PREDICTING FOUNDATION HEAT LOSSES: NEURAL NETWORKS VERSUS THE BASESIMP CORRELATIONS

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ABSTRACT

This paper describes a series of tests that were performed to determine whether a neural-network model could outperform a correlation-based model in representing foundation heat losses. The two models were *trained* with data generated by BASECALC, a finite-element-based program for modelling foundation heat losses. The two models are described along with details of the tests used to compare them.

The most important conclusion of this work is that although both models accurately represent the BASECALC data, the NN model outperforms the correlation-based model in the majority of the tests. This observation has greater implications in terms of time rather than accuracy. The use of neural networks rather than correlations could significantly reduce the development time of regression-based algorithms for building energy programs. Although correlation techniques may be preferable for some applications due to their closed-form nature, neural network models should be given due consideration.

INTRODUCTION

A regression-based residential-foundation heat-loss algorithm named BASESIMP (Beausoleil-Morrison and Mitalas 1997; Beausoleil-Morrison 1996b) has been created for implementing into whole-building programs. BASESIMP represents both above- and below-grade time-dependent foundation heat losses.

BASESIMP is a closed-form correlation model based on data generated with BASECALC, a finiteelement-based foundation heat-loss program (Beausoleil-Morrison et al 1995, Beausoleil-Morrison 1996a). 33 000 BASECALC parametric simulations were performed to generate the BASESIMP regression data. Location of the insulation, insulation resistance, structural material, height, depth, width, length, soil conductivity, and water-table depth were varied.

As is often necessary with regression methods, many constructs were examined before finalizing the form of Moncef Krarti Joint Centre for Energy Management University of Colorado Boulder, 80309-0428, USA krarti@khazi.colorado.edu

the BASESIMP correlation equations. This is a disadvantage of closed-form correlation methods: a great deal of time must be invested to establish the form of the correlation equations.

Neural Networks (NNs) offer an alluring alternative, despite their "black-box" nature. NNs have great flexibility as the user does not need to define the form of the correlation between the inputs and outputs. As such, it may be substantially less time consuming to apply NNs rather than correlation-based methods to develop regression-based algorithms. NNs also have the potential to represent data sets more accurately than correlation methods, even when the latter have well-developed forms.

Krarti (1995) explored the use of NNs to represent below-grade foundation heat losses. He compared the abilities of NNs and a correlation-based method to predict the annual mean heat loss, the annual amplitude of the heat loss, and the annual phase lag for a number of slabs-on-grade. He found the NNs outperformed the regression model and concluded that NNs offer an accurate method for predicting foundation heat losses.

In this paper, the ability of NNs to represent foundation heat losses is further tested. Specifically, the following question is posed: can an NN model represent foundation heat losses as (or more) accurately than the BASESIMP correlations?

The following sections succinctly review the BASESIMP algorithm and the NN model. A detailed description of the tests performed to compare BASESIMP and the NN model is presented and finally the full results of the comparison are given and conclusions drawn.

THE BASESIMP ALGORITHM

BASESIMP uses the principle of superpositioning to express the foundation's instantaneous heat loss by three components:

$$Q_{basement}(t) = Q_{above - grade}(t)$$

$$+Q_{below - grade, average}$$

$$+Q_{below - grade, harmonic}(t)$$
(1)

Each of the three components of the heat loss is related to its thermal boundary conditions and a three-dimensional shape factor:

$$Q_{above - grade}(t) = S_{ag}(T_{basement} - T_a)$$
 (2)

Qbelow - grade, average = Sbg, avg \cdot (Tbasement - Tg, avg)

$$Q_{below - grade, harmonic}(t) = S_{bg, var} \cdot T_{g, amp}$$

 $\cdot \sin(\omega t + PHASE - \pi / 2 - P_s)$

 S_{ag} is the three-dimensional shape factor for the above-grade component {W/K}; $S_{bg,avg}$ is threedimensional shape factor for the mean below-grade component {W/K}; $S_{bg,var}$ is the three-dimensional shape factor for the harmonic below-grade component {W/K}; PHASE is the thermal-response factor {radians}; $T_{basement}$ is the temperature of the space contained by the foundation {K}; $T_{g,avg}$ is the annualaverage ground-surface temperature {K}; $T_{g,amp}$ is the amplitude of the annual harmonic of the groundsurface temperature {K}; Ta is the exterior dry-bulb temperature {K}; and P_s is the phase lag of the ground-surface temperature cosine wave {radians}.

