hour basis and gives rise to a sinusoidal heat load that varies between 10 and 20 kW within a period of 4 hours. The objective of the simulation is to determine the daily variation in plant room temperature under warm ambient conditions.

Two separate simulation runs were performed, one utilizing conventional finite volume component models for the fan and duct components (ESP-r models 30 and 60, respectively) and the other utilizing component models built from PPs (models 560 and 580, respectively). The time step for both simulations was two minutes, which yielded consistently stable solutions. Figure 7 shows the predicted daily variation

Figure 6  Supply ventilation system network.

Figure 7  Comparison of daily temperature variations.
of supply and plant room temperatures. As can be seen, the results are almost identical.

Air-Conditioning System

Figure 8 shows the configuration of a cooling-only CAV system, with a remote AHU plant delivering supply air at a constant flow rate of 2.5 kg/s, resulting in a positive pressure inside the air-conditioned space. The return and outdoor airstreams mix at a fixed ratio. Chilled-water flow is proportionally controlled by a room thermostat set at 24°C. The objective of the simulation is to predict, for a warm condition, the transient response of room air temperature during plant start-up at 30°C and under a steady thermal load.

Two separate simulation runs were performed, one using HVACSIM+, a component modular simulation program (Clark 1985), and the other, the ESP-r program with the plant components built from the PP models. In the latter case, the cooling coil was represented by a 16-node explicit model comprising four interconnected air-to-water heat-transfer tubes in counterflow arrangement (Chow 1995b). Each heat-transfer tube is a four-node model. A time-step of one second was used. Figure 9 compares the simulation results. As can be seen, the predictions are in line.

CONCLUSIONS

Primitive part (PP) modeling provides a means to improve and control the integrity of component models. Plant component models developed from PPs are fundamental by nature, with explicit physical meaning and a clear mathematical structure. This paper has explained the theoretical basis of the PP approach in terms of fan and air duct models. The mathematical models that result have been shown to be equivalent to the corresponding models developed on the basis of formal conservation principles applied to finite volumes and to produce agreement with results from the HVACSIM+ program. The principal benefit of the approach lies in its flexibility and the prospect it affords for the synthesis of component models based on physical component descriptions.

REFERENCES


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 NOMENCLATURE

\[
\begin{align*}
A & = \text{area} \\
C & = \text{specific heat capacity} \\
Cc & = \text{moisture condensation rate} \\
Cp & = \text{specific heat at constant pressure} \\
g & = \text{moisture content} \\
H & = \text{specific enthapy} \\
h & = \text{heat transfer coefficient} \\
k & = \text{thermal conductivity} \\
Kd & = \text{mass diffusion conductivity} \\
l & = \text{length; spacing} \\
M & = \text{mass} \\
m & = \text{mass flow rate} \\
N_i & = \text{primitive part connection index} \\
Q & = \text{heat flux} \\
S & = \text{solid} \\
t & = \text{time} \\
U & = \text{overall heat transfer coefficient} \\
W & = \text{power} \\
x & = \text{dryness fraction} \\
\alpha & = \text{weighting factor} \\
\eta & = \text{efficiency} \\
\theta & = \text{temperature}
\end{align*}
\]

SUPERSCRIPT

\( * \) = present time row

SUBSCRIPTS

\( a \) = air (dry) \\
\( dew \) = dew point \\
\( e \) = ambient \\
\( fg \) = latent exchange \\
\( i \) = internal \\
\( m \) = mean \\
\( ma \) = moist air \\
\( o \) = external \\
\( s \) = solid \\
\( sat \) = saturated \\
\( v \) = water vapour \\
\( w \) = liquid water or wet

APPENDIX

Part No. 1.1 Thermal Conduction (Solid to Solid)

2 No. of Nodes:

\[
\begin{array}{c}
\begin{array}{c}
M
\end{array} \\
\begin{array}{c}
N
\end{array}
\end{array}
\]

