

# Numerical method: demand side



# Target domains

## Building

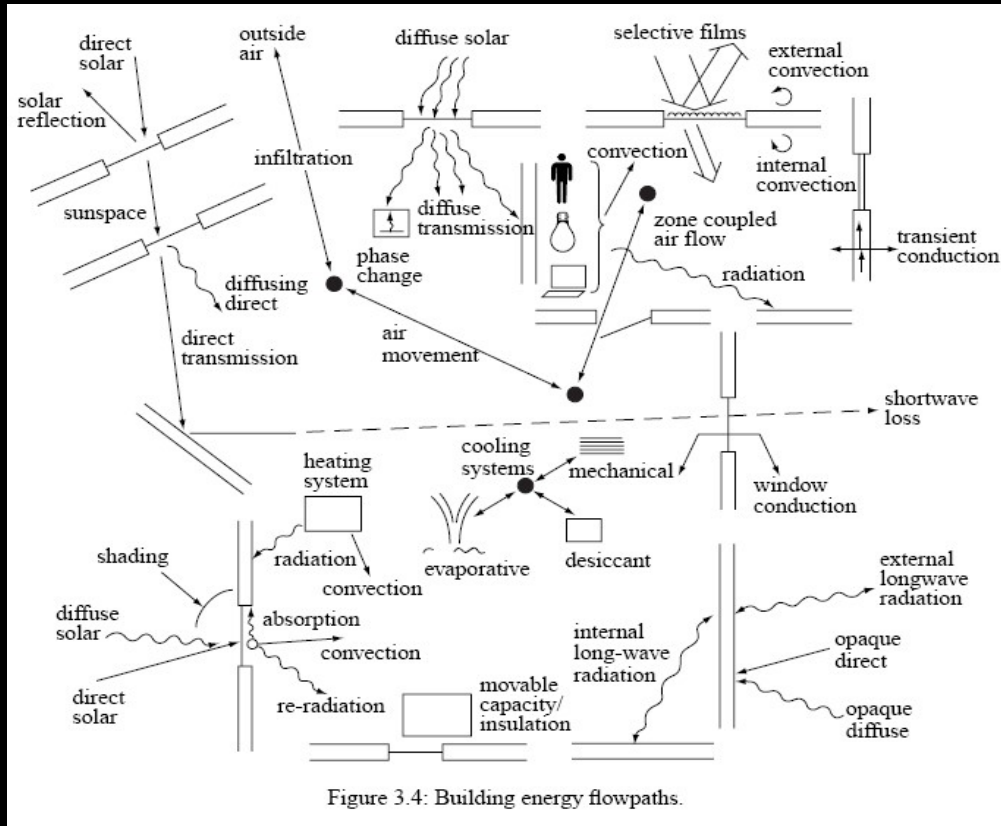


Figure 3.4: Building energy flowpaths.

## Technical integration

## Systems

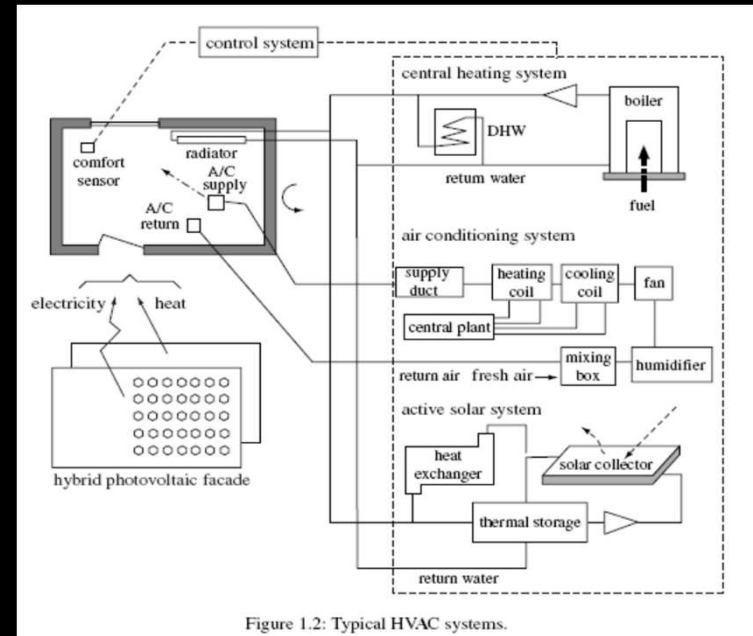


Figure 1.2: Typical HVAC systems.