The *corner-correction method* (Beausoleil-Morrison et al 1995b) is used to determine the threedimensional shape factors using two-dimensional calculations:

$$S_{ag} = SUMUO \cdot 2(length + width)$$
(5)
$$S_{bg, avg} = SUMUR \cdot \{2(length - width) + 4 \cdot width \cdot F_{cs}\}$$

 $S_{bg, var} = ATTEN \cdot \{2(length - width) + 4 \cdot width \cdot F_{cv}\}$

 $PHASE = PHASE \tag{8}$

 $F_{\rm cs}$ and $F_{\rm cv}$ are the scalar corner-correction factors, a function of foundation and site thermophysical properties. SUMUO, SUMUR, ATTEN are the results of 2D calculations.

Simple algebraic equations are used to determine SUMUO, SUMUR, ATTEN, and PHASE as a function of a foundation and site's thermal and physical variables. The equations for SUMUO, SUMUR, ATTEN, and PHASE were determined by correlating data generated by 33 000 finite-element-based BASECALC simulations. As is often necessary with regression methods, many constructs were examined before finalizing the form of the BASESIMP correlations. Establishing the appropriate form of the correlation equations is time consuming but critical. If an improper form is selected, the quality of the correlations will suffer. The equations finally selected for BASESIMP are given below:

$$SUMUO = \left\{ \frac{a + b_1(height - depth) + \frac{c_1}{soilk}}{rsi^{d_1}} \right\}$$

• $\left\{ \frac{1}{e_1 + (i_1)(overlap)^{f_1}(rsi)^{g_1}(height - depth)^{h_1}} \right\}$
+ $\{j_1\}$

$$SUMUR =$$

(3)

(4)

(6)

(7)

$$\frac{\{q_2 + r_2(width)\} \cdot \{u_2 + v_2(soilk)\} \cdot \{w_2 + x_2(depth)\}}{(wtable)^{s_2 + t_2(width) + y_2(depth)}}$$
$$+ \left[\frac{a_2(depth)^{b_2}(soilk)^{c_2}}{(wtable)^{d_2}(rsi)^{e_2 + f_2(soilk) + g_2(depth) + h_2(overlap)}}\right]$$

$$ATTEN = \left\{ a_3 + b_3(soilk) + c_3(depth) \right\}$$
$$+ \left\{ \frac{e_3 + f_3(soilk) + g_3(depth)}{(rsi)^{h_3 + i_3 \cdot overlap}} \right\}$$
(11)

$$PHASE = a_4 + \frac{b_4}{(rsi)^{c_4}}$$

(12)

For a given set of foundation and site thermophysical variables, the process of applying the BASESIMP algorithm is summarized below:

- Calculate SUMUO, SUMUR, ATTEN, and PHASE using equations (9) through (12).
- Correct for three-dimensional effects around corners using equations (5) through (8).
- Calculate the heat losses using equations (1) through (4).

A more detailed description of the BASESIMP algorithm is given in Beausoleil-Morrison and Mitalas (1997).

THE NN MODEL

There are several variations of neural network models applied to prediction problems. The approach used in this paper uses a multilayer feedforward neural network with the back-propagation learning algorithm. This approach has been successfully applied to predict short-term building loads (Anstett and Kreider 1993).

Neural networks provide an alternative approach to traditional modeling and statistical methods, and are used for a wide variety of learning tasks. General areas of application include classification, time-series prediction, function approximation, optimization, and control. NNs have inherent advantages over other traditional methods when the data are "fuzzy", have hidden patterns embedded, or exhibit nonlinearity.

The fundamental building block of an NN is the neuron. Each neuron has an output or activation, which is function of its input. This activation function is a threshold function that is nonlinear and easily differentiable. The input to a single neuron is the sum of the outputs for all other neurons connected to it multiplied by the weights connecting them.