Energy flow matrix:

\[
\begin{align*}
[A(11,1) & A(11,2)] \begin{bmatrix} \theta_m \end{bmatrix} = [B(11,1)] \\
[A(11,3) & A(11,4)] \begin{bmatrix} \theta_n \end{bmatrix} = [B(11,2)]
\end{align*}
\]

where

\begin{align*}
A(11,1) &= -\alpha C_{mn} - M_m C_m (N_i m \Delta t) \\
A(11,2) &= \alpha C_{mn} \\
A(11,3) &= \alpha C_{mn} \\
A(11,4) &= -\alpha C_{mn} - M_n C_n (N_i n \Delta t) \\
B(11,1) &= [(1-\alpha) C_{mn}^* - M_m^* C_m^* (N_i m \Delta t) \theta_m^* - (1-\alpha) C_{mn}^* \theta_n^*] \\
B(11,2) &= [(1-\alpha) C_{mn}^* - M_n^* C_n^* (N_i n \Delta t) \theta_n^* - (1-\alpha) C_{mn}^* \theta_n^*]
\end{align*}

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Mass flow matrices (for both first- and second-phase mass balances):

\[
\begin{align*}
A(11,1) &= 1/\dot{N}_i \\
A(11,2) &= 0 \\
A(11,3) &= 0 \\
A(11,4) &= 1/\dot{N}_o \\
B(11,1) &= 0 \\
B(11,2) &= 0.
\end{align*}
\]

Part No. 2.4 Surface Convection (with Ambient)

1 No. of Node:
\( S \) - solid (or fluid)
\( E \) - boundary fluid

Energy flow matrix:

\[
[A(24,1)] \cdot \begin{bmatrix} \theta_s \end{bmatrix} = [B(24,1)]
\]

where

\[
A(24,1) = -\alpha C_{\text{st}} - C_f/(\dot{N}_i \cdot \delta t)
\]

\[
B(24,1) = [(1-\alpha) C_{\text{st}} \theta_s - M_s \cdot C_p \cdot \theta_s \cdot (\dot{N}_i \cdot \delta t)] \theta_s - \alpha C_{\text{st}} \theta_s + (1-\alpha) C_{\text{st}} \theta_s^*.
\]

Part No. 2.1 Surface Convection (with Moist Air)

2 Nos. of Nodes:
\( S \) - solid
\( A \) - moist air

Energy flow matrix:

\[
\begin{bmatrix} A(21,1) & A(21,2) \\ A(21,3) & A(21,4) \end{bmatrix} \begin{bmatrix} \theta_s \\ \theta_a \end{bmatrix} = \begin{bmatrix} B(21,1) \\ B(21,2) \end{bmatrix}
\]

where

\[
A(21,1) = -\alpha C_{\text{st}} - M_s \cdot C_f/(\dot{N}_i \cdot \delta t)
\]

\[
A(21,2) = \alpha C_{\text{st}}
\]

\[
A(21,3) = \alpha C_{\text{st}}
\]

\[
A(21,4) = -\alpha C_{\text{st}} - M_s \cdot C_{\text{pm}}/(\dot{N}_i \cdot \delta t)
\]

\[
B(21,1) = [(1-\alpha) C_{\text{st}} \theta_s - M_s \cdot C_p \cdot \theta_s \cdot (\dot{N}_i \cdot \delta t)] \theta_s - \alpha C_{\text{st}} \theta_s + (1-\alpha) C_{\text{st}} \theta_s^* + [(1-\alpha) C_{\text{fg}} \theta_s^*] - \alpha C_{\text{st}} \theta_s^* + (1-\alpha) C_{\text{st}} \theta_s^*
\]

\[
B(21,2) = [(1-\alpha) C_{\text{st}} \theta_s - M_s \cdot C_{\text{pm}} \cdot \theta_s \cdot (\dot{N}_i \cdot \delta t)] \theta_s - \alpha C_{\text{st}} \theta_s^* + (1-\alpha) C_{\text{st}} \theta_s^*.
\]

and

\[
C_{\text{fg}} = -H_{\text{fg}} C_f.
\]

Mass flow matrices:

First-phase (dry air)  |  Second-phase (water vapor)
\[
\begin{align*}
A(21,1) &= 1/\dot{N}_i \\
A(21,2) &= 0 \\
A(21,3) &= 0 \\
A(21,4) &= 1/\dot{N}_o \\
B(21,1) &= 0 \\
B(21,2) &= 0
\end{align*}
\]

and

\[
C_c = \begin{cases} 0 & \text{for dry surface, i.e.,} \theta_s = 0, \theta_w = 0; \\
K_d A_{\text{st}} (g_s - g_d) & \text{for wet surface, i.e.,} \dot{M}_w > 0 \
\end{cases}
\]

where \( \dot{M}_w \) is the amount of condensate on the surface, \( g_s \) is the saturated moist content at \( \theta_s \).

