Energy Resources and Policy

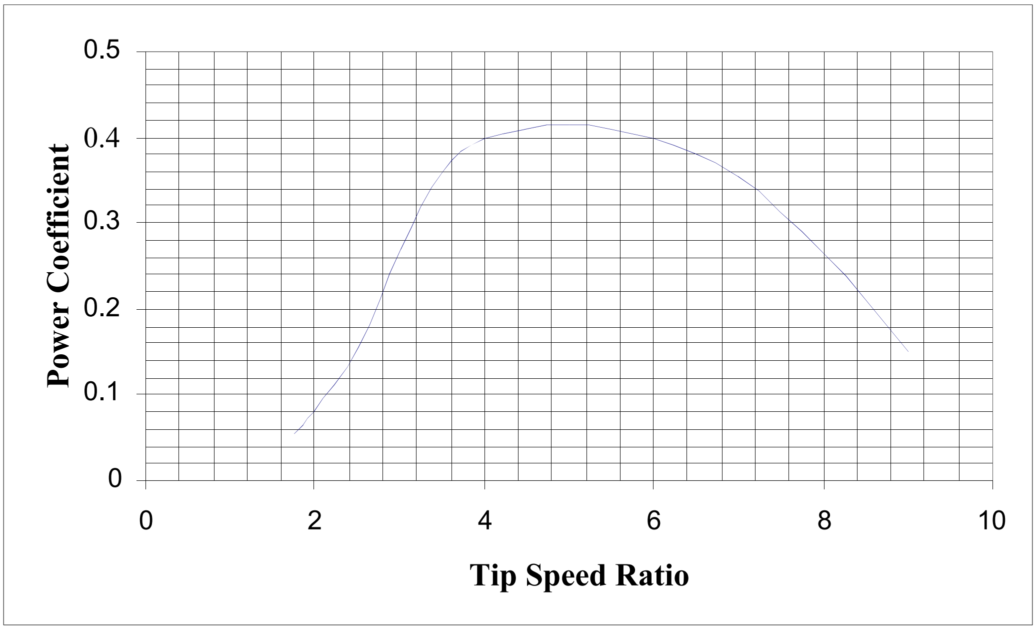
Tutorial:
Tidal power
Answers

Density of water is 1000 kg/m³ unless otherwise stated

1. A turbine to generate power from marine currents is based upon a wind turbine rotor specification, for which the variation of power coefficient C_P with tip speed ratio X_T is shown in the figure below. It is desired to produce 10 kW from a stream velocity of 2 m/s, with the turbine running at a tip speed ratio of 5. Specify the rotor diameter and its speed of rotation. [Hint: source the formula for tip speed ratio.]

The turbine is to operate at constant speed, and to shut down when its output falls below 2 kW. Use an iterative procedure to estimate the stream velocity at which this will occur. Assume a water density of 1060 kg/m³ throughout.

[2.69 m; 71.0 rev/min; 1.31 m/s]



$\lambda = \frac{\Omega R}{V} = 5$, so $\Omega R = 5V = 10$

Also, if $\lambda = 5$, $C_p = 0.415$

Power, $P = C_p.\frac{1}{2}\rho A V^3$

So $10x10^3 = 0.415x\frac{1}{2}x1060Ax8$; $A = \frac{10x10^3}{1.7596x10^3} = 5.683$ m²; $D = \sqrt{\frac{4}{\pi}A} = \underline{2.69 \text{ m}}$.

If $\Omega R = 10$, $\Omega = \frac{10x2}{2.69} = 7.435$ rad/s

Rotational speed $= \frac{60}{2\pi} = \underline{71.0 \text{ rev/min}}$.

Shutdown

$\Omega = 7.435$ rad/s and $\Omega R = 10$ as before.

Guess a value for λ and calculate power.

λ	8	7.2	7.6	7.65
V (m/s)	1.25	1.389	1.316	1.307
C_p	0.264	0.340	0.303	0.299
P (kW)	1.553	2.744	2.057	2.012

$\lambda = \frac{\Omega R}{V} = \frac{10}{V}$, so $V = \frac{10}{\lambda}$

Power = $\frac{1}{2}\rho A C_p V^3$

$= 3.012\, C_p V^3$ kW

Near miss – accept V = <u>1.31 m/s</u>.

2. At a site for a tidal current turbine, the water velocity varies sinusoidally with a period of 12 h 25 min. The turbine has a rotor of 15 m diameter and a rated power output of 600 kW. It has a cut-in current speed of 0.5 m/s.