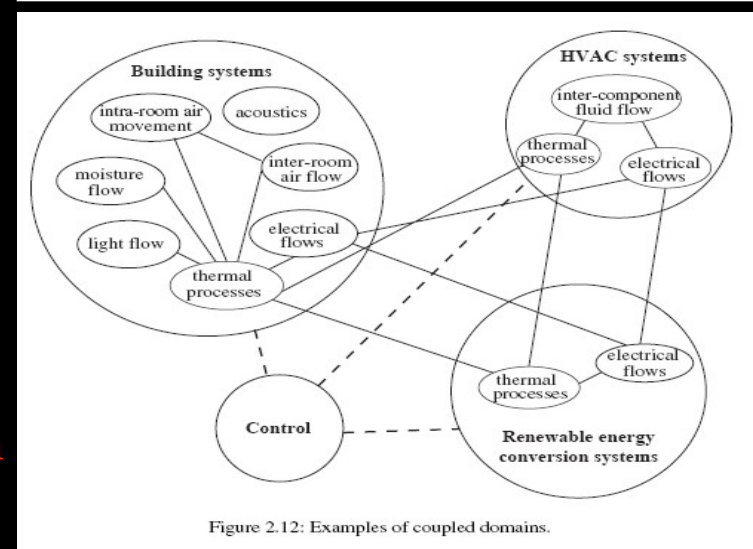
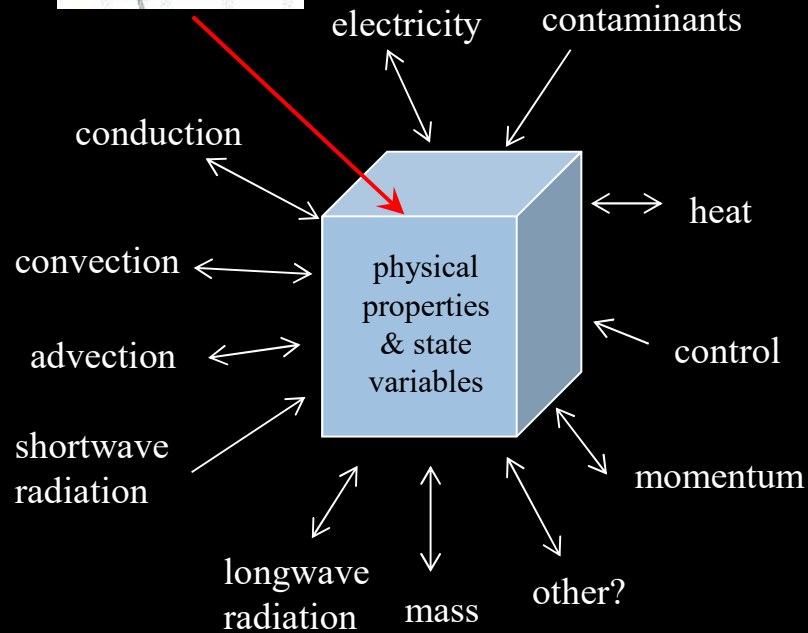
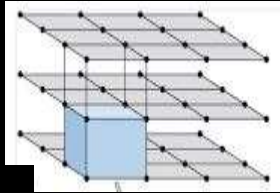


Figure 2.12: Examples of coupled domains.

