

# ESRU

## Technical Report



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### ESRU TRE 150/01

## Development of a Heated Water Storage Plant Component Model

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## Summary

The Energy Systems Research Unit has undertaken the development of a heated water storage unit (HWS) component for use in an advanced integrated mechanical system (AIMS) model. This process has involved:

- the development of a mathematical model of the component;
- the testing of this component;
- the modelling of the AIMS system using an ESP-r plant network (incorporating the developed component model).

Further, the AIMS system has been combined with a model of a typical Canadian building, forming an integrated building and HVAC model.

## AIMS System

An AIMS system combines the functions of domestic hot water storage and supply with mechanical ventilation and warm air heating. The basic layout of the system is shown below:

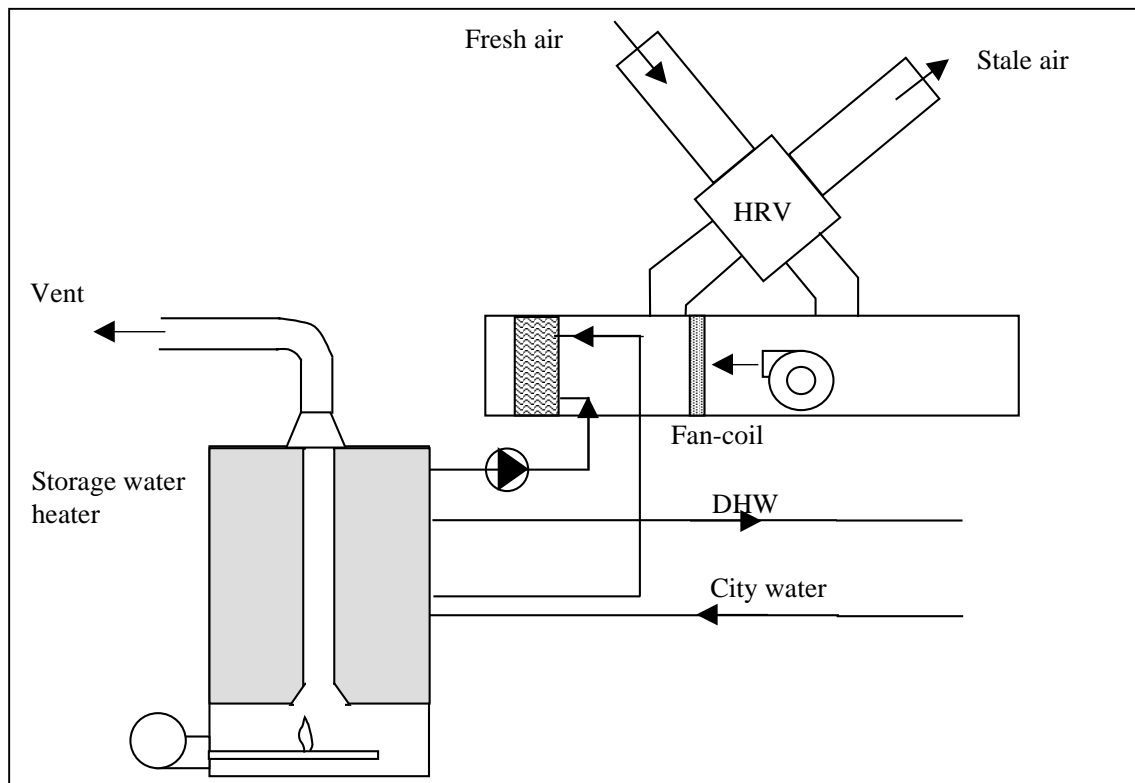


Figure 1: the aims system (from [1])

## Developing an HWS Component: Theory

ESP-r uses one or more control volumes to describe any physical entity: a control volume being a homogeneous region of space, to which the principles of conservation of mass, energy and momentum can be applied. In the case of the heated water storage unit, the model was developed using three control volumes (indicated by a node), these represent the main physical regions of the component:

- a massive volume, representing the stored water and casing;
- a volume representing the combustion chamber;
- a volume representing the flue, from which the heat is transferred to the stored water.

