Shortwave solar radiation

Calculating equation coefficients

Construction Conservation Equation

$$\left(2\rho_I(t+\delta t)C_I(t+\delta t)+\frac{2\delta t\,k(t+\delta t)}{\delta x_I^2}\right)\theta(I,t+\delta t)$$
$$-\frac{\delta t\,k(t+\delta t)}{\delta x_I^2}\theta(I-1,t+\delta t)-\frac{\delta t\,k(t+\delta t)}{\delta x_I^2}\theta(I+1,t+\delta t)-\frac{\delta t\,q_I(t+\delta t)}{\delta x_I\delta x_J\delta x_K}$$
$$=\left(2\rho_I(t)C_I(t)-\frac{2\delta t\,k(t)}{\delta x_I^2}\right)\theta(I,t)$$
$$+\frac{\delta t\,k(t)}{\delta x_I^2}\theta(I-1,t)+\frac{\delta t\,k(t)}{\delta x_I^2}\theta(I+1,t)+\frac{\delta t\,q_I(t)}{\delta x_I\delta x_J\delta x_K}.$$

Surface Conservation Equation

$$\left(2W_I(t+\delta t)+\frac{\delta t\,k'_{I-1,I}(t+\delta t)}{\delta x_{I-1,I}\delta_{I,I-1}}+\frac{\delta t\,h_{cI+1,I}(t+\delta t)}{\delta_{I,I-1}}+\frac{\delta t\sum_{s=1}^{N}h_{rs,I}(t+\delta t)}{\delta_{I,I-1}}\right)\theta(I,t+\delta t)$$
$$-\frac{\delta t\,k'_{I-1,I}(t+\delta t)}{\delta x_{I-1,I}\delta_{I,I-1}}\theta(I-1,t+\delta t)-\frac{\delta t\,h_{cI+1,I}(t+\delta t)}{\delta_{I,I-1}}\theta(I+1,t+\delta t)$$
$$-\frac{\delta t\sum_{s=1}^{N}h_{rs,I}(t+\delta t)\theta(s,t+\delta t)}{\delta_{I,I-1}}-\frac{\delta t\,q_{PI}(t+\delta t)}{\delta_{I,I-1}\delta_{J-1,J+1}\delta_{K-1,K+1}}$$
$$=\left(2W_I(t)-\frac{\delta t\,k'_{I-1,I}(t)}{\delta x_{I-1,I}\delta_{I,I-1}}-\frac{\delta t\,h_{cI+1,I}(t)}{\delta_{I,I-1}}-\frac{\delta t\sum_{s=1}^{N}h_{rs,I}(t)}{\delta_{I,I-1}}\right)\theta(I,t)$$
$$+\frac{\delta t\,k'_{I-1,I}(t)}{\delta x_{I-1,I}\delta_{I,I-1}}\theta(I-1,t)+\frac{\delta t\,h_{cI+1,I}(t)}{\delta_{I,I-1}}\theta(I+1,t)+\frac{\delta t\sum_{s=1}^{N}h_{rs,I}(t)\theta(s,t)}{\delta_{I,I-1}}$$
$$+\frac{\delta t[q_{PI}(t)+q_{SI}(t)+q_{RI}(t)+q_{SI}(t+\delta t)+q_{RI}(t+\delta t)]}{\delta_{I,I-1}\delta_{J-1,J+1}\delta_{K-1,K+1}}+\varepsilon.$$

Fluid Conservation Equation

$$\left(2W_I(t+\delta t)+\frac{\delta t\sum_{i=1}^{N}h_{ci,I}(t+\delta t)\delta A_{i,I}}{\delta V_I}+\frac{\delta t\sum_{j=1}^{M}v_{j,I}(t+\delta t)\bar{\rho}_{j,I}(t+\delta t)\bar{C}_{j,I}(t+\delta t)}{\delta V_I}\right)\theta(I,t+\delta t)$$
$$-\frac{\delta t\sum_{i=1}^{N}h_{ci,I}(t+\delta t)\delta A_{i,I}\theta(i,t+\delta t)}{\delta V_I}-\frac{\delta t\sum_{j=1}^{M}v_{j,I}(t+\delta t)\bar{\rho}_{j,I}(t+\delta t)\bar{C}_{j,I}(t+\delta t)\theta(j,t+\delta t)}{\delta V_I}$$
$$-\frac{\delta t\,q_I(t+\delta t)}{\delta V_I}=\left(2W_I(t)-\frac{\delta t\sum_{i=1}^{N}h_{ci,I}(t)\delta A_{i,I}}{\delta V_I}-\frac{\delta t\sum_{j=1}^{M}v_{j,I}(t)\bar{\rho}_{j,I}(t)\bar{C}_{j,I}(t)}{\delta V_I}\right)\theta(I,t)$$
$$+\frac{\delta t\sum_{i=1}^{N}h_{ci,I}(t)\delta A_{i,I}\theta(i,t)}{\delta V_I}+\frac{\delta t\sum_{j=1}^{M}v_{j,I}(t)\bar{\rho}_{j,I}(t)\bar{C}_{j,I}(t)\theta(j,t)}{\delta V_I}+\frac{\delta t\,q_I(t)}{\delta V_I}+\varepsilon$$

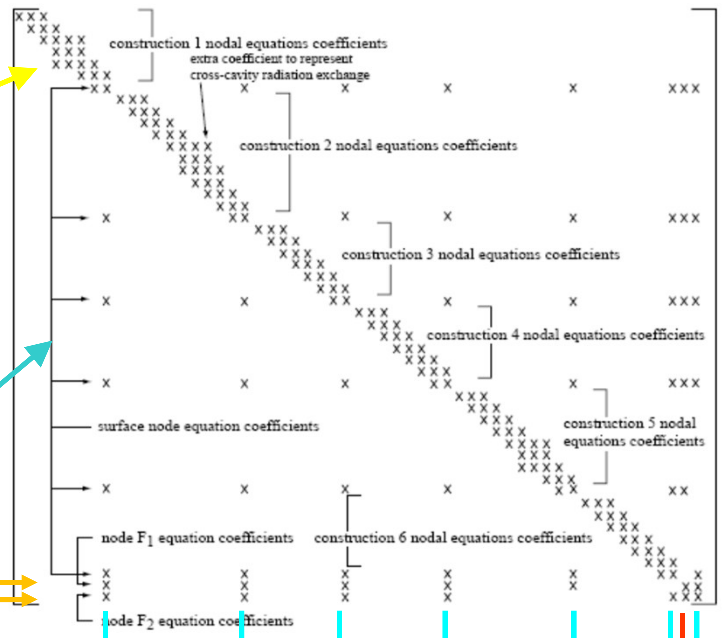
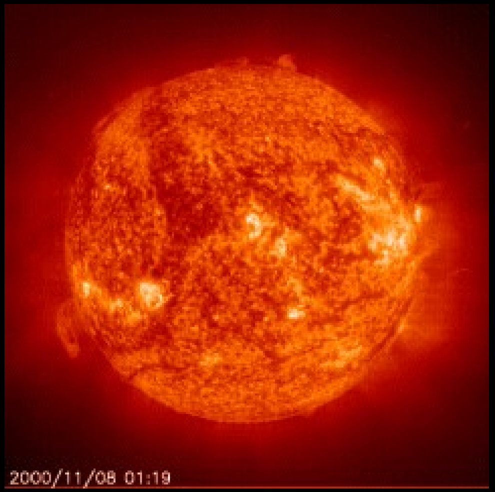


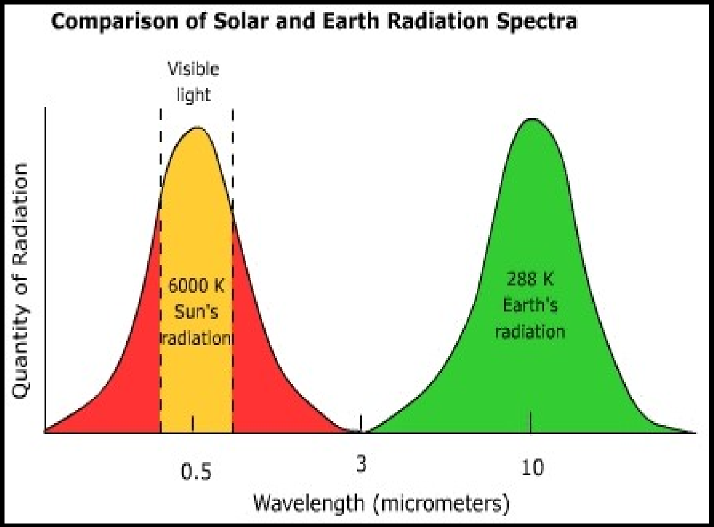
Figure 3.13: The future time-row coefficients matrix (**A**) of the single zone matrix equation $\mathbf{A}\theta_{n+1} = \mathbf{B}\theta_n + \mathbf{C}$.

needs flow estimation

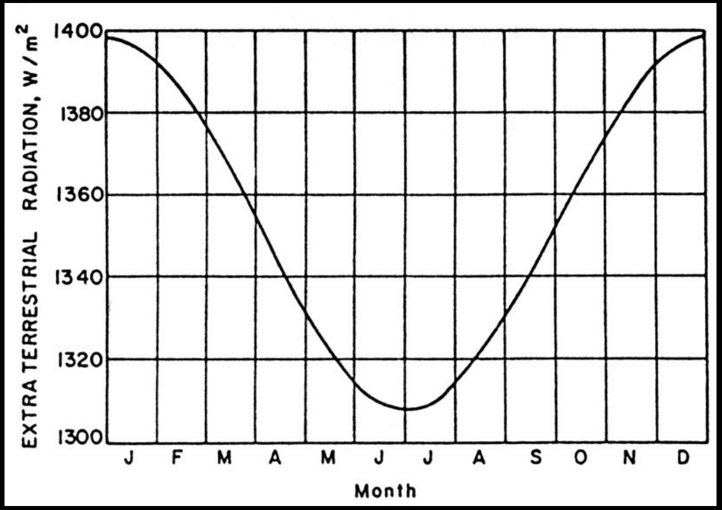
needs radiation and convection estimation