For a tidal steam with a maximum current velocity of 2.5 m/s, determine whether the turbine will reach its rated power output. Assume a power coefficient of 0.39 and a water density of 1025 kg/m^3.

Determine the energy captured by the turbine over a full tidal cycle, for the above conditions. Note that

$$\int \sin^3 \omega t \;=\; \frac{1}{\omega}\left[\frac{1}{3}\cos^3 \omega t \;-\; \cos \omega t\right].$$

During the tidal cycle, there are periods when the turbine produces zero output. These periods will be longest for neap tides. If the smallest neap tide at the site gives a maximum current velocity of 1.7 m/s, calculate the length of the corresponding zero-output period.

[No; 2907 kWh; 1h 10.8 min]

At 2.5 m/s, $P = C_p.\tfrac{1}{2}\rho A V^3 = 0.39x\tfrac{1}{2}x1025x\frac{\pi}{4}(15)^2(2.5)^3 = 551.9x10^3$ **W**

Therefore will <u>not</u> reach its rated power output.

In general $V = 2.5 \sin \omega t$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{745} = 0.008434$ **rad/min.**

At cut-in, $0.5 = 2.5 \sin \omega t$, **so** $\omega t = \sin^{-1}\left(\frac{0.5}{2.5}\right) = 11.5^{\circ} = 0.2014$ **rad.**

Then $t = T_1 = \frac{0.2014}{0.008434} = 23.87$ **minutes.**

Energy capture $= \int_{T_1}^{T/4} C_p.\tfrac{1}{2}\rho A V^3.dt = C_p.\tfrac{1}{2}\rho A \int_{T_1}^{T/4}[2.5\sin \omega t]^3.dt$ **for** $\frac{1}{4}$ **cycle.**

$= 0.39x\tfrac{1}{2}x1025x\frac{\pi}{4}(2.5)^3 \int_{T_1}^{T/4} \sin^3 \omega t.dt$

$= 551.9x10^3 x\frac{1}{0.008434}\left[\frac{1}{3}\cos^3 \omega t - \cos \omega t\right]_{T_1}^{T/4}$

$= 65.44x10^6\left[\frac{1}{3}x0 - 0 - \frac{1}{3}x0.9406 + 0.9798\right]$

$= 43.6x10^6$ **W-min** $= 43.6x10^3$ **kW-min = 727 kWh**

For a full cycle, capture = 4x727 = <u>2907 kWh</u>

When $t = T/4$, $\frac{\omega T}{4} = \frac{\pi}{2} = 90^{\circ}$

<u>Neap tide</u>

Here, $V = 1.7 \sin \omega t$ **and for cut-in condition,** $0.5 = 1.7 \sin \omega t$.

$\omega t = \sin^{-1}\left(\frac{0.5}{1.7}\right) = 17.1^{\circ} = 0.2985$ **rad.**

Then $t = \frac{0.2985}{0.008434} = 35.4$ **minutes.**

But total 'idle' time = 2t = 70.8 minutes = 1 h 10.8 min.

3. The energy produced per cycle in a tidal power plant, E is a function of the tidal range ($2a$), the tidal basin area, B, the water density, ρ, and the gravitational constant, g. A non-dimensional energy parameter Φ_E may be produced, where

$$\Phi_E = \frac{E}{(2a)^2 Bg\rho}.$$

For La Rance tidal barrage system, the mean tidal range is 8.0m and the basin area is 22 km^2. The annual energy produced from the plant is 544 GWh. Compute a value for Φ_E, and hence estimate the annual and time-averaged outputs from a system where the mean tidal range is 5.2 m and the basin surface area is 1.8 km^2. Take the water density as 1025 kg/m^3.

[0.197; 18.77 GWh; 2.143MW]

$$E = \frac{544x10^9}{365}x\frac{12.5}{24} = 776 \text{ MWh/cycle}$$

$$\Phi_E = \frac{776x10^6x3600}{64x22x10^6xgx1025} = \underline{0.197}$$

For proposed system:

$E = \Phi_E(2a)^2Bg\rho = 0.197\ (5.2)^2.1.8x10^6.g.1025 = 96.4 \times 10^9$ J

= 26.78 MWh/cycle

Number of cycles per year = 365 x 24/12.5 = 701,

∴ in one year, E = 26.78 x 701 MWh = 18.77 GWh

Time-averaged output $= \frac{18.77x10^9}{365x24} = 2.143x10^6$ **W = 2.143 MW**