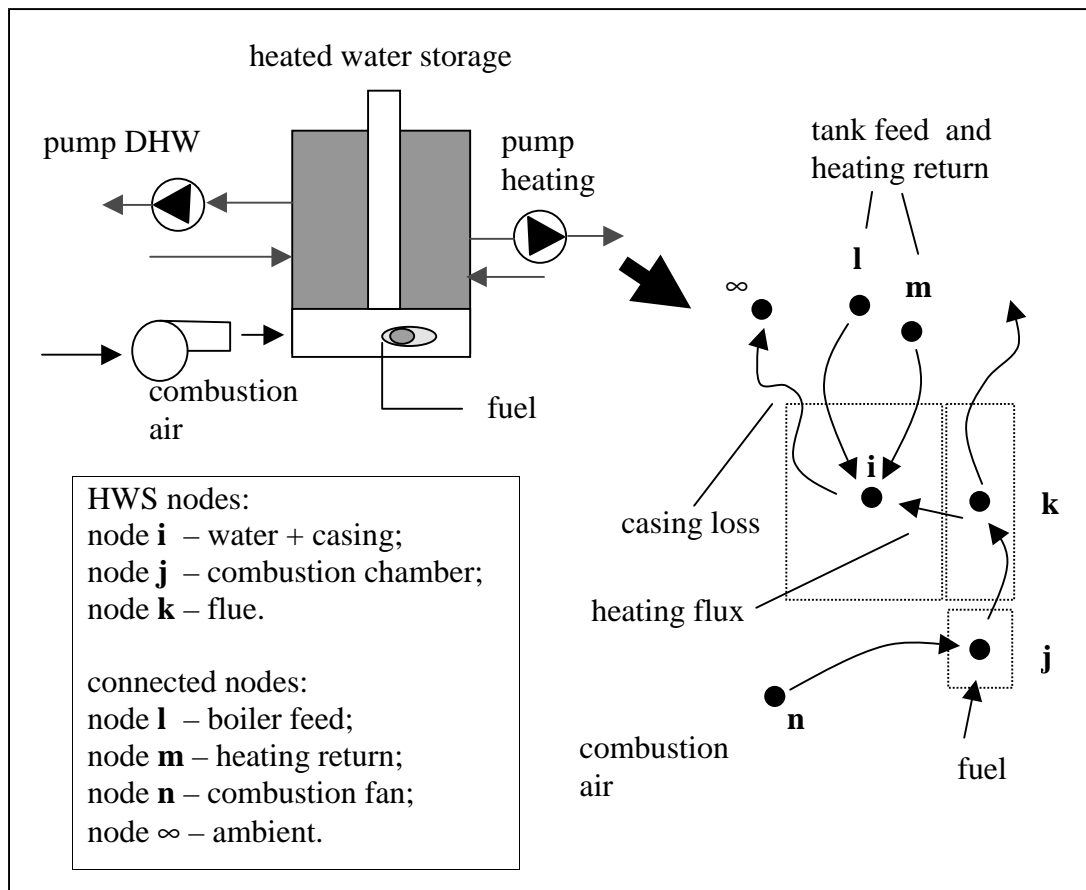


Figure 2: schematic and control volume representation of the HWS component

Each of the component's control volumes can be subjected to an energy and mass balance; these form the basis for the analysis of the component.

## Energy Balances

The three control volumes which constitute the HWS component are indicated by nodes  $i$ ,  $j$  and  $k$  (figure 2). The basic heat transfers occurring within the water and casing, node  $i$  are: heat loss to the environment, heat gain from the combustion gases, cold water make-up for domestic hot water consumption, return flow from the heating system and heat storage. Note that the properties (e.g. temperature) of a single control volume are homogeneous and only flows *into* it need be considered in the energy balances.

The energy balance for the water and casing (node  $i$ ) can be represented by the following partial differential expression:

$$UA_i(\theta_\infty - \theta_i) + \dot{m}_l c_l (\theta_l - \theta_i) + \dot{m}_m c_m (\theta_m - \theta_i) + \phi_{k,i} = M_i c_i \frac{\partial \theta_i}{\partial t} \quad (1)$$

The right hand side represents the heat storage of the water and casing, the terms on the left hand side relate to: the heat loss to the environment, ( $UA_i$  is the global heat loss coefficient (W/K) for the casing,  $\theta_\infty$  is the environment temperature ( $^{\circ}\text{C}$ ) ) and the loads on the tank (tank feed  $l$  and heating return  $m$ ). The  $\theta_x$  terms refer to the nodal temperatures ( $^{\circ}\text{C}$ );  $\dot{m}_x$  are nodal mass flow rates (kg/s);  $c_x$  are nodal specific heats (J/kgK). The  $\phi$  (W) term is the quantity of heat transferred to the water from the flue gases (node  $k$ )<sup>1</sup>.

