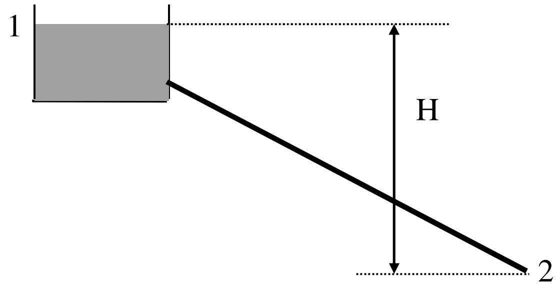
Energy Resources and Policy
Handout:
Power from water

For an application such as a hydro-electric scheme, the typical requirement is to evaluate the power delivered at the end of a pipeline as depicted in the following figure.



From the free surface in the reservoir to the pipe exit, the energy equation is

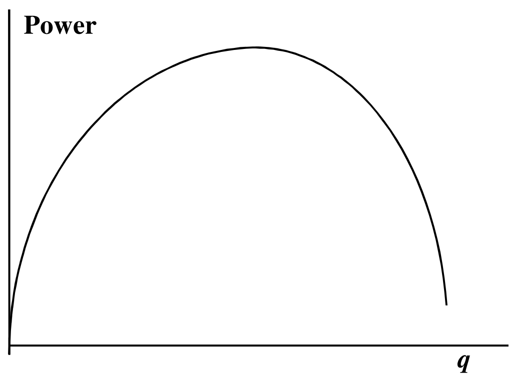
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2 + \frac{4 f . l}{d} . \frac{\overline{V}^2}{2} .$$

and the power available in the flow at pipe exit is

$$\rho q \left[\frac{P_2}{\rho} + \frac{V_2^2}{2} \right] = \frac{\pi}{4} \rho d^2 \overline{V} \left[g H - \frac{4 f . l}{d} . \frac{\overline{V}^2}{2} \right] ,$$

where $V_2 = \overline{V}$ = mean velocity in pipe. Clearly, power = 0 if $\overline{V} = 0$ and if $\overline{V}$ is large $\frac{4 f l}{d} . \frac{\overline{V}^2}{2} \rightarrow gH$ and so power tends to 0.

A graph of power against volume flow rate would look like the sketch shown here, with a maximum at a certain value of q.



For a maximum power condition, $\frac{d}{dV}(power) = 0$, giving

$$\frac{\pi}{4} \rho d^2 \left[g H - \frac{4 f l}{d} . \frac{3\overline{V}^2}{2} \right] = 0 .$$

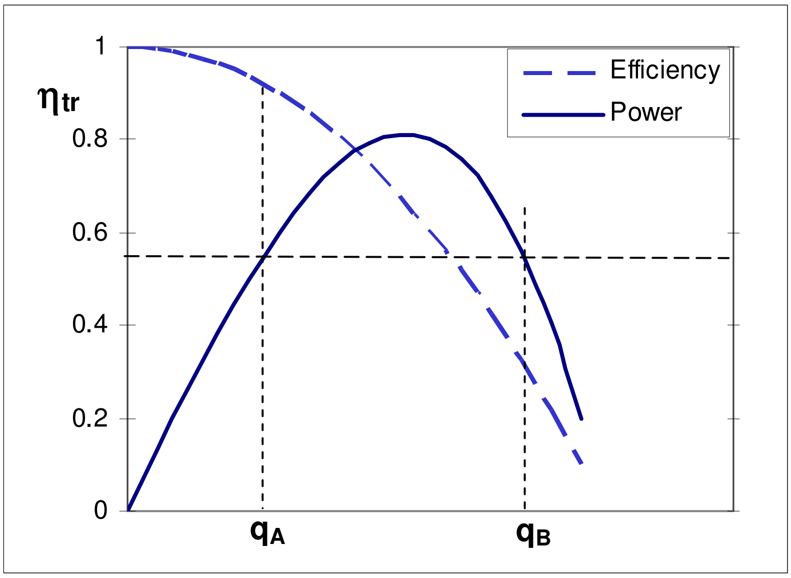
$$\therefore gH = 3 . \frac{4 fl}{d} . \frac{\overline{V}^2}{2} ; \text{ with friction loss } \boxed{\frac{4 \ fl}{d} . \frac{\overline{V}^2}{2} = \frac{1}{3} gH.}$$