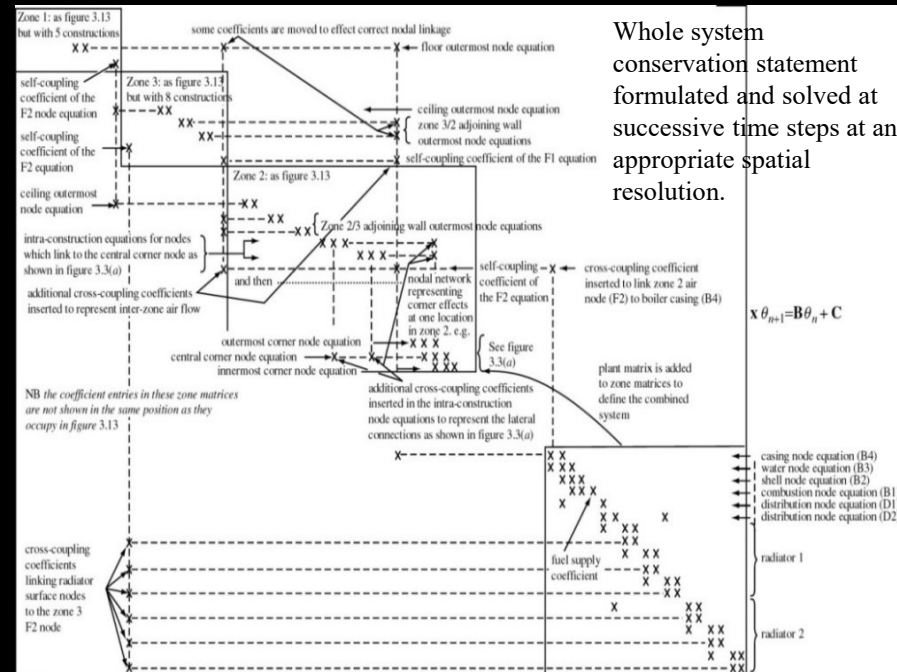
# Target domains

Discretised system



$$\frac{\partial}{\partial t} (\rho\phi) = \frac{\partial}{\partial x_i} \left( \Gamma_\phi \frac{\partial \phi}{\partial x_i} - \rho U_i \phi \right) + S_\phi$$

The rate of increase of  $\phi$  within a fluid element = the rate of increase of  $\phi$  due to diffusion - the net rate of flow of  $\phi$  out of the element + the rate of increase of  $\phi$  due to sources.



It is hubristic to suggest that the future can be predicted, only emulated to ensure resilience.

