Tutorial 4: Response function method

Q1. Outline the calculation steps of a time domain response function method.

1. A given heat transfer equation in the time domain is transformed into a subsidiary equation in an imaginary space using a Laplace transform;
2. this subsidiary equation is solved while imposing a unit boundary condition – this gives a Unit Response Function (URF);
3. actual boundary conditions (BC) are resolved into equivalent triangular approximations;
4. this BC representation is multiplied by the URF to give the specific system response; and
5. responses from different BC/URF pairs are added to give the overall response.

Q2. What is the principal difference between the time and frequency domain variants of the response function method?

The time domain response function method operates in the time domain and approximates continuous phenomena such as flux and temperature as a series of triangular pulses. The frequency domain response function method operates in the frequency domain and approximates continuous phenomena as a series of sine waves of increasing frequency and reducing amplitude.

Q3. Identify 3 principal assumptions underlying the Admittance Method and indicate how these assumptions will reduce accuracy.

1. Boundary condition is mean + 24 hour harmonic. This means that some climate time series might be poorly represented and that casual gains might be 'clipped'.
2. Problem parameters (e.g. heat transfer coefficients, material properties *etc*) are time-invariant where in reality they will vary as a function of temperature and moisture content. This will lead to an under- or over-estimation of heat flow depending on the departure of the system from the standard conditions implicit in the assumed parameter values.
3. Solar gain is handled by solar gain factors that embody assumptions on shortwave radiation penetration through windows and intra-zone distribution. This means that the complex shading devices and non-orthogonal room geometries are crudely represented.

Q4. Identify the three response factors as used in the Admittance Method and state the principal energy flow to which each relates.

Admittance: ratio of the swing in energy entering an internal surface to the corresponding temperature swing at the environmental point – surface convection.
Decrement: ratio of the cyclic flux transmission through a construction to the steady state flux transmission – transient conduction.
Surface Factor: ratio of the cyclic heat flux re-admitted from a surface to the total cyclic flux absorbed – solar radiation.

Q5. Describe the influence of the surface admittance factor on a room's response to a temperature fluctuation.

Materials with low admittance values (if located at the innermost position of a wall) cannot readily absorb any fluctuation in room temperature (insulation products generally have lower admittance values). Such designs will tend to overheat.

Q6. Using the data sheets provided, calculate the internal environmental temperature likely to occur at 15h00 on a sunny day in August in a West-facing office located at 51.7°N as described by the following data.

Internal dimensions: 6.5m x 5m x 3m high.
External wall: 6.5m x 3m, light external finish.
Window: 3.5m x 2m, open during day, closed at night, internal (dark) Venetian blind.
Occupancy: 4 persons for 8 hours at 85W (sensible) per person.
Lighting: 20 W/m^2 of floor area, ON 07h:00-18h00.
IT equipment: 7.5 W/m^2 of floor area, ON continuously.

Construction details:

	U-value W/m^{2}C	**Admittance W/m^{2}C**	**Decrement -**	**Lag h**	**Comment**
External wall	0.59	0.91	0.3	5	**220 mm brickwork 25 mm cavity 25 mm insulation 10 mm plasterboard**
Window	2.9	2.9	-	-	**Double glazed 12 mm air gap (ignore frame)**
Internal walls	1.9	3.6	-	-	**220 mm brickwork 13 mm light plaster**
Floor	1.5	2.9	-	-	**150 mm cast concrete 50 mm screed 25 mm wood block**
Ceiling	**1.5**	**6.0**	**-**	**-**	**As floor but reversed.**

1) Mean internal temperature

Mean solar gain:
$Q'_s = S' I'_T A_g = 0.56 \times 145 \times 3.5 \times 2 = 568.4$ W
S' is found in table A8.2 = 0.56
I'_T is found in table A8.1 = 145 W/m²
A_g is window area = 3.3 x 2 = 7 m²

Total mean heat gain:
$Q'_t = Q'_c + Q'_s$
$= 1223.4$ W

Mean casual gain:
$Q'_c = 113.3 + 297.9 + 243.8 = 655$ W
Persons: (4 x 85 x 8)/24 = 113.3 W
Lighting: (20 x 6.5 x 5 x 11)/24 = 297.9 W
IT equipment: (7.5 x 6.5 x 5 x 24)/24 = 243.8 W