Figure 1 Schematic of a Single-Hidden Layer NN Model

Figure 1 shows a schematic diagram of a singlehidden layer NN model. Between the input and the output layers, there are one or more hidden layers. The nodes between the layers are interconnected with weights W_{ij} . These weights are adjusted to minimize the error function *E* defined by the sum of squares of the predictions errors:

$$E = \frac{1}{2} \sum_{k=1}^{N} (t_k - O_k)^2$$

where, t_k is the actual or target value, and O_k is the predicted output using an activation function *f*:

$$O_k = f\left(\sum_{i=1}^{I} W_{ik}O_i\right)$$

The function *f* is typically a bounded monotone function such as f(x)=tanh(x). The weights W_{ij} of the neural network are adjusted to minimize the error function *E* using the gradient descent method by changing a weight from W_{ij} to $W_{ij} - \alpha \frac{\partial E}{\partial W_{ij}}$, where

the parameter α is called the learning rate. In addition to this learning rate parameter, a momentum term is also added to the weights to improve the convergence of the neural-network algorithm. This method of weight adjustment is called the backpropagation training procedure. When the error reaches a small value, it is customary to say that the network has *learned* the mapping between the input and output variables. However, this learning process is merely an error minimization routine.

The learning process of the NN used in this paper is summarized below:

- training Present network with examples consisting of patterns of input and target outputs. In this paper, the input data include the foundation and site's thermal and physical characteristics and the outputs are the coefficients SUMUO, SUMUR, ATTEN, and PHASE. output NNs are used in this analysis. Single
- Determine how closely the network output matches the target outputs (i.e, output-layer error).
- Adjust the connection weights by an amount which is proportional to the rate at which the output error changes as those weights change (error derivative).
- Continue the process of sending back the error signals through the network until the confidence level is achieved or a threshold number of iterations is reached.

Krarti (1995) provides more details on the training and testing methods used in this analysis.

NEURAL NETWORKS VERSUS BASESIMP

A series of tests were designed to answer the question posed at the start of this paper: can the NN model represent foundation heat losses as (or more) accurately than the BASESIMP correlations?

A single BASESIMP *system* was selected, namely concrete basements with interior full-height insulation (Figure 2). As discussed by Beausoleil-Morrison and Mitalas (1997), 1080 BASECALC parametric simulations were performed to generate the regression data for this system. The coefficients of equations (9) through (12) (a_1 , q_2 , etc.) were determined by fitting these 1080 data points. Hence, this set of BASECALC results is referred to as the *training* data.



Figure 2 Basement with interior full-height insulation

In order to compare BASESIMP and the NN model on equal footing, the NN model was *trained* with the same training data set—it *learned* the mapping between input and output variables.

Then, an additional 228 BASECALC simulations were performed to establish a test data set. Inputs for height, depth, width, water-table depth, soil conductivity, insulation resistance were and randomly-generated, producing combinations that neither BASESIMP the NN nor model had experienced. This is a blind test for the NN model: the modeller has no knowledge of the BASECALC outputs for the 228 test-data points. It is also a blind test for BASESIMP as the 228 test-data points encompass input combinations that were not used to develop the BASESIMP correlations.

Three tests (named A, B, and C) were then designed to determine how accurately BASESIMP and the NN model could represent the *test* data set. In each of the three tests, the 228 input combinations were provided to the models, each predicted the outputs, and the outputs were compared to the BASECALC results (the reference). Six criteria were used to assess the quality of the fits (shown here for BASESIMP and SUMUO):

- The mean of the absolute $\operatorname{errors:} err_{abs}^{avg} = \frac{\sum_{i=1}^{n} |SUMUO_{BASESIMP,i} - SUMUO_{BASECALC,i}|}{n}$
- The maximum of the absolute errors: $err_{abs}^{max} = max \left\{ SUMUO_{BASESIMP,i} - SUMUO_{BASECALC,i} \right\}$
- The root-mean square of the absolute errors: $err_{abs}^{ms} = \sqrt{\frac{\sum_{i=1}^{n} |SUMUO_{BASESIMP,i} - SUMUO_{BASECALC,i}|^{2}}{n}}$
- The mean of the relative $\frac{\sum_{i=1}^{n} \left[\frac{|SUMUO_{BASESIMP,i} - SUMUO_{BASECALC,i}|}{SUMUO_{BASECALC,i}} \right]}{n}$
- The maximum of the relative errors: $err_{rel}^{\max} = \max\left\{\frac{|SUMUO_{BASESIMP,i} - SUMUO_{BASECALC,i}|}{SUMUO_{BASECALC,i}}\right\}$
- The root-mean square of the relative errors:

$$err_{abs}^{rms} = \sqrt{\frac{\sum_{i=1}^{n} \left[\frac{|SUMUO_{BASESIMP,i} - SUMUO_{BASECALC,i}|}{SUMUO_{BASECALC,i}}\right]}{n}}$$