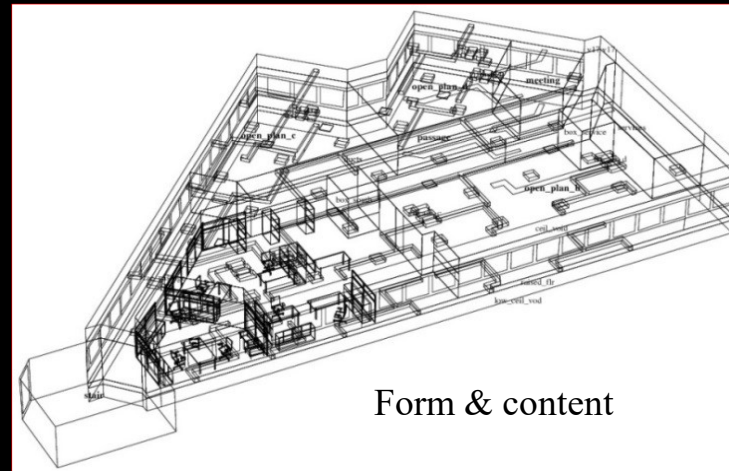


# High resolution commercial building models

Occupants

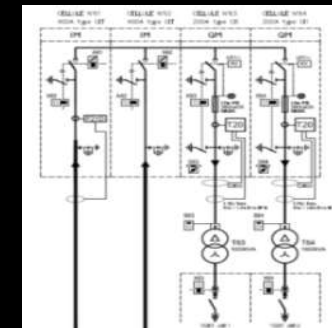


Renewable energy

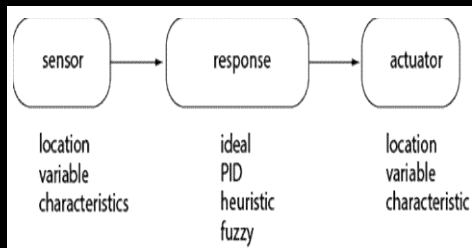


Form & content

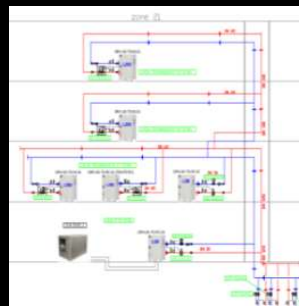
Electrical network



Control systems



HVAC systems



Lighting systems



## PDE approximation

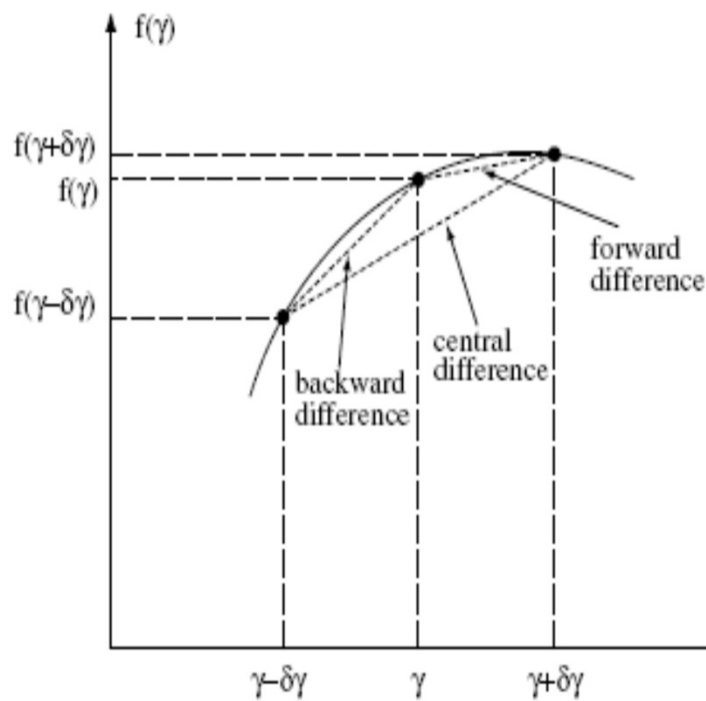


Figure 2.13: A continuous function of  $\gamma$ .

Central difference approximations

First forward difference approx.

First backward difference approx.

## Taylor series expansion:

$$f(\gamma + \delta\gamma) = f(\gamma) + \delta\gamma f^1(\gamma) + \frac{(\delta\gamma)^2}{2} f^2(\gamma) + \frac{(\delta\gamma)^3}{6} f^3(\gamma) + \dots$$

$$f(\gamma - \delta\gamma) = f(\gamma) - \delta\gamma f^1(\gamma) + \frac{(\delta\gamma)^2}{2} f^2(\gamma) - \frac{(\delta\gamma)^3}{6} f^3(\gamma) + \dots$$

## Add eqns:

$$f^2(\gamma) = \frac{f(\gamma + \delta\gamma) - 2f(\gamma) + f(\gamma - \delta\gamma)}{(\delta\gamma)^2} + \varepsilon[(\delta\gamma)^2]$$

## Subtract eqns:

$$f^1(\gamma) = \frac{f(\gamma + \delta\gamma) - f(\gamma - \delta\gamma)}{2\delta\gamma} + \varepsilon[(\delta\gamma)^2].$$

## Truncate eqn 1:

$$f^1(\gamma) = \frac{f(\gamma + \delta\gamma) - f(\gamma)}{\delta\gamma} + \varepsilon[(\delta\gamma)]$$

## Truncate eqn 2:

$$f^1(\gamma) = \frac{f(\gamma) - f(\gamma - \delta\gamma)}{\delta\gamma} + \varepsilon[(\delta\gamma)].$$

$$\frac{\partial^2 \theta(x, t)}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \theta(x, t)}{\partial t} - \frac{q}{k}$$

Fourier conduction eqn.

$$\frac{\theta(I+1, t) - 2\theta(I, t) + \theta(I-1, t)}{(\delta x)^2} = \frac{1}{\alpha} \frac{\theta(I, t + \delta t) - \theta(I, t)}{\delta t} - \frac{q(I, t)}{k}$$