The equation can be made linear by replacing the differential term by a forward difference expression. The explicit, linearised energy balance for some time  $t$  is:

$$UA_i(\theta_\infty^t - \theta_i^t) + \dot{m}_l^t c_l^t (\theta_l^t - \theta_i^t) + \dot{m}_m^t c_m^t (\theta_m^t - \theta_i^t) + \phi_{k,i}^t = M_i c_i \left[ \frac{\theta_i^{t+\Delta t} - \theta_i^t}{\Delta t} \right] \quad (2a)$$

Here  $\Delta t$  is the simulation time step(s).

The implicit expression at some time  $t+\Delta t$  is:

$$\begin{aligned} & UA_i(\theta_\infty^{t+\Delta t} - \theta_i^{t+\Delta t}) + \dot{m}_l^{t+\Delta t} c_l^{t+\Delta t} (\theta_l^{t+\Delta t} - \theta_i^{t+\Delta t}) + \dot{m}_m^{t+\Delta t} c_m^{t+\Delta t} (\theta_m^{t+\Delta t} - \theta_i^{t+\Delta t}) + \phi_{k,i}^{t+\Delta t} \\ & = M_i c_i \left[ \frac{\theta_i^{t+\Delta t} - \theta_i^t}{\Delta t} \right] \end{aligned} \quad (2b)$$

Multiplying the implicit expression by  $\alpha$  ( $0 < \alpha < 1$ ), the explicit expression by  $(1 - \alpha)$  and adding the two resulting expressions gives:

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<sup>1</sup> Currently this is calculated using a fixed efficiency; however the model can easily be extended if more detailed information regarding the heat and mass transfer processes within the flue becomes available (discussed later).

$$\begin{aligned}
& \left[ -\alpha(\dot{m}_l^{t+\Delta t} c_l^{t+\Delta t} + \dot{m}_m^{t+\Delta t} c_m^{t+\Delta t} + UA_i) - \frac{M_i c_i}{\Delta t} \right] \theta_i^{t+\Delta t} + \alpha \dot{m}_l^{t+\Delta t} c_l^{t+\Delta t} \theta_l^{t+\Delta t} + \alpha \dot{m}_m^{t+\Delta t} c_m^{t+\Delta t} \theta_m^{t+\Delta t} \\
& = \left[ (1-\alpha)(\dot{m}_l^t c_l^t + \dot{m}_m^t c_m^t + UA_i) - \frac{M_i c_i}{\Delta t} \right] \theta_i^t - (1-\alpha) \dot{m}_l^t c_l^t \theta_l^t - (1-\alpha) \dot{m}_m^t c_m^t \theta_m^t \\
& - (1-\alpha) UA_i \theta_\infty^t - \alpha UA_i \theta_\infty^{t+\Delta t} - (1-\alpha) \phi_{k,i}^t - \alpha \phi_{k,i}^{t+\Delta t}
\end{aligned} \tag{3}$$

The equation above is suitable for use in an overall matrix solution of the plant network containing the HWS model. In matrix form the coefficients which multiply nodal temperatures  $i, j$  or  $k$  are termed nodal *self-coupling coefficients* those associated with connected nodes  $l$  and  $m$  are *cross-coupling coefficients*, while those on the right-hand-side are *right-hand-side coefficients* made up of known and present time step terms.

The equations for the remaining control volumes are far simpler: the assumption is made that these nodes are effectively massless compared to the storage tank node. These equations therefore have no partial differential storage term associated with them.