The Sun

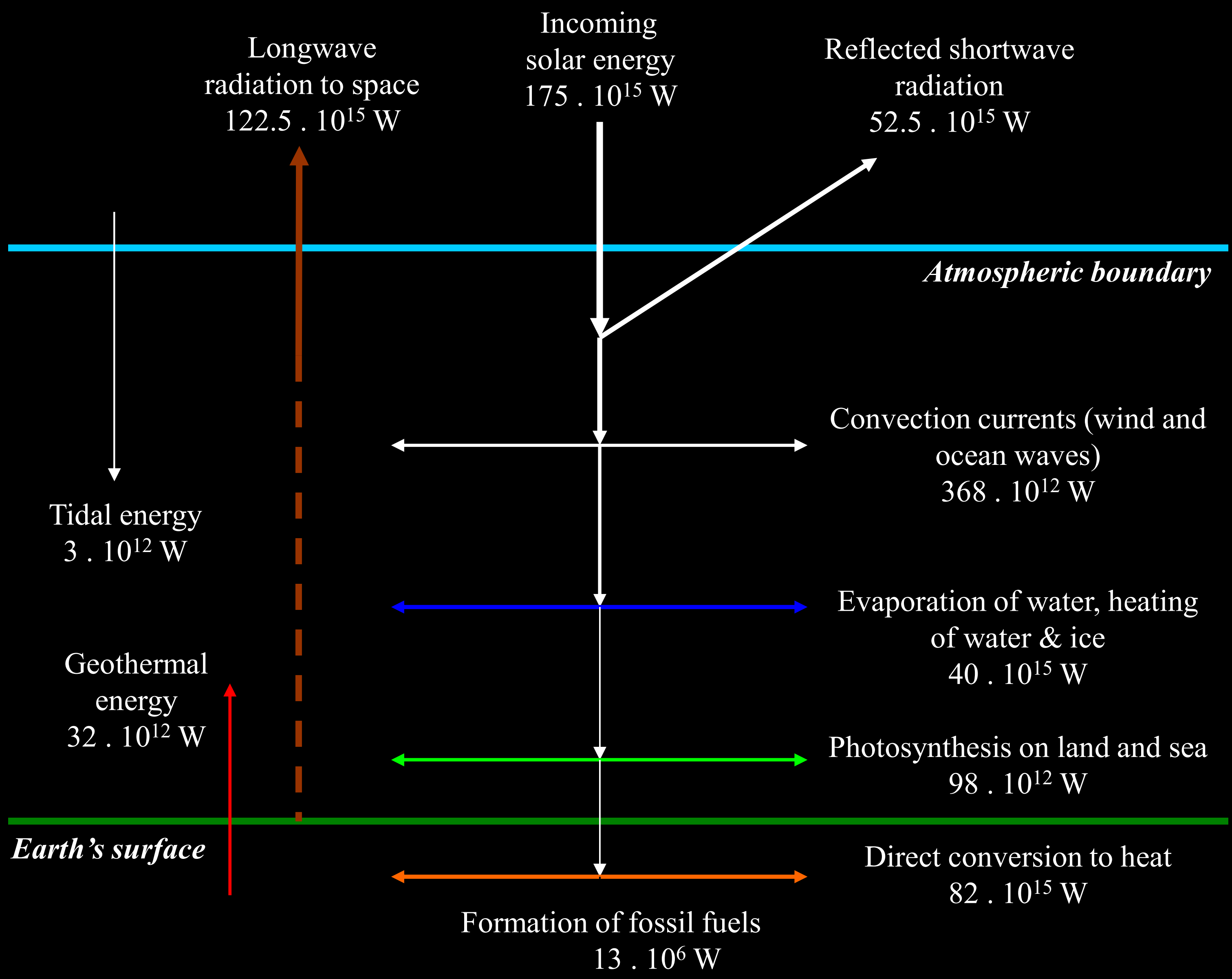




- Core temperature 8×10^6 to 40×10^6 K.
- Effective black body temperature of 6000 K.
- Solar constant: extraterrestrial flux from the sun received on a unit area perpendicular to the direction of propagation – mean Sun/Earth distance value is 1353 W/m^2.
- Actual extraterrestial radiation varies with time of year as earth-sun distance varies.

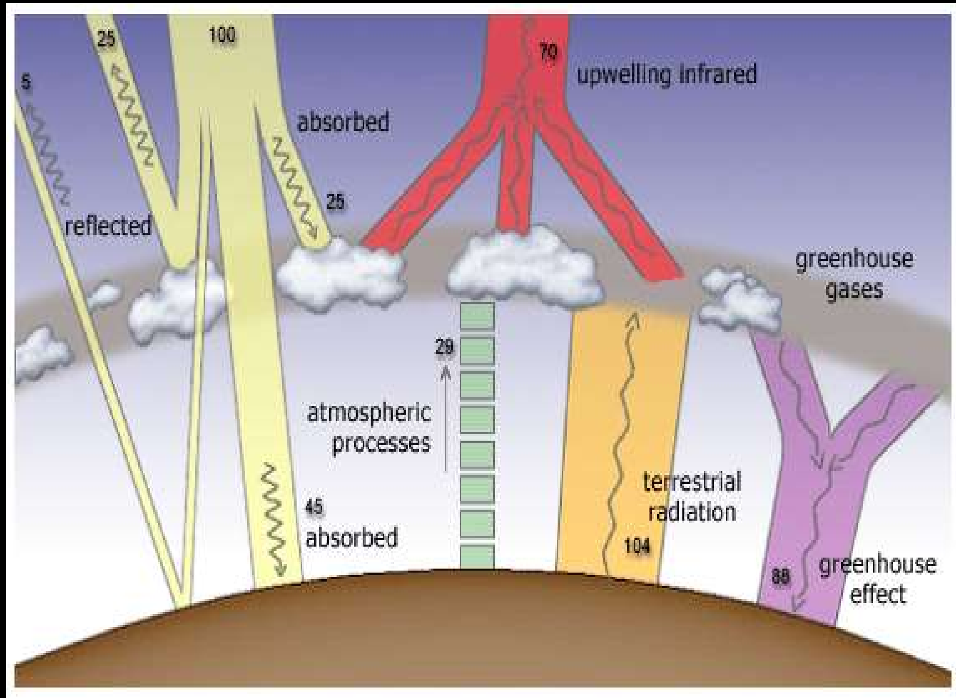


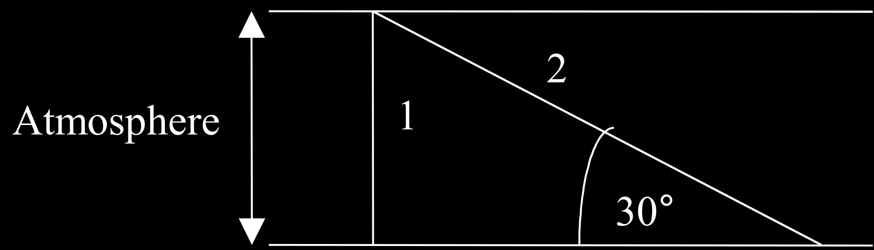
Energy from the sun



Atmospheric interactions

- The greater the distance that the radiation passes through the atmosphere, the greater is the frequency dependent scattering. Spectra at ground level are often referred to particular 'air masses'.
- Air Mass 1 is the thickness of the atmosphere vertically above sea level.
- Air Mass 2 is double this thickness (equivalent to direct solar radiation at an altitude of 30 degrees).



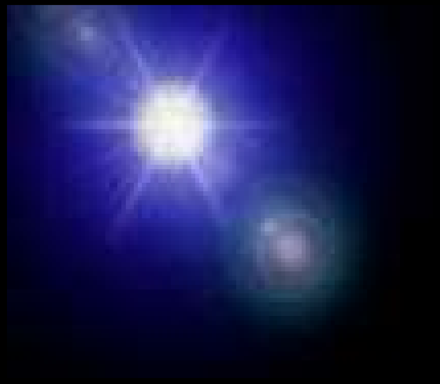


Direct and diffuse radiation

- Solar radiation reaches the Earth directly from the Sun) and diffusely after scattering in the atmosphere and reflected from surrounding objects.

- Only direct radiation can be focussed.

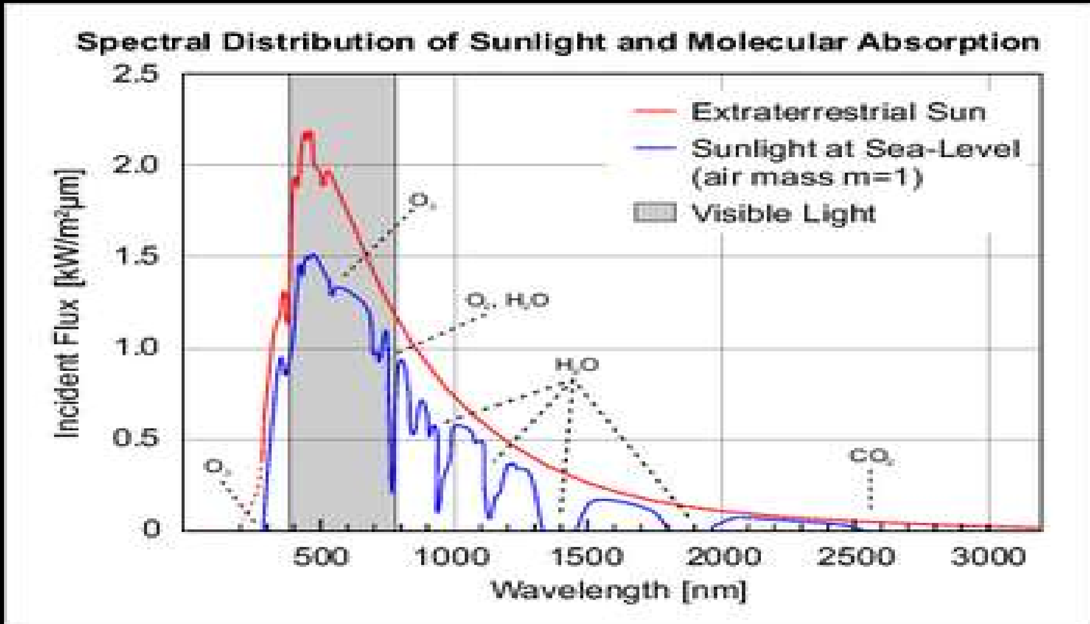
- The total radiation reaching a surface is the summation of the direct, sky diffuse and reflected components.