Explicit formulation  
(central and first forward)

$$1 - \frac{2k\delta t}{\rho C(\delta x)^2} < 0 \quad \frac{\alpha \delta t}{(\delta x)^2} \leq \frac{1}{2} \quad \text{or} \quad F \leq \frac{1}{2}$$

$$\theta(I, t + \delta t) = \frac{k\delta t}{\rho C(\delta x)^2} \theta(I+1, t) + \left(1 - \frac{2k\delta t}{\rho C(\delta x)^2}\right) \theta(I, t) + \frac{k\delta t}{\rho C(\delta x)^2} \theta(I-1, t) + \frac{q(I, t)\delta t}{\rho C}$$

$$\frac{\theta(I+1, t + \delta t) - 2\theta(I, t + \delta t) + \theta(I-1, t + \delta t)}{(\delta x)^2} = \frac{1}{\alpha} \frac{\theta(I, t + \delta t) - \theta(I, t)}{\delta t} - \frac{q(I, t + \delta t)}{k}$$

Implicit formulation (central  
and first backward)

$$\left(1 + \frac{2k\delta t}{\rho C(\delta x)^2}\right) \theta(I, t + \delta t) = \theta(I, t) + \frac{k\delta t}{\rho C(\delta x)^2} [\theta(I+1, t + \delta t) + \theta(I-1, t + \delta t)] + \frac{q(I, t + \delta t)\delta t}{\rho C}$$

$$(1 + 2WF)\theta(I, t + \delta t) = WF[\theta(I+1, t + \delta t) + \theta(I-1, t + \delta t)] + [1 - 2F(1 - W)]\theta(I, t) + (1 - W)F[\theta(I+1, t) + \theta(I-1, t)] + \frac{\delta t}{\rho C} [Wq(I, t + \delta t) + (1 - W)q(I, t)]$$

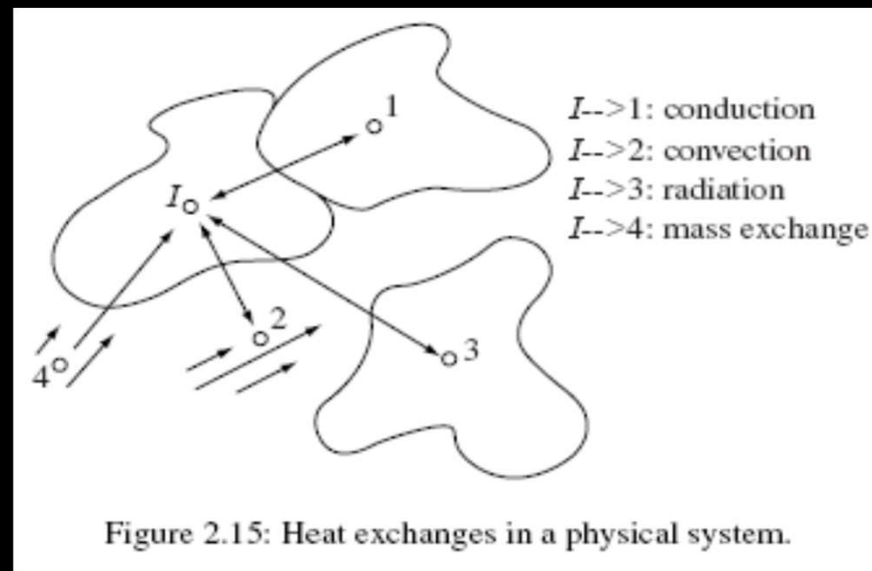
Weighted average scheme

$$F \leq \frac{1}{2} \left( \frac{1}{1 - W} \right)$$

$W < 0.5$  explicit  
 $W \geq 0.5$  implicit

## Application issues

- Complications to differencing by Taylor series expansion:
  - simultaneous presence of multiple heat transfer processes;
  - time and positional dependency of heat generation due to solar radiation, mechanical plant etc.;
  - discretisation leading to non-homogeneous, anisotropic finite volumes;
  - presence of multi-dimensional effects.
  
- Alternative approach: directly apply conservation principles to small control volumes.