The equation for the combustion chamber (node  $j$ ) is as follows:

$$(\dot{m}_n^t c_n^t \theta_n^t)_{air} + (\dot{m}_n^t c_n^t \theta_n^t + \dot{m}_n^t h_{fg})_{vap} - \dot{m}_j^t c_j^t \theta_j^t + \dot{m}_{(fuel)}^t HHV^t = 0 \tag{4}$$

$\dot{m}_{(fuel)}$  is the flow rate of fuel to the combustion chamber (kg), HHV is the fuel higher heating value (J/kg) and  $h_{fg}$  is the latent heat of vaporization of water (J/kgK). The flow of fuel into the component can be controlled.

The flow rate of combustion air (from connected node  $n$ ) into the combustion chamber is given by:

$$\dot{m}_n^t = \dot{m}_{(fuel)}^t \mu \lambda \tag{5}$$

Here  $\mu$  the stoichiometric air fuel ratio, while  $\lambda$  the excess air ratio. Rearranging equation (4) gives

$$-\dot{m}_j^t c_j^t \theta_j^t + [(\dot{m}_n^t c_n^t)_{air} + (\dot{m}_n^t c_n^t)_{vap}] \theta_n^t = -\dot{m}_{(fuel)}^t HHV^t - (\dot{m}_n^t h_{fg})_{vap} \tag{6}$$

Essentially this equation equates the enthalpy flow of the combustion products to the incoming fuel and air enthalpy streams plus the fuel heat release: (adiabatic combustion is assumed).

Similarly for the flue gas node  $k$ :

$$\dot{m}_k^t c_k^t \theta_k^t - \dot{m}_j^t c_j^t \theta_j^t - \phi_{k,i}^t = 0 \quad (7)$$

$\phi$  (W) is the heat transferred to the hot water from the combustion gases. The heat transferred to the water is a fixed fraction (i.e. a fixed efficiency) of the heat flux (W) entering the flue from the combustion chamber. The efficiency term (also incorporating the combustion efficiency) can be replaced by a more accurate equation (e.g. calculation of the efficiency as a function of load) at a later date. The term  $\dot{m}_j^t c_j^t \theta_j^t$  refers to the flow of combustion products from node  $j$ .

Rearranging (7) gives

$$-\dot{m}_j^t c_j^t \theta_j^t + \dot{m}_k^t c_k^t \theta_k^t = \phi_{k,i}^t \quad (8)$$

The three equations above are arranged in matrix form (along with all the other equations for the control volumes in a plant network) and solved simultaneously for temperature  $\theta$  at each simulation time step (see figure 3).

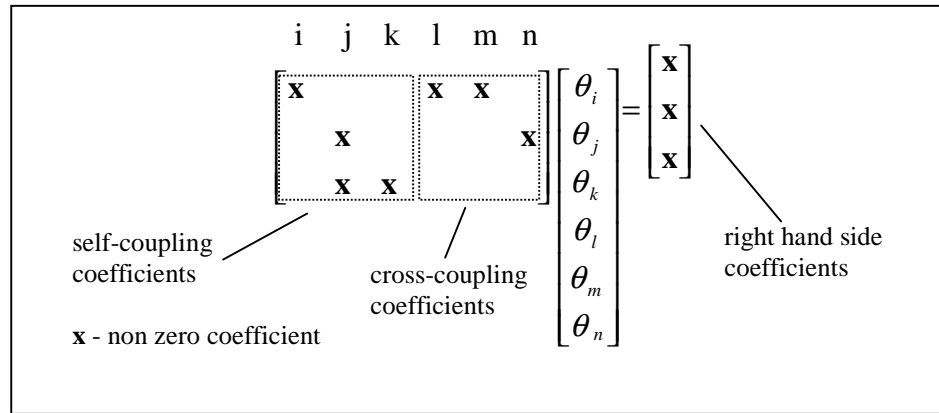


Figure 3 from of the partial matrix for HWS and connected components

### Flow Balances

The equations for flow through each component (using the existing plant flow regime) are as follows.

The water and casing node  $i$ :

$$\dot{m}_i^t + \dot{m}_m^t + \dot{m}_i^t = 0 \quad (9)$$

$\dot{m}_i^t$  is the flow of water out of node  $i$  to the supplied loads (DHW and heating),  $\dot{m}_i^t$  and  $\dot{m}_m^t$  are the cold water feed and heating return respectively.