On clear days around 90% of the total solar radiation is direct.

On heavily overcast days 100% of the solar radiation is diffuse.

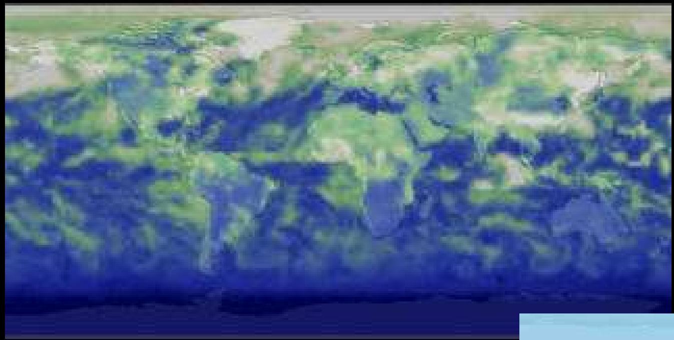
Spectral distribution of short-wave solar radiation



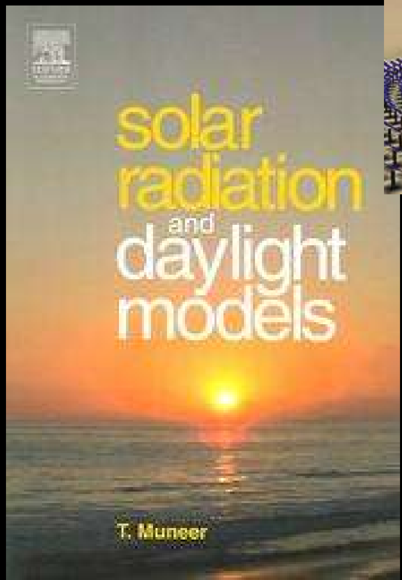
NASA/ASTM Standard Spectral Irradiance

	Wavelength (µm)		
	0 - 0.38	0.38 – 0.78 (visible range)	> 0.78
Fraction in range	0.07	0.47	0.46
Energy in range (W/m^2)	95	640	618