Part No. 3.1 Surface Radiation (with Local Surface)

2 Nos. of Nodes:
\( S \) - solid surface
\( T \) - solid surface

Energy flow matrix:

\[
\begin{bmatrix} A(31,1) & A(31,2) \\ A(31,3) & A(31,4) \end{bmatrix} \begin{bmatrix} \theta_s \\ \theta_t \end{bmatrix} = \begin{bmatrix} B(31,1) \\ B(31,2) \end{bmatrix}
\]

where

\[
A(31,1) = -\alpha C_{\text{st}} - C_f/(\dot{N}_i \cdot \delta t)
\]

\[
A(31,2) = \alpha C_{\text{st}}
\]

\[
A(31,3) = \alpha C_{\text{st}}
\]

\[
A(31,4) = -\alpha C_{\text{st}} - C_f/(\dot{N}_i \cdot \delta t)
\]

\[
B(31,1) = [(1-\alpha) C_{\text{st}} \theta_s - M_s \cdot C_p \cdot \theta_s \cdot (\dot{N}_i \cdot \delta t)] \theta_s - \alpha C_{\text{st}} \theta_s^* + (1-\alpha) C_{\text{st}} \theta_s^*.
\]

\[
B(31,2) = [(1-\alpha) C_{\text{st}} \theta_s - M_s \cdot C_{\text{pm}} \cdot \theta_s \cdot (\dot{N}_i \cdot \delta t)] \theta_s - \alpha C_{\text{st}} \theta_s^* + (1-\alpha) C_{\text{st}} \theta_s^*.
\]

Mass flow matrices (for both first- and second-phase):

\[
\begin{align*}
A(31,1) &= 1/\dot{N}_i \\
A(31,2) &= 0 \\
A(31,3) &= 0 \\
A(31,4) &= 1/\dot{N}_o \\
B(31,1) &= 0 \\
B(31,2) &= 0
\end{align*}
\]
Part No. 4.1 Flow Upon Surface (for Moist Air; 3 Nodes)

3 Nos. of Nodes:
- S - solid surface
- AM - moist air in contact
- A1 - leaving moist air

1 No. of Connection:
- A0 - incoming moist air

Energy flow matrix:

\[
\begin{bmatrix}
A(41,1) & A(41,2) & 0 \\
A(41,4) & A(41,5) & A(41,6) \\
0 & A(41,8) & A(41,9)
\end{bmatrix}
\begin{bmatrix}
\theta_{a_0} \\
\theta_{am} \\
\theta_{a_1}
\end{bmatrix}
= \begin{bmatrix}
B(41,1) \\
B(41,2) \\
B(41,3)
\end{bmatrix}
\]

where

\[
A(41,1) = -\alpha C_{as} - (M_s C_f/N_i_0) \delta_t
\]

\[
A(41,2) = \alpha C_{as}
\]

\[
A(41,4) = \alpha C_{as}
\]

\[
A(41,5) = -\alpha C_{as} - M_a C_{pm}/N_i_{am} \delta_t
\]

\[
A(41,6) = -\alpha C_{a_1}
\]

\[
A(41,8) = 1
\]

\[
A(41,9) = 1
\]

\[
A(41,11) = \alpha C_{a_0}
\]

\[
A(41,12) = 0
\]

\[
B(41,1) = [(1-\alpha) C_{as} - M_s C_f/N_i_0] \delta_t + \alpha C_{as} + (1-\alpha) C_{as}
\]

\[
B(41,2) = [(1-\alpha) C_{as} - M_a C_{pm}/N_i_{am}] \delta_t
\]

\[
B(41,3) = 0
\]

and

\[
C_{fs} = -H_{fs} (m_{a_0} - m_{a_1}) = -H_{fs} C_c
\]

For dry surface, \( m_{a_1} = m_{a_0} \), so \( C_{fs} = 0 \).