The combustion chamber node  $j$ :

$$\dot{m}_n^t + \dot{m}_{(fuel)}^t + \dot{m}_j^t = 0 \quad (10)$$

$\dot{m}_j^t$  is the flow of gas out of node  $j$  to the flue node  $k$ ,  $\dot{m}_n^t$  is the flow of combustion air and  $\dot{m}_{(fuel)}^t$  is the flow of fuel.

The flue gas node  $k$ :

$$\dot{m}_j^t + \dot{m}_k^t = 0 \quad (11)$$

$\dot{m}_k^t$  is the flow of flue gas out of node  $k$  to ambient,  $\dot{m}_j^t$  is as previously described.

### Developing an HWS Model: Implementation in ESP-r

The implementation of the model in ESP-r involved several stages of development:

1. development of a plant component database entry;
2. development of a coefficient generator;
3. development of a component static template.

### Plant Component Database Entry

ESP-r has a plant component database entry for each type of component it can support. The HWS model therefore also required such an entry. Each entry holds the information necessary for the analysis of the plant component: component parameters, the type of each control volume (solid, water or moist air) and the component connectivity information (how many connections can be made to each node). In the case of the HWS component the parameters are as follows:

ADATA variables:

1	Component total mass (kg)
2	Average specific heat of water and casing (J/kgK)
3	UA modulus (node i to surroundings) (W/K)

BDATA variables:

1	Nominal burner capacity when ON (W)
2	Nominal burner capacity when OFF (standby) (W)
3	UAx value for heat transfer to water (W/K)
4	Fuel heating value @ nominal AF ratio (J/kg fuel)
5	Stoichiometric air/fuel mass ratio (-)
6	Excess air (%)

7	Fuel specific heat capacity (J/kgK)
8	Nominal combustion efficiency (%)
9	Nominal specific heat of combustion gases (J/kgK)
10	Mass of water in the combustion gas per kg fuel (kg/kg a)

Each parameter can be altered as required by the user (e.g. if a different fuel is used or a bigger tank is required). The database entry for the component is contained in /usr/esru/databases/plantc.db1; the entry can be viewed using ESP-r's plant database management utility.

### **Coefficient Generator**

The function of the coefficient generator is to calculate the self-coupling, cross-coupling and right-hand-side coefficients associated with each control volume of a plant component model. These are calculated using the parameters read from the database, combined with the heat balance equations embedded within the coefficient generator, enabling the determination of temperature dependent properties and inter-nodal heat and mass exchanges. The coefficients calculated for each component are later used in the simultaneous energy solution of the plant network.

In a similar fashion, coefficient generator also calculates the coefficients associated with the component nodal mass balance equations. As with the energy equation coefficients, these are used in the simultaneous solution of the plant network fluid flows. The coefficient generator for the hot water heating component is contained within esruplt/pcomp2.F and is named CMP34C

### **Static Template**

The static template checks if the component has a) the correct number of data items b) the correct number of connections to it and c) if the nodes within the component are connected to appropriate nodes in connected components e.g. that a water node is connected to another water node. The static template is contained within esruplt/pcomps.F and is named CMP34S

### **System Model**

The development of the HWS component enables the whole AIMS system to be modeled using a network of ESP-r plant components. The core AIMS system model comprises 13 individual components linked together as shown in figure 4. If this network model is compared with the systems diagram of figure 1 it can be seen that all the functionality of the original system is present: hot water heating, hot water supply, heat recovery and warm air heating. Note that the developed heated water storage component model supplies both a heating coil and the hot water demand of the building. A connected fan supplies the combustion air for the same component.