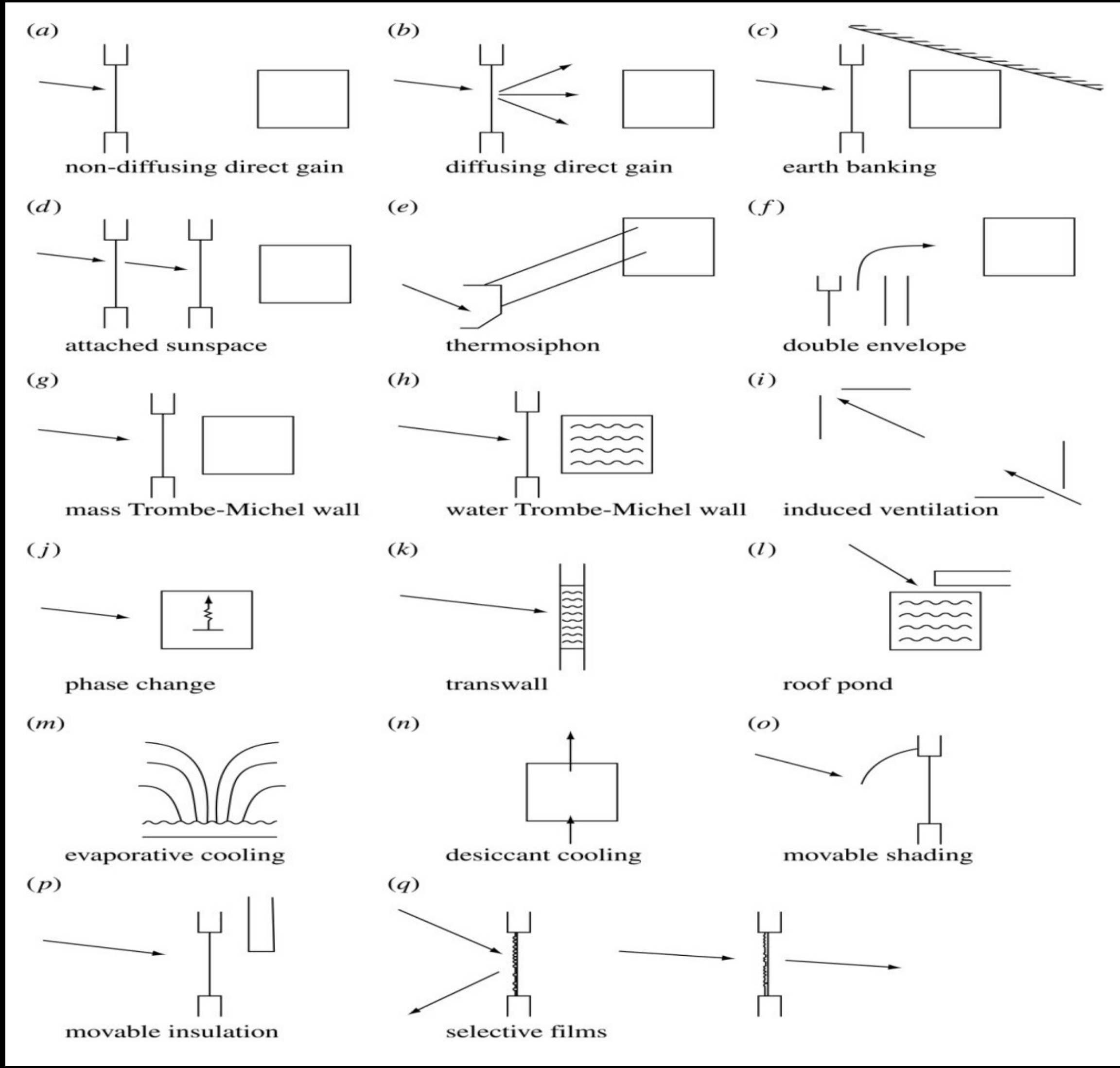
Short-wave radiation impacts



Jul 31 2000

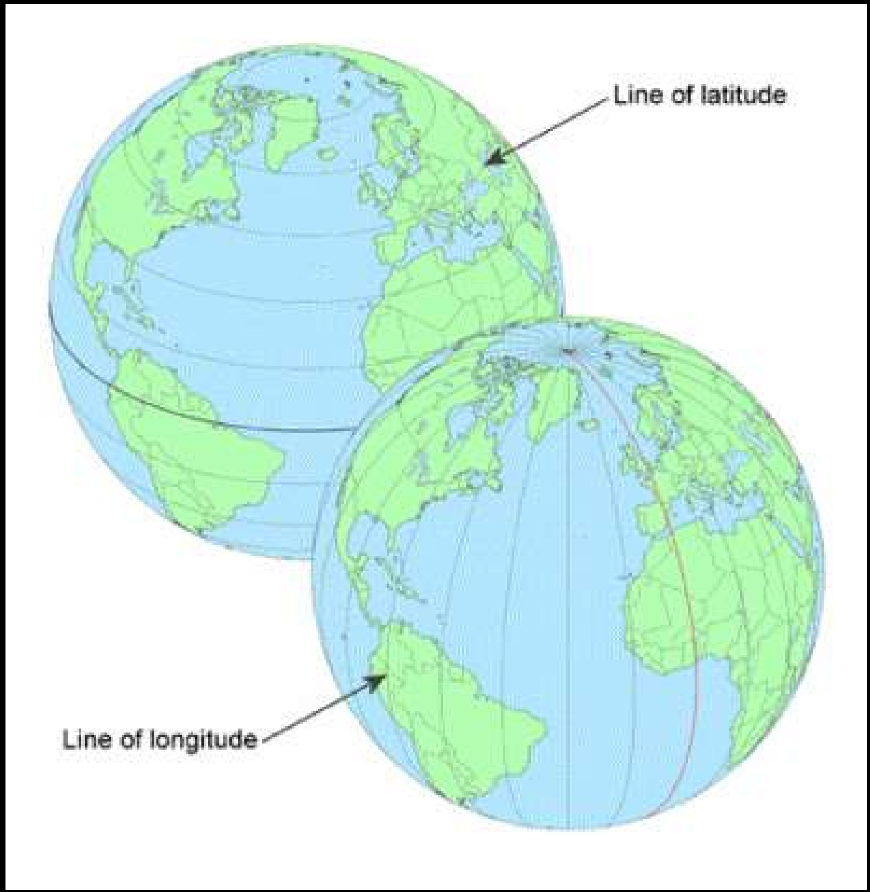


Passive utiulisation

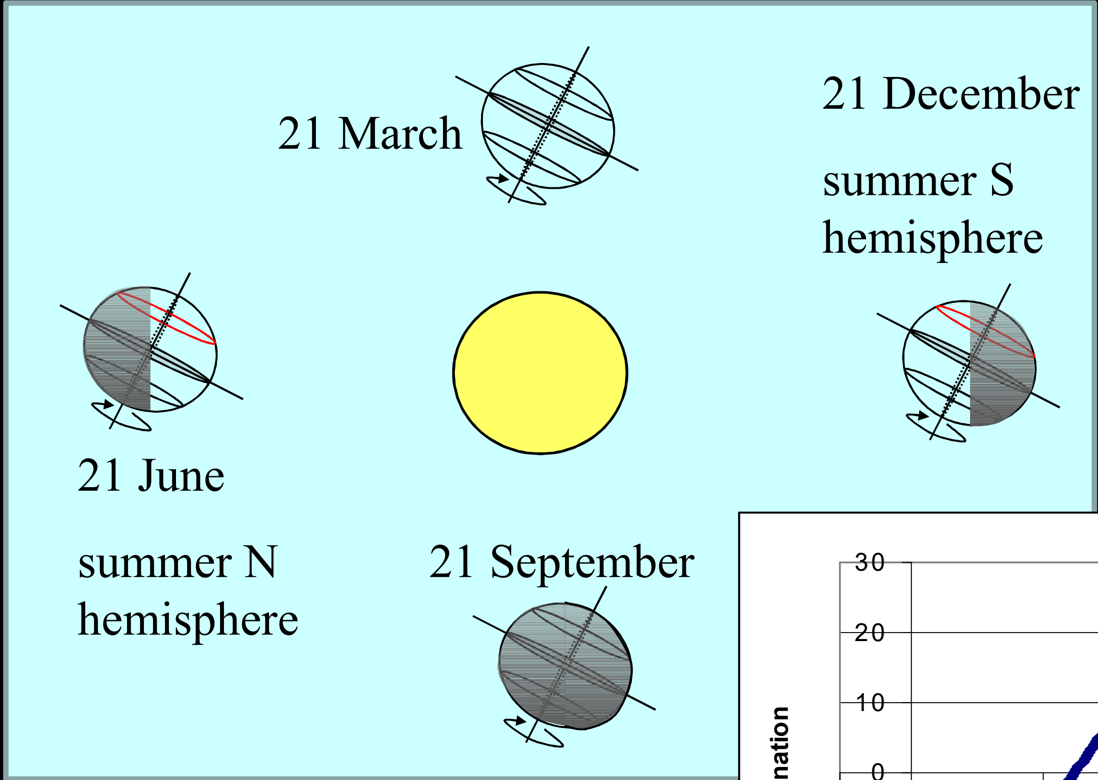


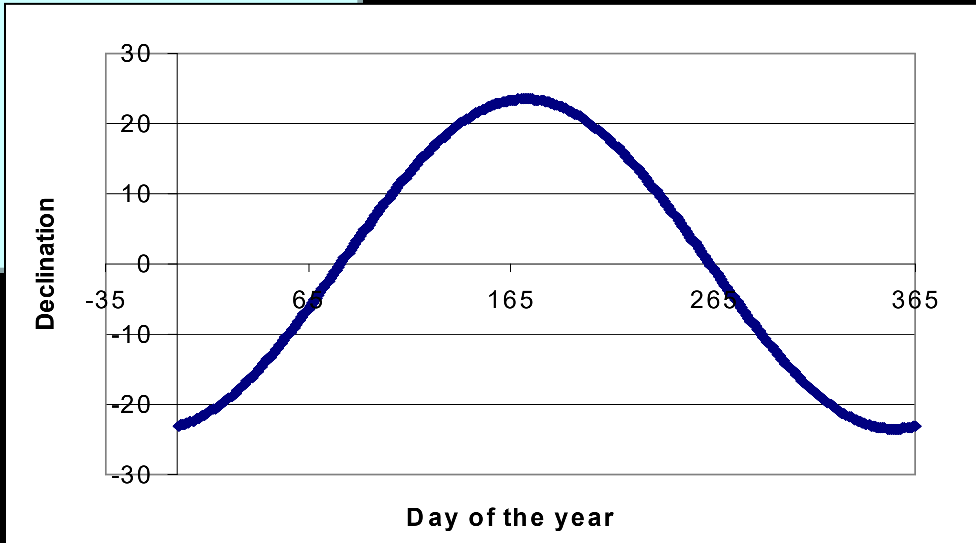
Location coordinates

- latitude - angle N or S above or below equator.
- longitude – angle E or W from prime meridian (Greenwich).
- Longitude difference – angle from location to local time zone reference meridian (west –ve).



Solar declination



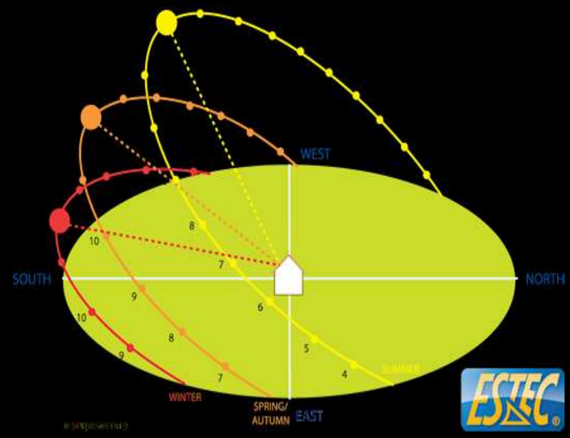


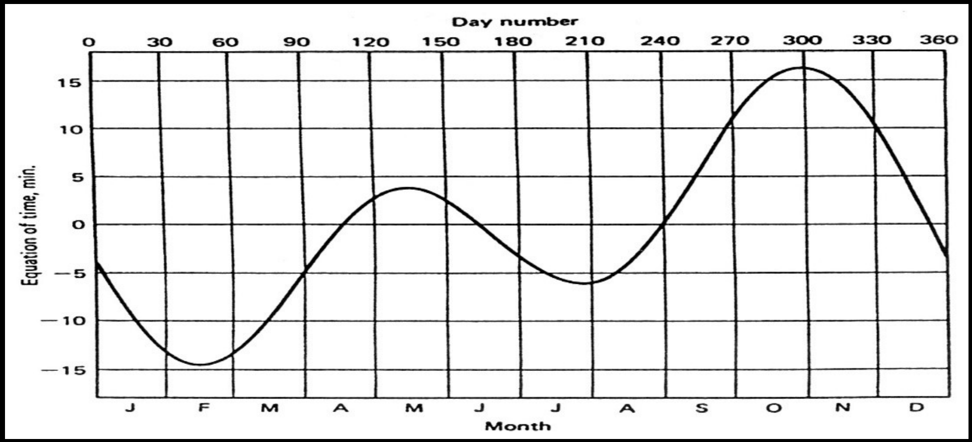
Solar time

$$t_s - t_m = \pm L_{diff}/15 + (e_t/60) + d_s$$

where,

t_s = solar time
t_m = local time
L_{diff} = longitude difference
e_t = equation of time
d_s = daylight saving time





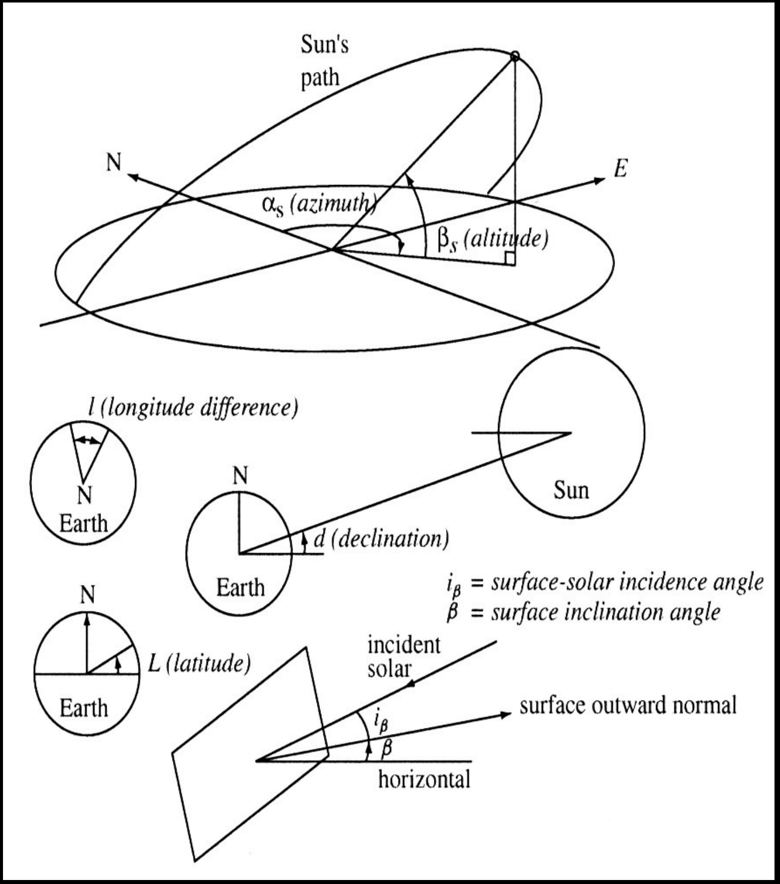
Solar geometry

- Declination
  $d = 23.45 \sin(280.1 + 0.9863\,Y)$
  where Y = year day number (January 1 =1,
  December 31 = 365)

- Altitude
  $\beta_s = \sin^{-1}[\cos L \cos d \cos \theta_h + \sin L \sin d]$
  where L is site latitude,
  θ_h is hour angle = $15\,(12 - t_s)$

- Azimuth
  $\alpha_s = \sin^{-1}[\cos d \sin \theta_h / \cos \beta_s]$

- Incidence angle
  $i_\beta = \cos^{-1}[\sin \beta_s \cos(90-\beta_f) + \cos \beta_s \cos \omega \sin(90-\beta_f)]$
  where ω = azimuth angle between sun and surface normal,
  β_f = surface inclination angle



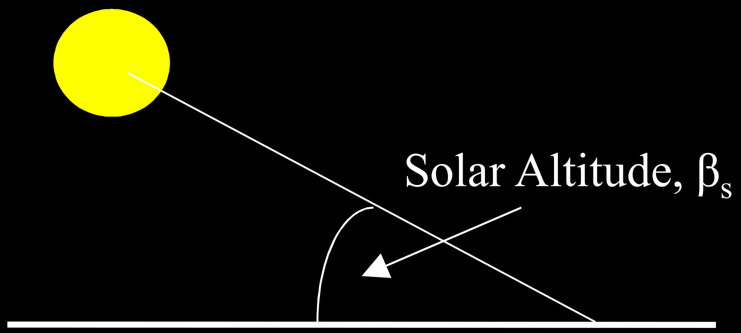
Solar radiation prediction (all W/m^2)

I_{dn} - direct normal or "beam" (pyrheliometer)
I_{dh} - direct horizontal $I_{dh} = I_{dn}\sin\beta_s$
I_{fh} - diffuse horizontal (pyranometer with shadow band)
I_{gh} - global horizontal (pyranometer or solarimeter)
r_g - ground reflectivity