Mean internal temperature:
$Q'_t = (A_g U_g + C_V)(T'_{ei} - T'_{ao}) + A_f U_f \ (T'_{ei} - T'_{eo})$ (1)
$T'_{ao} = 16.5°C$, $T'_{eo} = 19°C$ (Table A8.3)
$1/C_V = 1/(0.33 \ N \ V) + 1 / (4.8 \ \Sigma A)$
N = 3/h (Table A8.4) , V = 6.5 x 5 x 3 = 97.5 m³
ΣA = (6.5 x 5 +6.5 x 3 + 5 x 3) x 2 = 134 m²
$1/C_V = 1/(0.33 \times 3 \times 97.5) + 1/(4.8 \times 134) \Rightarrow C_V =$ 83.9 W/K
$Q't = 1223.4 = (3.5 \times 2 \times 2.9 + 83.9) \times (T'_{ei} - 16.5) + (6.5 \times 3 - 7) \times 0.59 \times (T'_{ei} - 19)$

=> **$T'_{ei} = 27.6$ °C**

2) Swing in internal temperature

Solar gain swing:
$Q_{s,swing} = S_a A_g (I_p - I'_T)$
S_a (Table A8.6) = 0.57
I_p (Table A8.1) = 460 W/m² (value at 14h00 to account for a 1 hour lag)
$Q_{s,swing} = 0.57 \times 3.5 \times 2 \times (460 - 145)$
$= 1256.9$ W

Casual gain swing:
$Q_{c,swing} = Q_c - Q'_c$
$= 4 \times 85 + 6.5 \times 5 \times 20 + 6.5 \times 5 \times 7.5 - 656.5$
$= 577.3$ W

Total swing in gains:
$Q_{t,swing} = Q_{s,swing} + Q_{struc,swing} + Q_{c,swing} + Q_{a,swing}$
$= 1256.9 + 6.6 + 577.3 + 521$
$= 2355.2$ W

Structural gain swing:
$Q_{struc,swing} = f \ A \ U \ (T_{eo} - T'_{eo})$
f (decrement) = 0.3, U=0.59 W/m²
T_{eo} = 19°C (Table A8.3, value at 7h00 to account for the 5 hour time lag)
$Q_{struc,swing} = 0.3 \times (6.5 \times 3 - 3.5 \times 2) \cdot 0.59 \cdot (19-19)$
$= 0$ W

Swing in gain, air-to-air:
$Q_{a,swing} = (A_g U_g + Cv) \ T_{ao,swing}$ (2)
$T_{ao,swing} = T_{ao} - T'_{ao} = 21.5 - 16.5 = 5$ °C (Table A8.3)
$Q_{a,swing} = (3.5 \times 2 \times 2.9 + 83.9) \times 5$
$= 521$ W

Swing in internal temperature:
$Q_{t,swing} = (\Sigma AY + C_V) \ T_{ei,swing}$ (3)
ΣAY = 32.5 x 2.9 (floor) + 32.5 x 6 (ceiling) + 7 x 2.9 (window) + (6.5 x 3 – 7) x 0.91 (external wall) + 49.5 x 3.6 (internal walls)
= 499.13
$T_{ei,swing} = 2355.2 / (499.13 + 83.9)$

=> **$T_{ei,swing} = 4.0$°C**

3) Peak internal temperature

$T_{ei} = T'_{ei} + T_{ei,swing} = 27.6 + 4.0 =$ **31.6 °C**

Q7. Following on from the previous question, and by calculation, determine which of the following design modifications will have the greatest impact: replacing the internal blind with a dark, external, louvered shading device; or leaving the window open at night.

i) Effect of external shading device:

Mean solar gain factor is 0.1 and therefore T'_{ei} falls to **23.4ºC**

Alternating solar gain factor is 0.07 and therefore $T_{ei,swing}$ falls to **2.1ºC**

=> T_{ei} = 25.5ºC

ii) Effect of night ventilation:

C_v rises to 214.5 (i.e. an air change rate of 10 hr^{-1}).

Substitution in (1) gives T'_{ei} = **21.6ºC**.

Substitution in (2) gives new $Q_{a,swing}$ (1174 W) and therefore $Q_{t,swing}$ as 3008.2 W.

Substitution of new $Q_{t,swing}$ in (3) gives $T_{ei,swing}$ = **4.2ºC**

=> T_{ei} = **25.8ºC**

Therefore overheating is best reduced by external shading although the benefit is marginal.