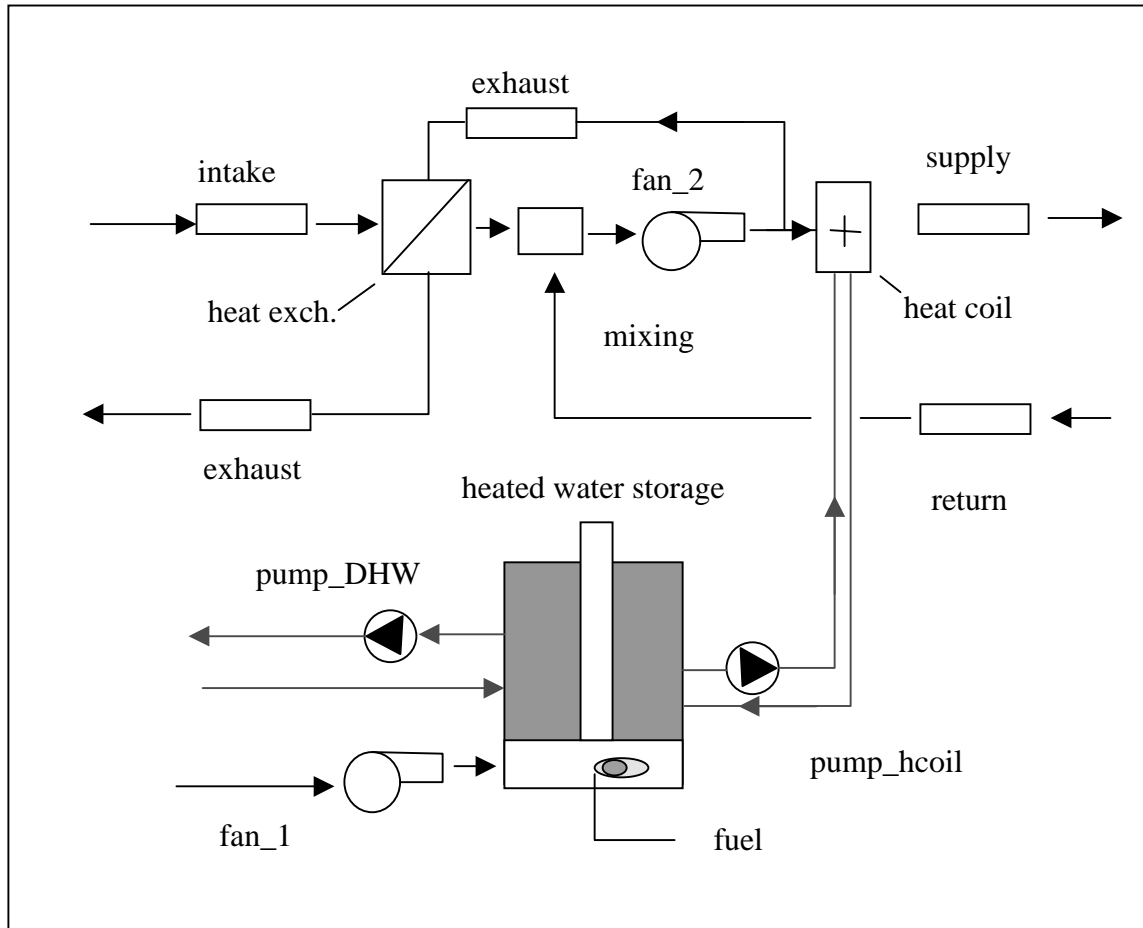


Figure 4: the network representation of the AIMS system.

The AIMS system model has been integrated with a model of a typical Canadian building (see figure 5). This building comprises 3 storeys: basement; main floor and second floor. The AIMS system is used to heat the main and second floors. To achieve this multi-zone linkage, extra ducting components are added.

To form a complete model of the integrated system a series of control loops are added enabling the plant network to control the temperature of the air inside the building. The details of the control loops are as follows:

<b>loop 1:</b> ON/OFF control of the burner, keeping the temperature of the water to 50 °C +/-15 °C
<b>loop 2:</b> ON/OFF control for the combustion fan; ON when the burner is triggered
<b>loop 3:</b> ON/OFF control for the heating coil pump; control is based on the temperature of the main floor, which the system tries to maintain at 19 °C +/- 1 °C; this control action is subjected to timer action
<b>loop 4:</b> TIMER control for the supply fan

The complete system model including control loops and connected building model is shown in figure 5

## **Further Work**

The component model described is extensible in numerous areas, notably in the modelling of the heat transfer between the flue gas node and the water node. Currently a fixed efficiency is used to calculate this: the heat transfer to the water is a fraction of the inlet enthalpy stream. The existing model can be improved so that the heat transfer coefficient between the flue gases and casing of the water store is explicitly calculated, based on inter nodal temperature differences, combustion gas flow rate and properties. Similarly, algorithms for the identification and analysis of condensation of the flue gases can be added. Note however that the evidence from the current model indicates that flue gas condensation does not occur.