} known

$I_{d\beta}$ - direct radiation on a surface of inclination β_f
$I_{s\beta}$ - sky diffuse radiation incident on a surface of inclination β_f
$I_{r\beta}$ - ground reflected radiation incident on a surface of inclination β_f

} unknown



$$I_{gh} = I_{dh} + I_{fh}$$
$$= I_{dn} \sin \beta_s + I_{fh}$$

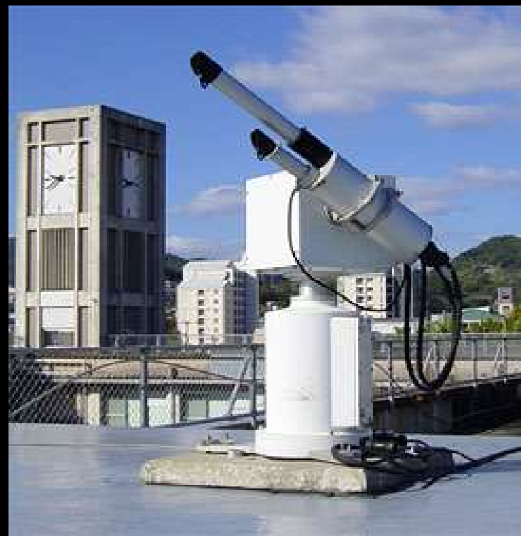
Solar data for simulation:

either: I_{gh} and I_{fh} or I_{dn} and I_{fh}

Solar radiation measurement

- Pyranometer measures the total solar irradiance on a planar surface.

- Pyrheliometer measures direct beam solar radiation by tracking the sun's position throughout the day.



Solar radiation measurement

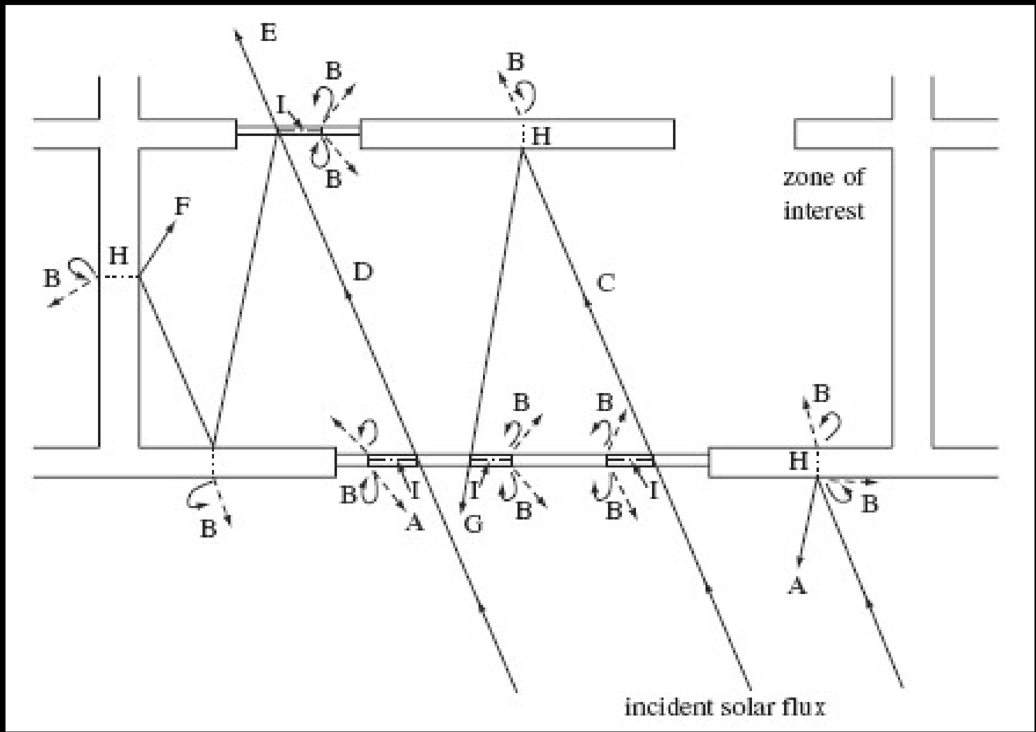
- Shaded pyranometer measures diffuse solar irradiance on a (usually horizontal) surface.

- The shade blocks direct radiation and some diffuse radiation (so need to adjust readings).

- Integrated pyranometer measures both total and diffuse radiation on a (usually horizontal) surface.

- Diffuse is calculated based on shading patterns from internal shades

Short-wave flow-paths



A - reflected shortwave flux
B - flux emission by convection and longwave radiation
C - shortwave flux transmission to cause opaque surface insolation
D - shortwave transmission to cause transparent surface insolation
E - shortwave transmission to adjacent zone
F - enclosure reflections
G - shortwave loss
H - solar energy penetration by transient conduction
I - solar energy absorption prior to retransmission by the processes of B.

Short-wave radiation calculation

Intensity of direct radiation on surface of inclination β:

$I_{d\beta} = I_{dh} \cos i_{\beta} / \sin \beta_s$

Intensity of diffuse radiation on same surface

ground reflected: $I_{r\beta} = 0.5\,[1 - \cos(90 - \beta_f)]\,(I_{dh} + I_{fh})\,r_g$
where r_g is the ground reflectance

sky component: $I_{s\beta} = 0.5\,[1 + \cos(90 - \beta_f)]\,I_{fh}$
assuming an isotropic diffuse sky

i_β - angle between the incident beam and the surface normal vector

ω - surface-solar azimuth ($= |\alpha_s - \alpha_f|$)

α_f, β_f - surface azimuth and inclination respectively

α_s, β_s - solar azimuth and elevation respectively

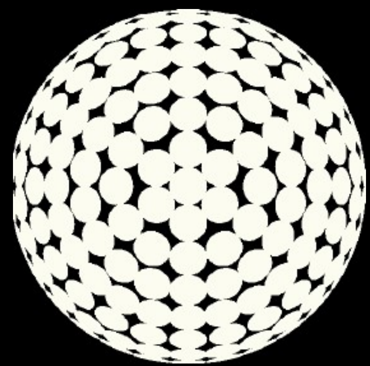
In practice the sky is not isotropic and so empirically-based models that correct for circumsolar and horizon brightening are employed:

sky component:

$$I_{s\beta} = I_{fh}\left(\frac{1 + \cos(90 - \beta_f)}{2}\right) \times \left(1 + \left[1 - \left(\frac{I_{fh}^2}{I_{gh}^2}\right)\right]\sin^3\left(\frac{\beta_f}{2}\right)\right) \times \left(1 + \left[1 - \left(\frac{I_{fh}^2}{I_{gh}^2}\right)\right]\cos^2(i_\beta)\sin^3(90 - \beta_s)\right)$$

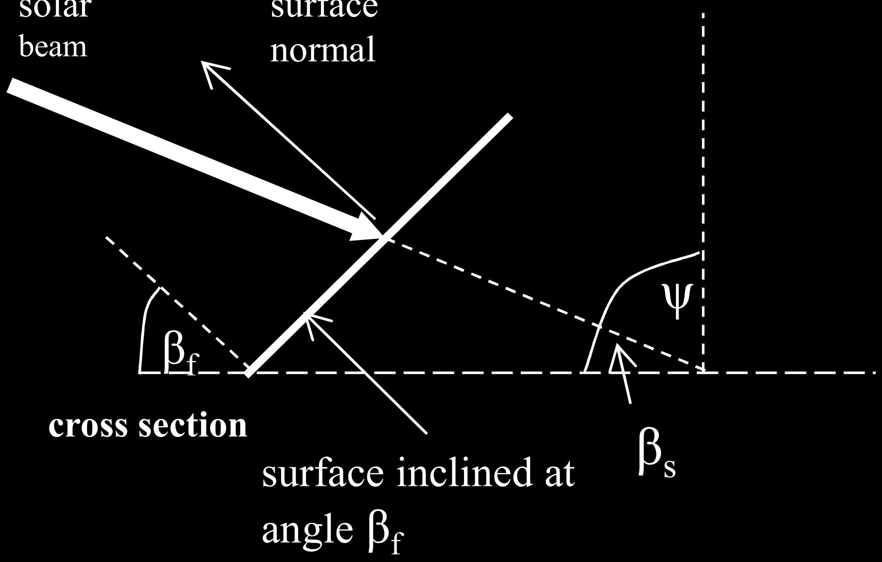
Angle of incidence:

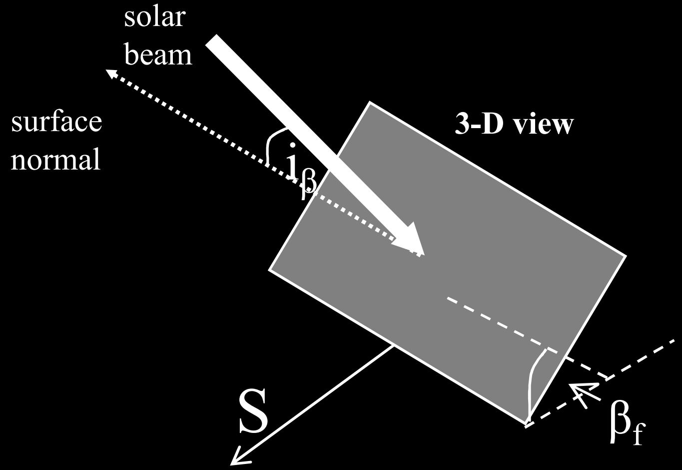
$$i_\beta = \cos^{-1}\left(\sin\beta_s \cos(90 - \beta_f) + \cos\beta_s \cos\omega \sin(90 - \beta_f)\right)$$