In Test A the ability to predict SUMUO, SUMUR, ATTEN, and PHASE was determined. BASESIMP applies equations (9) through (12) to predict these quantities while they are outputs of the NN model.

The ability to predict heat loss from 2D crosssections is assessed in Test B. In this test the BASESIMP- and NN-predicted SUMUO, SUMUR, ATTEN, and PHASE (from Test A) are combined with weather data for three climates (Montréal, Edmonton, and Vancouver) to predict the instantaneous heat loss at two points during the heating season: November 10 and February 20. Errors resulting from the predictions of SUMUO, SUMUR, ATTEN, and PHASE could propagate when these factors are combined with weather data. Hence, this is more demanding than Test A.

Since the heat loss from 2D cross-sections was being compared, the corner-correction method is not applied. Rather the following relation is used to estimate the 2D heat loss:

$$Q' \text{ basement}(t) = SUMUO(T\text{ basement} - Ta) + SUMUR(T\text{ basement} - Tg, avg) + \begin{cases} ATTEN \cdot Tg, amp \\ \cdot \sin(\omega t + PHASE - \pi/2 - Ps) \end{cases}$$

The two methods' accuracy in predicting heatingseason heating loads from whole basements (including 3D effects around corners) is assessed in Test C. The full BASESIMP algorithm is employed in this test:

- SUMUO, SUMUR, ATTEN, and PHASE are predicted using equations (9) through (12)
- adjustments are made for the three-dimensional effects using the corner-correction method— equations (5) through (8).
- the heat loss is expressed using equations (1) through (4).
- equation (1) is integrated from October 1 to April 30 to produce the heating-season heating load.

This procedure is not used for the NN model; rather the heating-season energy loss is the output of the NN model, having been trained to the BASECALCpredicted heating-season energy loss.

Weather data for three cities are used in this test: Montréal, Edmonton, and Vancouver.

TEST A RESULTS

The following two figures (Figure 3 and Figure 4) illustrate how BASESIMP and the NN model perform in predicting SUMUR for the test data set. If agreement were perfect, all data points would lie on the diagonal line whose slope is unity. As can be seen, both models produce good agreement, although there is slightly less scatter with the NN model.



Figure 3 BASESIMP SUMUR predictions



Figure 4 NN SUMUR predictions

The following tables present the six error criteria for SUMUO, SUMUR, ATTEN, and PHASE, for both BASESIMP and the NN model. As can be seen, BASESIMP slightly outperforms the NN model in predicting SUMUO, whereas the NN model produces lower error criteria for SUMUR, ATTEN, and PHASE.

	SUMUO		SUMUR	
	BASESIMP	NN	BASESIMP	NN
err _{abs} ^{avg}	0.011 W/mK	0.011 W/mK	0.062 W/mK	0.042 W/mK
err _{abs} ^{max}	0.031 W/mK	0.040 W/mK	0.277 W/mK	0.199 W/mK
err _{abs}	0.013 W/mK	0.013 W/mK	0.077 W/mK	0.058 W/mK
err _{rel} ^{avg}	5.6%	6.3%	3.2%	2.1%
err _{rel} ^{max}	19.7%	25.5%	10.3%	8.9%
err _{rel} rms	6.8%	8.3%	3.8%	2.6%

	ATTEN		PHASE	
	BASESIMP	NN	BASESIMP	NN
err _{abs} ^{avg}	0.020 W/mK	0.016 W/mK	0.062 rad	0.043 rad
err _{abs} ^{max}	0.070 W/mK	0.066 W/mK	0.170 rad	0.131 rad
err _{abs}	0.024 W/mK	0.020 W/mK	0.076 rad	0.054 rad
$\operatorname{err}_{\operatorname{rel}}^{\operatorname{avg}}$	3.5%	2.6%	2.3%	1.6%
err _{rel} ^{max}	15.8%	9.9%	6.6%	4.9%
err _{rel} rms	4.5%	3.2%	2.9%	2.0%

TEST B RESULTS

The next two figures (Figure 5 and Figure 6) illustrate how BASESIMP and the NN model have performed in predicting the heat loss on November 10 in Edmonton. Once again, there is less scatter associated with the NN model.