Finally, it would be possible to add the calculation of the high temperature specific heat of the combustion gases (including dissociation effects), if this information was available.

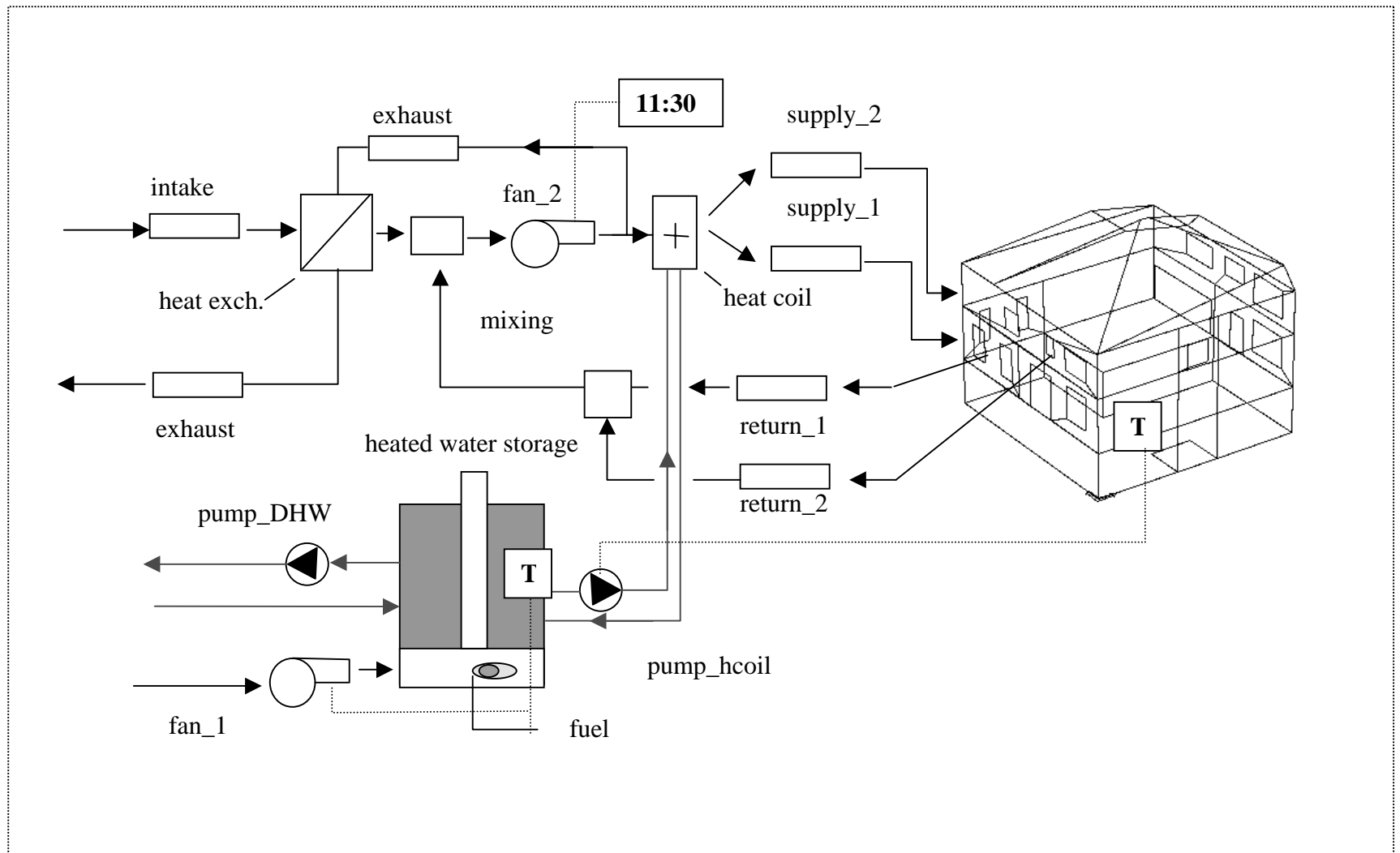


Figure 5: the aims system model connected to a typical Canadian dwelling

## **References**

[1] Parent M., *Combo/AIMS Model Description*, Technosim Report for the Buildings Group, May 2000.