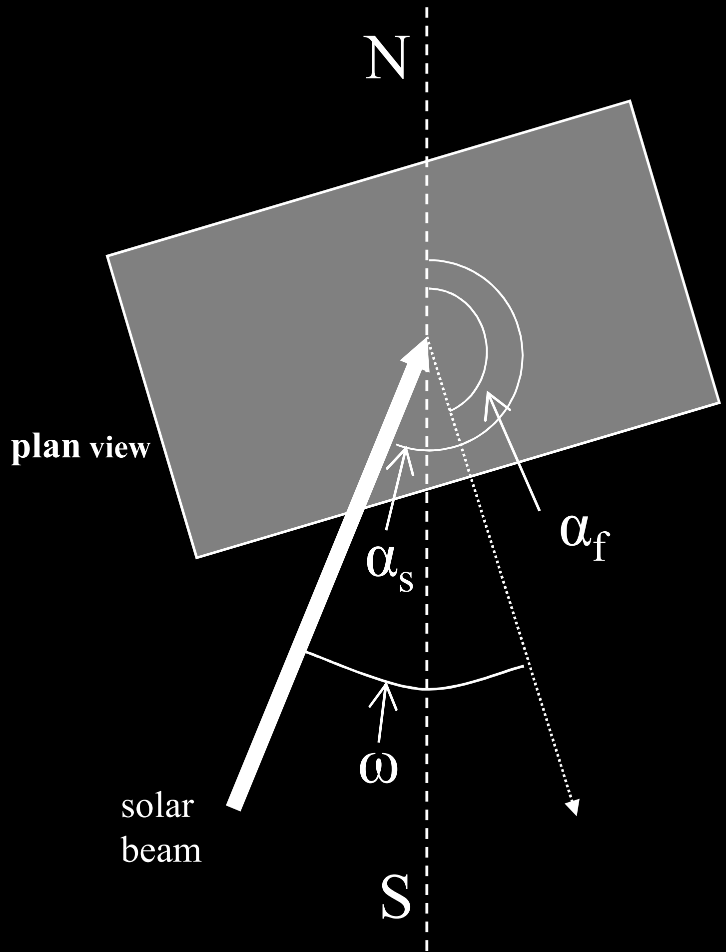


Numerical approach using 145 sky vault patches.

Surface-solar angles







Solar angle tables (altitude & azimuth)

North Latitude	Sun Time	Jan. 21		Feb. 21		Mar. 21		Apr. 22		May 22		June 21		July 23		Aug. 22		Sept. 22		Oct. 22		Nov. 22		Dec. 21		Sun Time
		Alt	Az	Alt	Az	Alt	Az	Alt	Az	Alt	Az	Alt	Az	Alt	Az	Alt	Az	Alt	Az	Alt	Az	Alt	Az	Alt	Az	
	06					0	90	8	81	13	74	15	72	13	74	8	81	0	90							06
	07			4	108	11	100	19	90	24	83	26	80	24	83	19	90	11	100	4	108					07
	08	8	125	15	118	22	110	31	100	36	92	37	89	36	92	31	100	22	110	15	118	8	125	6	127	08
	09	17	136	24	130	33	123	42	112	47	104	49	100	47	104	42	112	33	123	24	130	17	136	14	138	09
	10	24	149	32	145	42	138	52	128	58	118	60	114	58	118	52	128	42	138	32	145	24	149	21	151	10
	11	28	164	37	161	48	157	59	150	67	142	69	138	67	142	59	150	48	157	37	161	28	164	25	165	11
40°	12	30	180	39	180	50	180	62	180	70	180	74	180	70	180	62	180	50	180	39	180	30	180	27	180	12
	13	28	196	37	199	48	203	59	210	67	218	69	222	67	218	59	210	48	203	37	199	28	196	25	195	13
	14	24	211	32	215	42	222	52	232	58	242	60	246	58	242	52	232	42	222	32	215	24	211	21	209	14
	15	17	224	24	230	33	237	42	248	47	256	49	260	47	256	42	248	33	237	24	230	17	224	14	222	15
	16	8	235	15	242	22	250	31	260	36	268	37	271	36	268	31	260	22	250	15	242	8	235	6	233	16
	17			4	252	11	260	19	270	24	277	26	280	24	277	19	270	11	260	4	252					17
	18					0	270	8	279	13	286	15	288	13	286	8	279	0	270							18
	06					0	90	8	81	14	75	16	73	14	75	8	81	0	90							06
	07			3	108	10	101	19	92	25	85	27	83	25	85	19	92	10	101	3	108					07
	08	5	125	12	120	21	112	30	103	35	96	37	93	35	96	30	103	21	112	12	120	5	125	2	127	08
	09	13	137	21	132	30	125	40	116	46	108	48	105	46	108	40	116	30	125	21	132	13	137	10	139	09
	10	19	150	28	146	38	141	48	133	55	125	58	121	55	125	48	133	38	141	28	146	19	150	16	152	10
	11	24	165	32	162	43	159	55	154	62	148	65	146	62	148	55	154	43	159	32	162	24	165	20	165	11
45°	12	25	180	34	180	45	180	57	180	65	180	68	180	65	180	57	180	45	180	34	180	25	180	22	180	12
	13	24	195	32	198	43	201	55	206	62	212	65	214	62	212	55	206	43	201	32	198	24	195	20	195	13
	14	19	210	28	214	38	219	48	227	55	235	58	239	55	235	48	227	38	219	28	214	19	210	16	208	14
	15	13	223	21	228	30	235	40	244	46	252	48	255	46	252	40	244	30	235	21	228	13	223	10	221	15
	16	5	235	12	240	21	248	30	257	35	264	37	267	35	264	30	257	21	248	12	240	5	235	2	233	16
	17			3	252	10	259	19	268	25	275	27	277	25	275	19	268	10	259	3	252					17
	18					0	270	8	279	14	285	16	287	14	285	8	279	0	270							18

Solar tables (I_{dv} & I_{dh})

Table A2.35 *(m)* Basic direct solar irradiances on vertical, I_{DV}, and horizontal, I_{DH}, surfaces and basic diffuse (cloudy and clear sky) solar irradiances on horizontal surfaces, I_{dH}, (W/m²).

55°N

Date	Orientation	Daily mean	Sun Time 03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21
June 21	N	35		95	175	135	25	0	0	0	0	0	0	0	0	0	25	135	175	95	
	NE	85		160	385	485	470	365	205	20	0	0	0	0	0	0	0	0	0	0	
	E	145		130	365	550	640	630	545	395	210	0	0	0	0	0	0	0	0	0	
	SE	145		20	135	290	435	530	565	540	455	325	160	0	0	0	0	0	0	0	
	S	115		0	0	0	0	115	255	365	435	465	435	365	255	115	0	0	0	0	
	SW	145		0	0	0	0	0	0	0	160	325	455	540	565	530	435	290	135	20	
	W	145		0	0	0	0	0	0	0	0	0	210	395	545	630	640	550	365	130	
	NW	85		0	0	0	0	0	0	0	0	0	0	20	205	365	470	485	385	160	
	H	290		10	80	195	335	465	585	675	735	755	735	675	585	465	335	195	80	10	
Diff (Cldy)		115		20	55	95	140	180	225	260	285	295	285	260	225	180	140	95	55	20	
Diff (Clr)		50		15	45	60	75	80	90	95	100	100	100	95	90	80	75	60	45	15	
July 23 and May 22	N	25		25	135	110	0	0	0	0	0	0	0	0	0	0	0	110	135	25	
	NE	75		45	310	445	445	345	185	0	0	0	0	0	0	0	0	0	0	0	
	E	135		35	305	520	625	630	545	400	210	0	0	0	0	0	0	0	0	0	
	SE	150		5	120	290	445	545	585	565	480	350	185	0	0	0	0	0	0	0	
	S	130		0	0	0	0	145	285	395	470	495	470	395	285	145	0	0	0	0	
	SW	150		0	0	0	0	0	0	0	185	350	480	565	585	545	445	290	120	5	
	W	135		0	0	0	0	0	0	0	0	0	210	400	545	630	625	520	305	35	
	NW	75		0	0	0	0	0	0	0	0	0	0	0	185	345	445	445	310	45	
	H	265		0	50	160	295	430	550	640	700	720	700	640	550	430	295	160	50	0	
Diff (Cldy)		110		5	40	85	125	170	210	245	270	280	270	245	210	170	125	85	40	5	
Diff (Clr)		50		5	35	55	70	80	90	95	100	100	100	95	90	80	70	55	35	5	
August 22 and April 22	N	5			20	45	0	0	0	0	0	0	0	0	0	0	0	45	20		
	NE	45			60	295	355	285	135	0	0	0	0	0	0	0	0	0	0		
	E	115			65	370	555	605	540	400	215	0	0	0	0	0	0	0	0		
	SE	155			30	230	430	570	630	620	540	410	240	50	0	0	0	0	0		
	S	160			0	0	50	200	350	470	550	580	550	470	350	200	50	0	0		
	SW	155			0	0	0	0	0	50	240	410	540	620	630	570	430	230	30		
	W	115			0	0	0	0	0	0	0	0	215	400	540	605	555	370	65		
	NW	45			0	0	0	0	0	0	0	0	0	0	135	285	355	295	60		
	H	205			0	65	185	320	445	540	600	620	600	540	445	320	185	65	0		
Diff (Cldy)		85			5	50	95	135	175	205	230	235	230	205	175	135	95	50	5		
Diff (Clr)		40			5	40	60	70	80	85	90	90	90	85	80	70	60	40	5		

PV power output

A simple model: $$P_{mp} = P_{STC}\frac{J_{tot}}{1000}\left(1-\beta[T-25]\right)\times p$$