Figure 5 BASESIMP 2D heat-loss predictions for November 10 in Edmonton



Figure 6 NN 2D heat-loss predictions for November 10 in Edmonton

The following three tables presents the six error criteria for the heat loss predictions on November 10 and February 20 for both BASESIMP and the NN model. There is a separate table for each city.

As can be seen, the NN model outperforms BASESIMP in Test B. In all cases, the NN model produces lower error criteria.

	Montréal November 10		Montréal February 20	
	BASESIMP	NN	BASESIM P	NN
err _{abs} ^{avg}	1.2 W/m	0.8 W/m	1.0 W/m	0.6 W/m
err _{abs} ^{max}	3.8 W/m	3.1 W/m	4.2 W/m	2.6 W/m
err _{abs} ^{rms}	1.5 W/m	1.0 W/m	1.2 W/m	0.7 W/m
$\operatorname{err}_{\operatorname{rel}}^{\operatorname{avg}}$	4.2 %	2.6 %	2.3 %	1.3 %
$\operatorname{err}_{\operatorname{rel}}^{\max}$	11.4 %	10.3 %	8.7 %	6.8 %
$\operatorname{err}_{\operatorname{rel}}^{\operatorname{rms}}$	5.1 %	3.2 %	2.9 %	1.7 %

	Edmonton		Edmonton	
	November 10		February 20	
	BASESIMP	NN	BASESIMP	NN
err _{abs} ^{avg}	1.3 W/m	0.8 W/m	1.0 W/m	0.6 W/m
err _{abs} ^{max}	4.0 W/m	3.2 W/m	4.3 W/m	2.7 W/m
err _{abs}	1.6 W/m	1.0 W/m	1.3 W/m	0.8 W/m
err _{rel} ^{avg}	3.8 %	2.2 %	2.4 %	1.3 %
err _{rel} ^{max}	10.0 %	9.5 %	8.6 %	6.8 %
err _{rel}	4.6 %	2.8 %	2.9 %	1.8 %

	Vancouver November 10		Vancouver February 20	
	BASESIMP	NN	BASESIMP	NN
err _{abs} ^{avg}	0.8 W/m	0.5 W/m	0.6 W/m	0.3 W/m
err _{abs} ^{max}	2.4 W/m	2.0 W/m	2.7 W/m	1.7 W/m
err _{abs}	1.0 W/m	0.6 W/m	0.8 W/m	0.5 W/m
err ^{avg}	4.1 %	2.5 %	2.3 %	1.3 %
err _{rel} ^{max}	10.8 %	10.0 %	8.9 %	6.9 %
err _{rel} ^{rms}	4.9 %	3.0 %	2.8 %	1.7 %

TEST C RESULTS

The next two figures (Figure 7 and Figure 8) illustrate how BASESIMP and the NN model have performed in predicting the heating-season heating load in Montréal. This time, BASESIMP produces slightly better agreement than the NN model.







Figure 8 NN predictions of heating-season heating load for Montréal

The following two tables presents the six error criteria for the heating-season heating load for both BASESIMP and the NN model.

In this test there is no clear winner. For all three locations, the two methods have similar average and root-mean square error numbers: in some cases BASESIMP is better while in others the NN model is better. However, BASESIMP consistently produces lower maximum errors, indicating that the NN model has greater errors for a few points

To place these numbers in context, space heat from a mid-efficiency gas furnace costs in the order of 7\$Can/GJ in Canada (4.50 ECU). Therefore, the average error for both models is in the order of 4.50\$Can/year (2.90 ECU). The greatest errors are less than 16\$Can (10.30 ECU) for BASESIMP and less than 25\$Can (16.10 ECU) for the NN model.