Thus the energy available at the end of the pipe = $\frac{2}{3}gH$ and the transmission efficiency, η_{tr}, for the pipeline is defined as

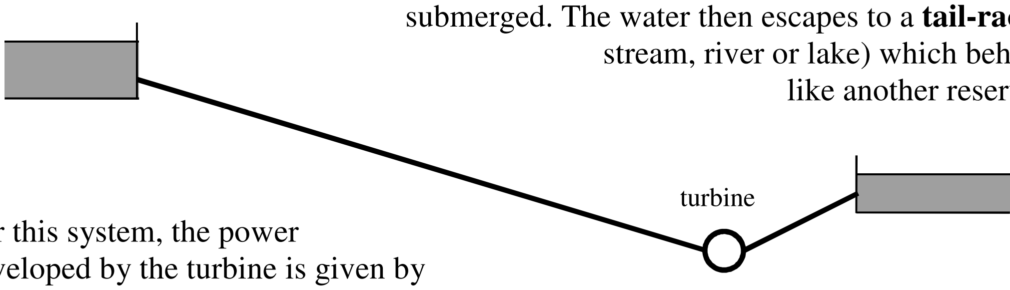
$$\frac{energy\ extracted}{total\ energy\ available} = \frac{\frac{2}{3}gH}{gH} = \frac{2}{3}.$$

Note that in general, $\eta_{tr} = \dfrac{gH - losses}{gH} = 1 - \dfrac{losses}{gH}$.

So η_{tr} will be 1.0 for q = 0, falling to zero where losses = gH. An accurate plot of power and efficiency characteristics is shown in the graph below.



Practical hydro-electric plants have two possible configurations, depending on the type of turbine used. Systems with **reaction** turbines (Francis, Kaplan) are the more common - there is a substantial pressure drop across the turbine, which is fully submerged. The water then escapes to a **tail-race** (a stream, river or lake) which behaves like another reservoir.



For this system, the power developed by the turbine is given by

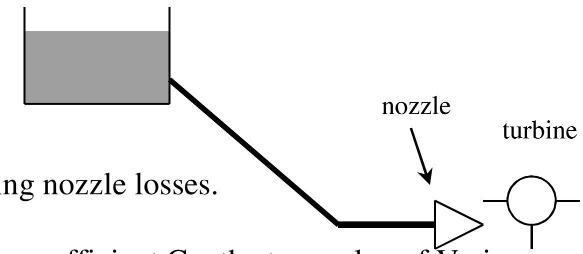
$$\text{Power} = \rho q \,.\, W_{out} = \rho q\left[gH - \Sigma\text{losses}\right] = \rho q\left[gH - \frac{32\,flq^2}{\pi^2 d^5}\right].$$

For a given system, there is a maximum power which can be made available to the turbines, its value depending on H, f, l and d.

A hydro-electric plant will normally run at less than maximum power, say at a level indicated by the dashed line on the graph. If this power is known, a cubic equation for q is produced, which in general will have two real solutions q_A and q_B, as illustrated in the graph. The transmission efficiencies are very different; the system will of course be run at the lower flow rate, q_A.

Sometimes an **impulse** turbine is used: a Pelton Wheel, Turgo or cross-flow design. Here, the energy of the water is converted entirely into kinetic energy using a variable nozzle or spear valve. The water is discharged to atmosphere with a velocity V_N, and the energy equation becomes

$$gH = \frac{V_N^2}{2} + \frac{4fl}{d}.\frac{\overline{V}^2}{2}.$$



So $V_N = \sqrt{2\left(gH - \frac{4fl}{d}.\frac{\overline{V}^2}{2}\right)}$, ignoring nozzle losses.

If these losses are represented by a velocity coefficient C_V, the true value of V_N is

$$C_V\sqrt{2\left(gH - \frac{4fl}{d}.\frac{\overline{V}^2}{2}\right)}.$$

The power in the water jet is

$$\rho q.\frac{V_N^2}{2} = \frac{\pi}{4}\rho d_N^2 V_N.C_v^2\left(gH - \frac{4fl}{d}.\frac{\overline{V}^2}{2}\right)$$

$$= \frac{\pi}{4}\rho d_N^2 C_v^2\left(\frac{d}{d_N}\right)^2 \overline{V}\left(gH - \frac{4fl}{d}.\frac{\overline{V}^2}{2}\right)$$

For maximum power,

$$\frac{d}{d\overline{V}}(power) = 0 = \frac{\pi}{4}\rho d_N^2 C_v^2\left(\frac{d}{d_N}\right)^2\left(gH - \frac{4fl}{d}.\frac{3\overline{V}^2}{2}\right),$$

so the friction loss $= \frac{1}{3}gH$ as before.