In case the DELAY flag is ON, the following coefficients will be revised as

\[
A(41,4) = 0
\]

\[
A(41,5) = 1
\]

\[
A(41,6) = 0
\]

\[
A(41,8) = 0
\]

\[
A(41,9) = 0
\]

\[
A(41,11) = 0
\]

\[
B(41,2) = \text{DELAY}(\theta_{am})
\]

\[
B(41,3) = \text{DELAY}(\theta_{a_1})
\]

Mass flow matrices:

- First-phase (dry air): \( 1/N_i_s \)
- Second-phase (water vapor): \( 1/N_i_r \)

- First-phase (dry air): \( 0 \)
- Second-phase (water vapor): \( 0 \)

\[
A(41,5) = 1/N_i_{am}
\]

\[
A(41,6) = 0
\]

\[
A(41,8) = -1
\]

\[
A(41,9) = 1
\]

\[
A(41,11) = -1
\]

\[
B(41,1) = 0
\]

\[
B(41,2) = 0
\]

\[
B(41,3) = 0
\]

\[
C_c = 0
\]

for dry surface, i.e.,

\[
\theta_s = \theta_{a_0,\text{sw}} \text{ and } M_{a_0} = 0;
\]

\[
= K_d A_{as} (g_{am} - g_s) \text{ for wet surface, i.e., } M_{a_0} > 0
\]

where \( M_{a_0} \) is the amount of condensate on the surface,

\[
g_s \text{ is the saturated moist content at } \theta_s.
\]

Part No. 4.4 Flow Upon Surface (for Moist Air; 2 Nodes)

2 Nos. of Nodes:
- S - solid surface
- A1 - leaving moist air

1 No. of Connection:
- A0 - incoming moist air

Energy flow matrix:

\[
\begin{bmatrix}
A(44,1) & A(44,2) & 0 \\
A(44,3) & A(44,4) & A(44,6)
\end{bmatrix}
\begin{bmatrix}
\theta_{a_1} \\
\theta_{a_0}
\end{bmatrix}
= \begin{bmatrix}
B(44,1) \\
B(44,2)
\end{bmatrix}
\]

where

\[
A(44,1) = -\alpha C_{as} - M_s C_f/N_i_0 \delta_t
\]

\[
A(44,2) = \alpha C_{as}
\]

\[
A(44,3) = \alpha C_{as}
\]

\[
A(44,4) = -\alpha (C_{a_1} + C_{as}) - (M_a C_{pm}/N_i_{am}) \delta_t
\]

\[
A(44,6) = \alpha C_{a_0}
\]

\[
B(44,1) = [(1-\alpha) C_{as} - M_s C_f/N_i_0] \delta_t + \alpha C_{as} + [(1-\alpha) C_{as}]^*
\]

\[
B(44,2) = [(1-\alpha)(C_{a_1} + C_{as}) - M_a C_{pm}/(N_i_{am}) \delta_t] \theta_{a_0}^* - (1-\alpha) C_{a_0} \theta_{a_0}^*
\]

and

\[
C_{fs} = -H_{fs} (m_{a_0} - m_{a_1}) = -H_{fs} C_c
\]

For dry surface, \( m_{a_1} = m_{a_0} \), so \( C_{fs} = 0 \).

Mass flow matrices:

- First-phase (dry air): \( 1/N_i_s \)
- Second-phase (water vapor): \( 1/N_i_r \)

- First-phase (dry air): \( 0 \)
- Second-phase (water vapor): \( 0 \)

\[
A(44,1) = 1/N_i_s
\]

\[
A(44,2) = 0
\]

\[
A(44,3) = 0
\]

\[
A(44,4) = 1/N_i_{am}
\]

\[
A(44,5) = 1/N_i_{am}
\]

\[
A(44,6) = 0
\]

\[
A(44,8) = -1
\]

\[
A(44,9) = 1
\]

\[
A(44,11) = -1
\]

\[
B(44,1) = 0
\]

\[
B(44,2) = 0
\]

\[
B(44,3) = 0
\]

\[
C_c = 0
\]

for dry surface, i.e.,

\[
\theta_s = \theta_{a_0,\text{sw}} \text{ and } M_{a_0} = 0;
\]

\[
= K_d A_{as} (g_{am} - g_s) \text{ for wet surface, i.e., } M_{a_0} > 0
\]