	Montréal		Edmonton	
	BASESIMP	NN	BASESIMP	NN
err _{abs} ^{avg}	0.6 GJ	0.6 GJ	0.6 GJ	0.6 GJ
err _{abs} ^{max}	2.1 GJ	3.5 GJ	2.2 GJ	3.4 GJ
err _{abs}	0.8 GJ	0.8 GJ	0.8 GJ	0.8 GJ
$\operatorname{err}_{\operatorname{rel}}^{\operatorname{avg}}$	2.8 %	2.9%	2.8 %	2.4 %
err _{rel} ^{max}	8.2 %	14.0 %	8.4 %	11.8 %
err _{rel} rms	3.5%	3.9 %	3.4 %	3.2 %

	Vancouver		
	BASESIMP	NN	
err abs	0.4 GJ	0.3 GJ	
err _{abs} ^{max}	1.3 GJ	1.8 GJ	
err _{abs}	0.5 GJ	0.4 GJ	
$\operatorname{err}_{\operatorname{rel}}^{\operatorname{avg}}$	2.8 %	2.3%	
$\operatorname{err}_{\operatorname{rel}}^{\max}$	8.2 %	12.8 %	
$\operatorname{err}_{\operatorname{rel}}^{\operatorname{rms}}$	3.4%	3.2 %	

DISCUSSION AND CONCLUSIONS

Two regression-based approaches for representing foundation heat losses have been presented. The first is a closed-form correlation model (BASESIMP) while the second is a multilayer feedforward neural network model with the back-propagation learning algorithm (the NN model).

Both models were *trained* to represent heat losses from basements with interior full-height insulation. The training data was a set of results derived from 1080 BASECALC simulations. An additional 228 BASECALC simulations were performed to generate a *test* data set and a series of tests executed to compare the two models' accuracy in representing the *test* data.

In all tests, both BASESIMP and the NN model accurately represented the *test* data. However, with a few exceptions, the NN model outperformed BASESIMP. This indicates that either approach can be used to accurately represent foundation heat losses; however, the NN model can lead to greater accuracy.

These results have significant implications for the development of regression-based models for representing foundation heat losses—and for the entire field of building energy analysis. Compared to traditional correlation-based techniques, NN models

offer the potential for improved accuracy. Additionally—and much more significantly—their use can substantially reduce model development time. Many constructs were examined before the functional form of the BASESIMP correlations was established: this was a time consuming task. In contrast, because the modeller did not have to specify functional forms, a small fraction of the this time was required to develop the NN model.

Correlation-based techniques may be preferable for some applications due to their closed-form—as opposed to "black-box"—nature. Notwithstanding, NN approaches should be given due consideration when developing regression-based algorithms.

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NOMENCLATURE

 $Q_{\text{basement}}(t) = \text{total heat loss from foundation (W)}$

 $\boldsymbol{Q}_{above\text{-}grade}(t)$ = heat loss from foundation to ambient air (W)

 $Q_{below-grade,average}(t) = mean-annual heat loss from foundation to soil (W)$

 $\mathbf{Q}_{\text{below-grade,harmonic}}(t)$ = annual harmonic of heat loss from foundation to soil (W)

 S_{as} = 3D shape factor for above-grade heat loss (W/K)

 $S_{bg,avg}$ = 3D shape factor for mean-annual below-grade heat loss (W/K)

 $S_{\rm _{bg,var}}$ = 3D shape factor for annual harmonic below-grade heat loss (W/K)

PHASE = thermal-response factor (radians)

 $T_{\scriptscriptstyle basement}$ = temperature of the space contained by the foundation (K)

 $T_{g,avg}$ = annual-average ground-surface temperature (K)

 $T_{g,amp}$ = amplitude of the annual harmonic of the ground-surface temperature (K)

T_a = exterior dry-bulb temperature (K)

 P_s = phase lag of the ground-surface temperature cosine wave (radians)

t = time (weeks)

 $\varpi = 2^1 \text{ rad/year}$

length = length of foundation (m)

width = width of foundation (m)

height = height of foundation wall (m)

depth = depth of foundation wall (m)

soilk = thermal conductivity of soil (W/mK)

rsi = thermal resistance of insulation (m^2K/W)

wtable = depth of water table below grade (m)

 a_1 , q_2 , etc = correlation coefficients