Example 1

Calculate the power output from a PV panel at 60°C with 840 W/m^2 incident solar radiation if the same panel produces 150 W at STC (1000W/m^2 & 25°C). β is measured at 0.003 W/K

$$P = P_{STC}\frac{\boldsymbol{J}_{tot}}{1000}\left[1-\beta(T-25)\right]$$

$$P = 150\times\frac{840}{1000}\left[1-0.003(60-25)\right]$$

$$=112.8\,W$$

Example 1

For the same situation calculate the power output if the temperature was 30°C. β is again measured at 0.003 W/K

$$P = P_{STC}\frac{\boldsymbol{J}_{tot}}{1000}\left[1-\beta(T-25)\right]$$

$$P = 150\times\frac{840}{1000}\left[1-0.003(30-25)\right]$$

$$=124.1\,W$$


Longwave Radiation
Exchange
23

Calculating equation coefficients

Construction Conservation Equation

$$\left(2\rho_I(t+\delta t)C_I(t+\delta t) + \frac{2\delta t\, k(t+\delta t)}{\delta x_I^2}\right)\theta(I,t+\delta t) - \frac{\delta t\, k(t+\delta t)}{\delta x_I^2}\theta(I-1,t+\delta t) - \frac{\delta t\, k(t+\delta t)}{\delta x_I^2}\theta(I+1,t+\delta t) - \frac{\delta t\, q_I(t+\delta t)}{\delta x_I \delta x_J \delta x_K}$$
$$= \left(2\rho_I(t)C_I(t) - \frac{2\delta t\, k(t)}{\delta x_I^2}\right)\theta(I,t) + \frac{\delta t\, k(t)}{\delta x_I^2}\theta(I-1,t) + \frac{\delta t\, k(t)}{\delta x_I^2}\theta(I+1,t) + \frac{\delta t\, q_I(t)}{\delta x_I \delta x_J \delta x_K}.$$

Surface Conservation Equation

$$\left(2W_I(t+\delta t) + \frac{\delta t\, k'_{I-1,I}(t+\delta t)}{\delta x_{I-1,I}\delta_{I,I-1}} + \frac{\delta t\, h_{cI+1,I}(t+\delta t)}{\delta_{I,I-1}} + \frac{\delta t \sum_{s=1}^{N} h_{rs,I}(t+\delta t)}{\delta_{I,I-1}}\right)\theta(I,t+\delta t)$$
$$- \frac{\delta t\, k'_{I-1,I}(t+\delta t)}{\delta x_{I-1,I}\delta_{I,I-1}}\theta(I-1,t+\delta t) - \frac{\delta t\, h_{cI+1,I}(t+\delta t)}{\delta_{I,I-1}}\theta(I+1,t+\delta t)$$
$$- \frac{\delta t \sum_{s=1}^{N} h_{rs,I}(t+\delta t)\theta(s,t+\delta t)}{\delta_{I,I-1}} - \frac{\delta t\, q_{PI}(t+\delta t)}{\delta_{I,I-1}\delta_{J-1,J+1}\delta_{K-1,K+1}}$$
$$= \left(2W_I(t) - \frac{\delta t\, k'_{I-1,I}(t)}{\delta x_{I-1,I}\delta_{I,I-1}} - \frac{\delta t\, h_{cI+1,I}(t)}{\delta_{I,I-1}} - \frac{\delta t \sum_{s=1}^{N} h_{rs,I}(t)}{\delta_{I,I-1}}\right)\theta(I,t)$$
$$+ \frac{\delta t\, k'_{I-1,I}(t)}{\delta x_{I-1,I}\delta_{I,I-1}}\theta(I-1,t) + \frac{\delta t\, h_{cI+1,I}(t)}{\delta_{I,I-1}}\theta(I+1,t) + \frac{\delta t \sum_{s=1}^{N} h_{rs,I}(t)\theta(s,t)}{\delta_{I,I-1}}$$
$$+ \frac{\delta t[q_{PI}(t) + q_{SI}(t) + q_{RI}(t) + q_{SI}(t+\delta t) + q_{RI}(t+\delta t)]}{\delta_{I,I-1}\delta_{J-1,J+1}\delta_{K-1,K+1}} + \varepsilon.$$

Fluid Conservation Equation

$$\left(2W_I(t+\delta t) + \frac{\delta t \sum_{i=1}^{N} h_{ci,I}(t+\delta t)\delta A_{i,I}}{\delta V_I} + \frac{\delta t \sum_{j=1}^{M} v_{j,I}(t+\delta t)\bar{\rho}_{j,I}(t+\delta t)\bar{C}_{j,I}(t+\delta t)}{\delta V_I}\right)\theta(I,t+\delta t)$$
$$- \frac{\delta t \sum_{i=1}^{N} h_{ci,I}(t+\delta t)\delta A_{i,I}\theta(i,t+\delta t)}{\delta V_I} - \frac{\delta t \sum_{j=1}^{M} v_{j,I}(t+\delta t)\bar{\rho}_{j,I}(t+\delta t)\bar{C}_{j,I}(t+\delta t)\theta(j,t+\delta t)}{\delta V_I}$$
$$- \frac{\delta t\, q_I(t+\delta t)}{\delta V_I} = \left(2W_I(t) - \frac{\delta t \sum_{i=1}^{N} h_{ci,I}(t)\delta A_{i,I}}{\delta V_I} - \frac{\delta t \sum_{j=1}^{M} v_{j,I}(t)\bar{\rho}_{j,I}(t)\bar{C}_{j,I}(t)}{\delta V_I}\right)\theta(I,t)$$
$$+ \frac{\delta t \sum_{i=1}^{N} h_{ci,I}(t)\delta A_{i,I}\theta(i,t)}{\delta V_I} + \frac{\delta t \sum_{j=1}^{M} v_{j,I}(t)\bar{\rho}_{j,I}(t)\bar{C}_{j,I}(t)\theta(j,t)}{\delta V_I} + \frac{\delta t\, q_I(t)}{\delta V_I} + \varepsilon$$

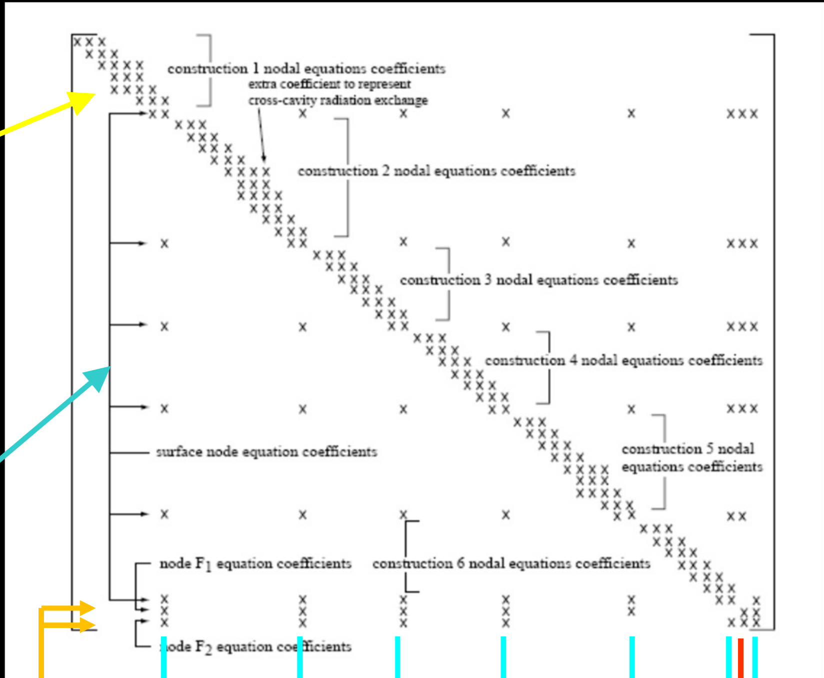


Figure 3.13: The future time-row coefficients matrix (**A**) of the single zone matrix equation $\mathbf{A}\theta_{n+1} = \mathbf{B}\theta_n + \mathbf{C}$.

needs flow estimation

needs radiation and convection estimation

Internal long-wave radiation – calculation

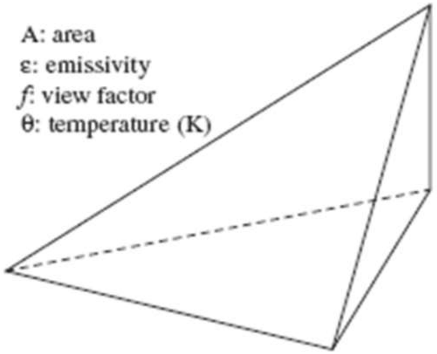


Figure 7.18: Four grey surfaces bounding an enclosure.

$$q_1 = \varepsilon_1 \sigma A_1 \theta_1^4 \qquad q_2 = \varepsilon_2 \sigma A_2 \theta_2^4$$
$$q_3 = \varepsilon_3 \sigma A_3 \theta_3^4 \qquad q_4 = \varepsilon_4 \sigma A_4 \theta_4^4$$

$$\begin{aligned}
a_1' &= & & +q_2 f_{2\to1}\varepsilon_1 & +q_3 f_{3\to1}\varepsilon_1 & +q_4 f_{4\to1}\varepsilon_1 \\
a_2' &= & +q_1 f_{1\to2}\varepsilon_2 & & +q_3 f_{3\to2}\varepsilon_2 & +q_4 f_{4\to2}\varepsilon_2 \\
a_3' &= & +q_1 f_{1\to3}\varepsilon_3 & +q_2 f_{2\to3}\varepsilon_3 & & +q_4 f_{4\to3}\varepsilon_3 \\
a_4' &= & +q_1 f_{1\to4}\varepsilon_4 & +q_2 f_{2\to4}\varepsilon_4 & +q_3 f_{3\to4}\varepsilon_4 &
\end{aligned}$$

$$r_i' = a_i'(1-\varepsilon_i)/\varepsilon_i \quad ; i = 1, 2, 3, 4$$

$$\begin{aligned}
a_1'' &= a_1' & & +r_2' f_{2\to1}\varepsilon_1 & +r_3' f_{3\to1}\varepsilon_1 & +r_4' f_{4\to1}\varepsilon_1 \\
a_2'' &= a_2' & +r_1' f_{1\to2}\varepsilon_2 & & +r_3' f_{3\to2}\varepsilon_2 & +r_4' f_{4\to2}\varepsilon_2 \\
a_3'' &= a_3' & +r_1' f_{1\to3}\varepsilon_3 & +r_2' f_{2\to3}\varepsilon_3 & & +r_4' f_{4\to3}\varepsilon_3 \\
a_4'' &= a_4' & +r_1' f_{1\to4}\varepsilon_4 & +r_2' f_{2\to4}\varepsilon_4 & +r_3' f_{3\to4}\varepsilon_4 &
\end{aligned}$$

$$r_1'' = (a_1'' - a_1')(1-\varepsilon_1)/\varepsilon_1 \qquad r_2'' = (a_2'' - a_2')(1-\varepsilon_2)/\varepsilon_2$$
$$r_3'' = (a_3'' - a_3')(1-\varepsilon_3)/\varepsilon_3 \qquad r_4'' = (a_4'' - a_4')(1-\varepsilon_4)/\varepsilon_4$$

