Tutorial 5: Numerical method - buildings

Q1. Identify three principal differences between a response function method and a numerical method when both are employed to represent an energy system.

Response function methods require simplifying assumptions in order to permit an analytical solution. Such assumptions are generally not required by a numerical method. For example:

- the equations representing each energy flow-path must be linearised;
- system parameters (equation coefficients) are assumed to be time invariant; and
- the boundary conditions are simplified (e.g. steady cyclic).

Q2. What is the role of a Taylor Series expansion in solving a partial differential equation?

The expansion allows the first and higher order derivatives to be replaced by an approximate difference representation that in many cases will produce an equation that can be readily manipulated and solved.

Q3. Describe two error sources that occur in a numerical approximation of the Fourier partial differential heat equation.

Discretisation errors associated with the replacement of a continuous system with discrete space and time steps.
The truncation error associated with the replacement of the derivatives with a Taylor series expansion.

Q4. What is the difference between an implicit and explicit method of representing a partial differential equation relating to an energy flow-path (such as the Fourier heat equation relating to transient conduction for example). Identify in your answer the limitations of each approach..

Explicit method: the second order derivative is replaced by a central difference approximation and the first order derivative by a first forward difference approximation.
Implicit method: the second order derivative is replaced by a central difference approximation but using the unknown temperature values at some future time-row (rather than the known values at some present time-row as in the explicit case); the first order derivative is represented by the first backward difference approximation.

The explicit method has related stability criteria that restricts the space and time steps that may be used in a simulation. The implicit method is unconditionally stable but of lower accuracy.

Q5. Outline the principal steps involved in establishing a numerical representation of a building as used in the simultaneous approach. For each step identify a major associated issue.

System discretisation into finite volumes (FV) representing parts of system (fluid volumes, constructional material, heat transfer surfaces *etc.*). Issue from: balance between the required resolution (number of FVs) and computational efficiency; control of space and time discretisation errors.

Conservation equation development for each inter-FV connection and transfer type (e.g. heat, mass, momentum, electricity *etc.*). Issue from: absence of appropriate theories to represent the transfer types connecting the FVs; absence of appropriate parameter values (e.g. heat/mass transfer coefficients).

Simultaneous solution of the entire equation-set for successive time-steps and under evolving boundary conditions. Issue from: sparseness of the complete system equation-set requiring specialised solvers; the different domain time constants requiring different solution frequencies (or that all equations be solved at the highest frequency); the presence of non-linear equations and complex domain interdependencies requiring iteration.

Q6. Identify the three principal energy conservation equations required to establish a energy systems simulation model and identify the finite volume types to which each relates.

Material energy conservation equation relates to:
- opaque intra-construction;
- transparent intra-construction;
- phase change;
- boundary between materials; and
- lumped capacity regions.

Surface energy conservation equation relates to:
- room surfaces;
- plant component surfaces; and
- the ground.

Fluid energy conservation equation relates to:
- room air;
- construction air gaps; and
- plant component working fluids.

Q7. Why is it best to use different equation solvers for the equation sets corresponding to discrete technical domains (heat conduction, air flow, radiation exchange etc.) as opposed to subjecting all equations to a single solver type?

Because the domain-specific equation sets will relate to system parts of different time constants, be of different equation types (linear, non-linear) and possess various degrees of sparsity. Using a single solver would be inefficient because all equations would need to be solved at the lowest common frequency and the solver would need to be of type iterative to cope with non-linear equations.

Q8. For an energy component of your choice, outline a simple and detailed discretisation scheme and say for each the analyses that would typically be enabled.

Consider a solar thermal collector.
Discretisation as a single node would allow an estimate of the variation in the energy output over time as well as the typical temperature of the supplied air or water.

Discretisation that explicitly represents the collecting surface, glass cover and contained fluid each as several nodes would, in addition, allow an assessment of temperatures gradients, energy enhancing flow regimes, heat loss (convective and radiative) minimization, condensation potential *etc*.