where \( M_{a_0} \) is the amount of condensate on the surface,

\[
g_s \text{ is the saturated moist content at } \theta_s.
\]
\[ A(44.6) = -1 \]
\[ B(44.1) = 0 \]
\[ B(44.2) = 0 - C_c \]
where
\[ C_c = 0 \]
for dry surface, i.e., when \( \theta_i > \theta_{\text{dew}} \) and \( M_w = 0 \);
\[ = K_d A_{\text{at}} (e_{\text{at}} - g_s) \]
for wet surface, i.e., \( M_w > 0 \)
where \( M_w \) is the amount of condensate on the surface, \( g_s \) is the saturated moist content at \( \theta_i \).

**Part No. 5.3 Flow Inducer (for All Fluid Types)**

1 Nos. of Nodes:
- R1 - leaving fluid
1 No. of Connection:
- R0 - incoming fluid

Energy flow matrix:

\[
[A(53,1) \ A(53,2)] \cdot \begin{bmatrix} \theta_{R1} \\ \theta_{R0} \end{bmatrix} = [B(53,1)]
\]

where
\[ A(53,1) = 1 \]
\[ A(53,2) = -1 \]
\[ B(53,1) = 0. \]

Mass flow matrices:

<table>
<thead>
<tr>
<th>First phase</th>
<th>Second phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A(53,1) = 1 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( A(53,2) = 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( B(53,1) = k_1 m_i )</td>
<td>( k_2 m_i )</td>
</tr>
</tbody>
</table>

Note:

1. \( m_i \) is the induced mass flow rate.
2. The values of \( k_1 \) and \( k_2 \) are fluid-type dependent such that:

<table>
<thead>
<tr>
<th>Fluid-type</th>
<th>Moist Air</th>
<th>Liquid</th>
<th>Wet Vapor</th>
<th>Dry Vapor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>1</td>
<td>1</td>
<td>1-( \lambda )</td>
<td>0</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>( g_{w0} )</td>
<td>0</td>
<td>( x )</td>
<td>1</td>
</tr>
</tbody>
</table>

where \( g_{w0} = m_{w0}/m_{w} \) and \( x = m_{v}/(m_{w0} + m_{v}) \).

**Part No. 10.1 Heat Injection (to Solid)**

1 No. of Node:
- S - solid
1 No. of Excitation:
- Q - heat input

Energy flow matrix:

\[
[A(101,1)] \cdot \begin{bmatrix} \theta_s \end{bmatrix} = [B(101.1)]
\]

where
\[ A(101.1) = -M_s \cdot C_p/(N_{i_s} \cdot \delta t) \]
\[ B(101.1) = -[M_s \cdot C_p \cdot (N_{i_s} \cdot \delta t)] \theta_s - [\alpha Q + (1-\alpha)Q^*]. \]

Mass flow matrices (for both first- and second-phase):
\[ A(101,1) = 1/N_{i_s} \]
\[ B(101,1) = 0. \]

**Part No. 10.3 Heat Injection (to Moist Air)**

1 No. of Node:
- A - moist air
1 No. of Excitation:
- Q - heat input

Energy flow matrix:

\[
[A(103,1)] \cdot \begin{bmatrix} \theta_a \end{bmatrix} = [B(103.1)]
\]

where
\[ A(103.1) = -[\alpha C_a + (1-\alpha)C_a^*]/(N_{i_a} \cdot \delta t) \]
\[ B(103.1) = -[\alpha C_a + (1-\alpha)C_a^*] \theta_a + (N_{i_a} \cdot \delta t) - [\alpha Q + (1-\alpha)Q^*] - [\alpha H_{fg} \cdot C_c + (1-\alpha)H_{fg}^* \cdot C_c^*] \]

and
\[ C_c = \text{condensation rate} = (M_a^* - M_a)/\delta t, \]
\[ C_a = M_a \cdot C_{p_a} + M_v \cdot C_{p_v}. \]

Note: Check \( g_a = M_a / M_a^* \) against \( g_a^* \) at each time step;
if \( g_a > g_a^* \), then \( M_a = M_a^* \cdot g_a^* \).

Mass flow matrices (for both first- and second-phase):

\[ A(103,1) = 1/N_{i_a} \]
\[ B(103,1) = 0. \]

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