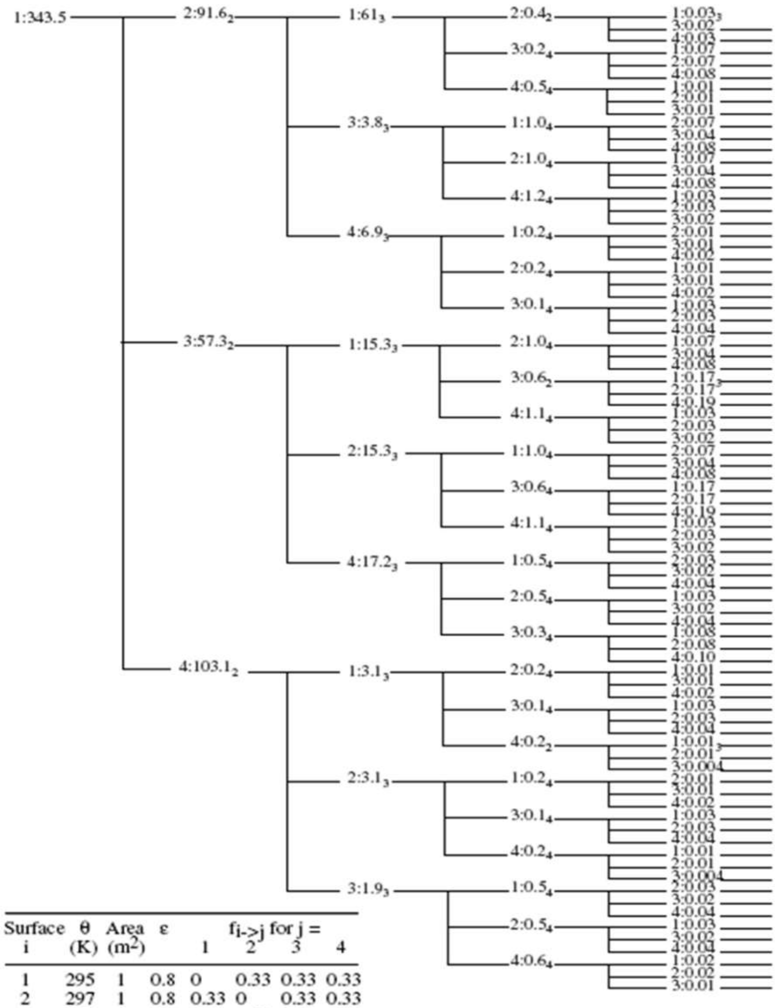
$$\left.\begin{aligned}
a_i^n &= a_i^{n-1} + \sum_{j=1}^{N} r_j^{n-1} f_{j\to i}\varepsilon_i \\
r_i^n &= (a_i^n - a_i^{n-1})(1-\varepsilon_i)/\varepsilon_i
\end{aligned}\right\} \begin{aligned} 1 \le n \le \infty \\ a_i^0 = 0 \\ r_i^0 = q_i \\ f_{i\to i} = 0 \end{aligned}$$

Internal long-wave radition

$$Q_e = \varepsilon\, \sigma\, A\, \theta^4$$

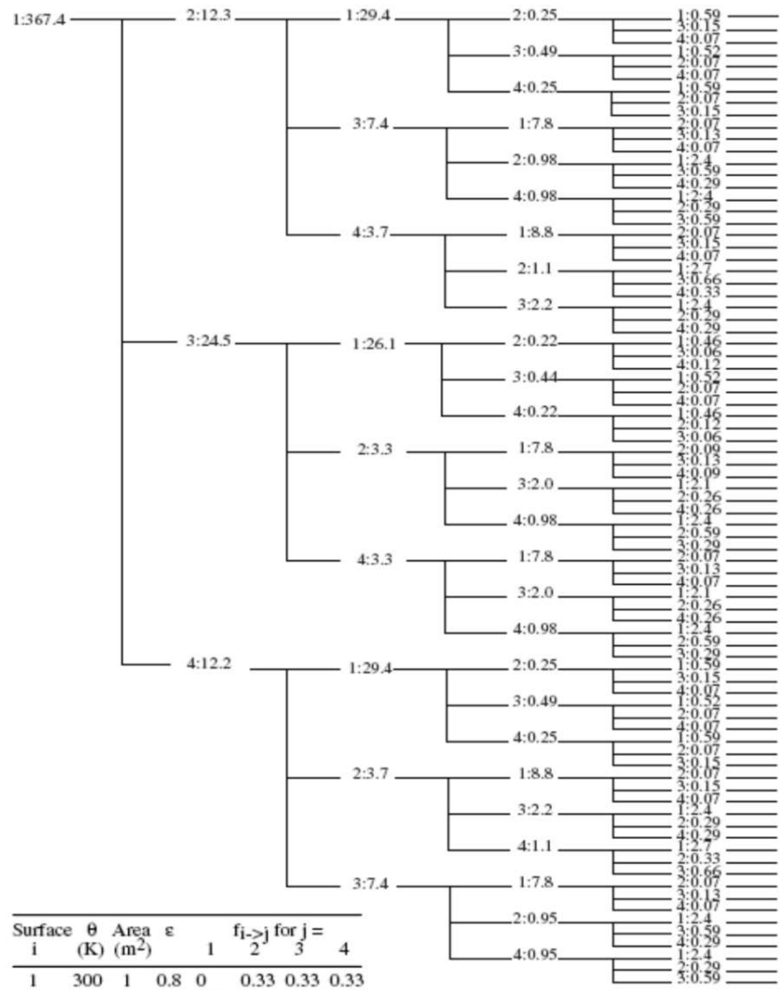
$$Q_{1\to 2} = h_r\, A\, \Delta\theta$$

Table 7.13: Application of the recursive techniques to the problem of figure 7.17.



Surface i	θ (K)	Area (m^2)	ε	$f_{i\to j}$ for j = 1	2	3	4
1	295	1	0.8	0	0.33	0.33	0.33
2	297	1	0.8	0.33	0	0.33	0.33
3	293	1	0.5	0.33	0.33	0	0.33
4	295	1	0.9	0.33	0.33	0.33	0

Table 7.14: Application of the recursive techniques to low emissivity surfaces.



Surface i	θ (K)	Area (m^2)	ε	$f_{i\to j}$ for j = 1	2	3	4
1	300	1	0.8	0	0.33	0.33	0.33
2	290	1	0.1	0.33	0	0.33	0.33
3	290	1	0.2	0.33	0.33	0	0.33
4	290	1	0.1	0.33	0.33	0.33	0

Internal long-wave radiation – numerical method

- Surfaces divided into finite elements and a unit hemisphere superimposed on each element.
- Unit hemisphere's surface divided into patches representing the radiosity field of the associated finite element.
- 'Energy rays' are formed by connecting the centre point of the finite element and all surface patches.
- Each ray is projected to determine an intersection with another surface.
- At this intersection a surface response model is invoked to determine the energy absorption and the number and intensity of exit rays – these are continually added to the stack of rays queued for processing.
- Ray processing is discontinued when the inherent energy level falls below a threshold.
- The energy absorptions for each finite element are then summated as appropriate to give the final net longwave radiation exchanges for the enclosure.

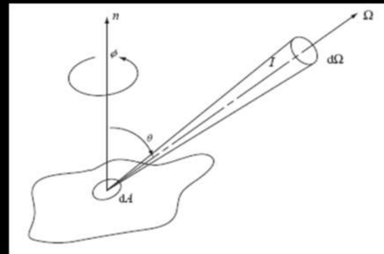


Figure 7.23: View factor prediction.

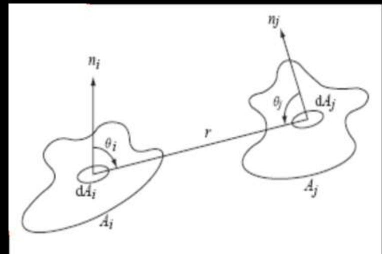


Figure 7.24: Two communicating elemental areas.

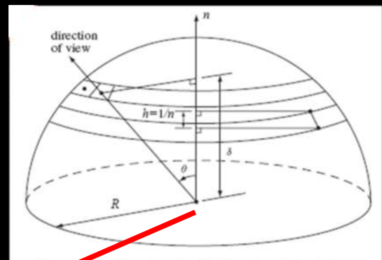
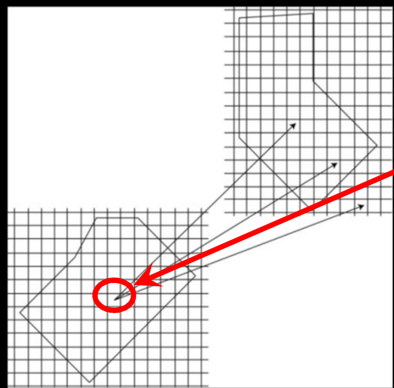


Figure 7.25: Strip and patch subdivision of a unit hemisphere.



External long-wave radiation

$$q = A_s \varepsilon \sigma (\theta_e^4 - \theta_s^4)$$

$$\theta_e^4 = f_s \theta_{sky}^4 + f_g \theta_{grd}^4 + f_u \theta_{sur}^4$$

Table 7.15: Representative values of sky, ground and obstructions view factors.

Location	f_s	f_g	f_u
City centre: surrounding buildings at same height, vertical surface	0.36	0.36	0.28
City centre: surrounding buildings higher, vertical surface	0.15	0.33	0.52
Urban site: vertical surface	0.41	0.41	0.18
Rural site: vertical surface	0.45	0.45	0.10
City centre: sloping roof	0.50	0.20	0.30
Urban site: sloping roof	0.50	0.30	0.20
Rural site: isolated	0.50	0.50	0.00