## Control volume method – heat balance

Heat flux:

$$Q_{J,I} = K_{J,I}(\theta_J - \theta_I) ; J = 1, 2, 3, 4$$

Heat storage:

$$Q_s = \frac{\rho_I(\xi)C_I(\xi)\delta V_I(\xi)}{\delta t} [\theta(I, t + \delta t) - \theta(I, t)]$$

Energy balance:

$$\frac{\rho_I(\xi)C_I(\xi)\delta V_I(\xi)}{\delta t} [\theta(I, t + \delta t) - \theta(I, t)] = \sum_{j=1}^N K_{j,I}(\theta_j - \theta_I) \Big|_{t=\xi} + q_I \Big|_{t=\xi}$$

General form:

$$C_s(t+\delta t)\theta(I, t+\delta t) - \sum_{i=1}^N C_{ci}(t+\delta t)\theta(i, t+\delta t) - \frac{\delta t q_I(t+\delta t)}{\delta V_I} \\ = C_s(t)\theta(I, t) + \sum_{i=1}^N C_{ci}(t)\theta(i, t) + \frac{\delta t q_I(t)}{\delta V_I} + \varepsilon$$

$$\frac{\partial^2 \theta(x, t)}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \theta(x, t)}{\partial t} - \frac{q}{k}$$

in the limit

Always the same!

Note:

- equation can be applied to building and plant components;
- number of coefficients will vary;
- analogous considerations for mass and momentum balance.



## Formulating a numerical model

- Continuous system made discrete by the placement of nodes at points of interest:
  - nodes represent homogeneous or non-homogeneous physical volumes (comprising fluids, opaque and transparent surfaces, constructional elements, plant component parts, room contents *etc.*).

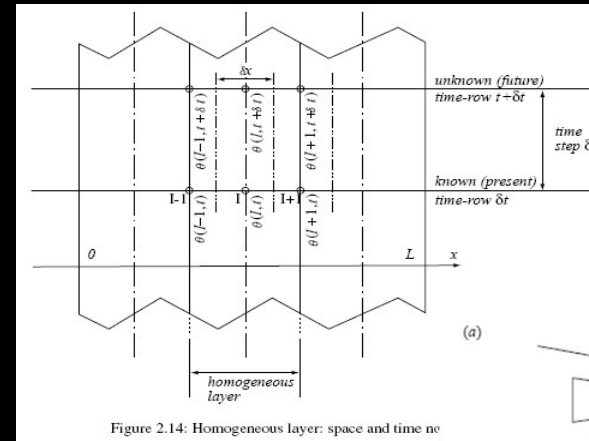


Figure 2.14: Homogeneous layer: space and time nodes

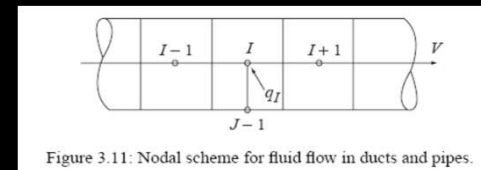
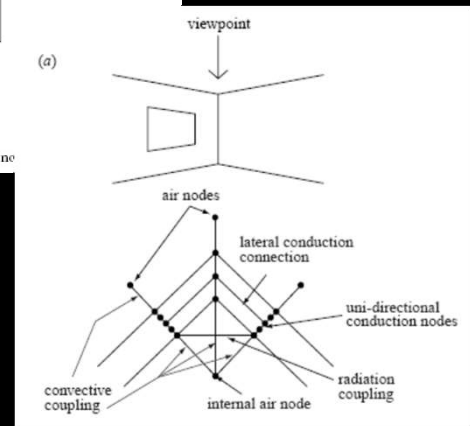


Figure 3.11: Nodal scheme for fluid flow in ducts and pipes.



- For each node, and in terms of all surrounding nodes representing regions deemed to be in thermodynamic contact, conservation equations are developed:
  - represents the nodal condition and the inter-nodal transfers of energy, mass and momentum.
- The entire equation-set is solved simultaneously for successive time steps:
  - gives the future time-row nodal state variables as a function of present time-row states and prevailing boundary conditions at both time-rows.