Note that η_{tr} is now $\frac{2}{3}C_v^2$.

In general, $V_N^2 = 2C_v^2\left(gH - \frac{4fl}{d}.\frac{\overline{V}^2}{2}\right)$, and for maximum power conditions:

$$V_N^2 = 2C_v^2\left(2.\frac{4fl}{d}.\frac{\overline{V}^2}{2}\right) = C_v^2.\frac{8fl}{d}.\overline{V}^2.$$

Also of course,

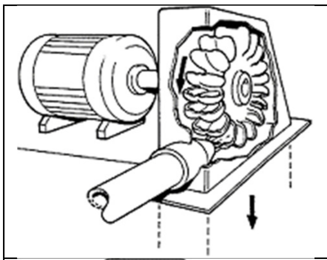
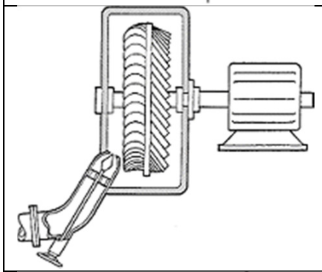
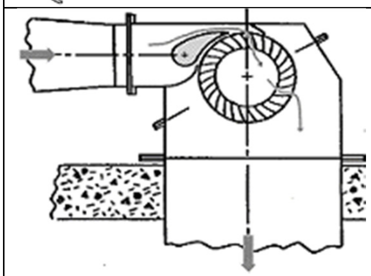
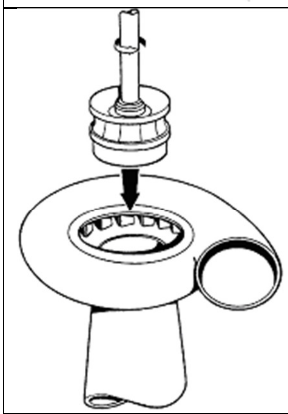
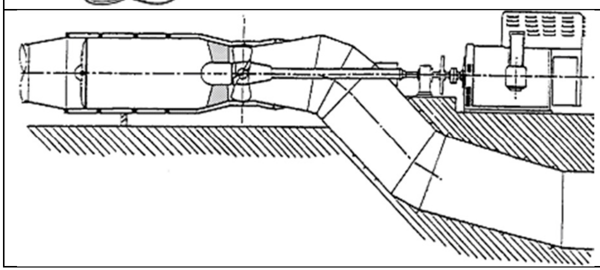
$$V_N^2 = \left(\frac{d}{d_N}\right)^4\overline{V}^2,$$

$$\therefore \quad \frac{d^4}{d_N^4} = \frac{8\,f\,l}{d}\,.\,C_v^2 \quad giving \quad d_N^4 = \frac{d^5}{8\,f\,l\,C_v^2}$$

or water jet diameter $d_N = \sqrt[4]{\dfrac{d^5}{8\,f\,l\,C_v^2}}$ for maximum power.

Hydraulic turbines

Impulse – all energy converted to kinetic before impact with runner.

	**Pelton wheel:** - jet is split symmetrically by blades on the wheel; wheel can have up to 6 jets if mounted on vertical shaft.
	**Turgo:** - jet passes through the wheel from left to right; it may have a larger diameter than the jet for a Pelton wheel of the same size.
	**Cross-flow (or Banchi or Ossberger):** has a cylindrical runner, and water passes through the blade ring twice. Runner may have several segments on the same axis, to deliver high efficiency for a wide range of flow rates. Runner has simple geometry, may be fabricated from sheet metal.
	**Francis:** fed by water from a spiral casing, fitted with a ring of guide vanes immediately upstream of the runner. These may be adjusted to vary the volume flow rate through the machine. Water exits through a gently expanding draft tube.
	**Propellor**: an axial-flow machine, normally fitted with guide vanes as shown. If it has adjustable-pitch blades, it is known as a **Kaplan** turbine. It may also be installed with its shaft vertical.

All reaction turbines will operate efficiently as pumps; impulse turbines will not!