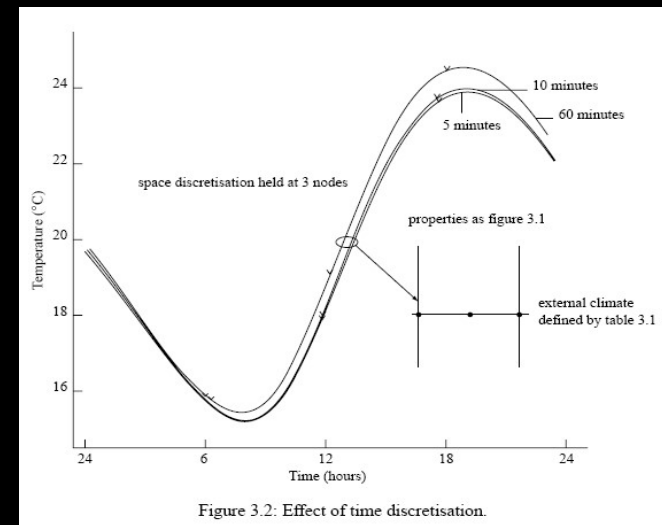
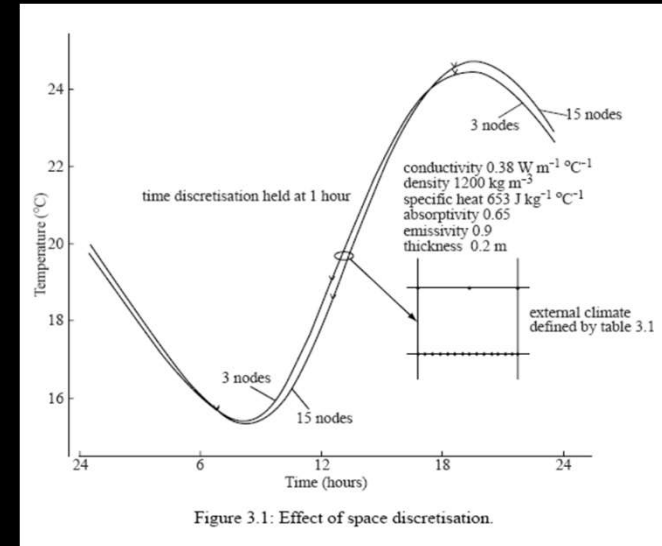
## Modelling issues

- ❑ Not possible to prescribe a spatial discretisation scheme in advance — depending on the problem, model parts may require high resolution (many nodes) or low resolution (few nodes).
- ❑ Discretised conservation equations will have a variable number of coefficients depending on the node type. This will require a carefully designed matrix coefficient indexing scheme to facilitate efficient equation solution.
- ❑ Because different system parts will have different time constants and coupling strengths, equation processing must be structured to allow these effects to be reconciled whilst not enforcing a lowest common denominator processing frequency.
- ❑ Since different domain equations possess different characteristics (e.g. some are highly non-linear — an approach that depends on several co-operating solvers will be more computationally efficient than an approach that attempts to coerce the disparate equation-sets into a single solver type.

## Numerical errors

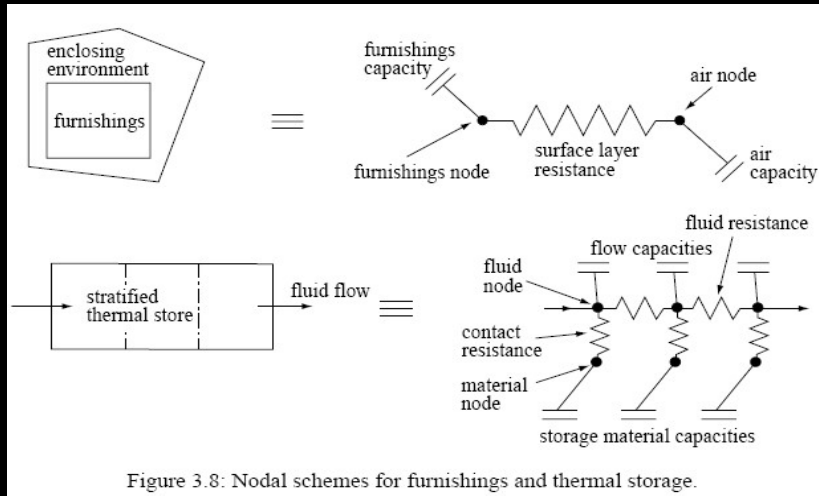
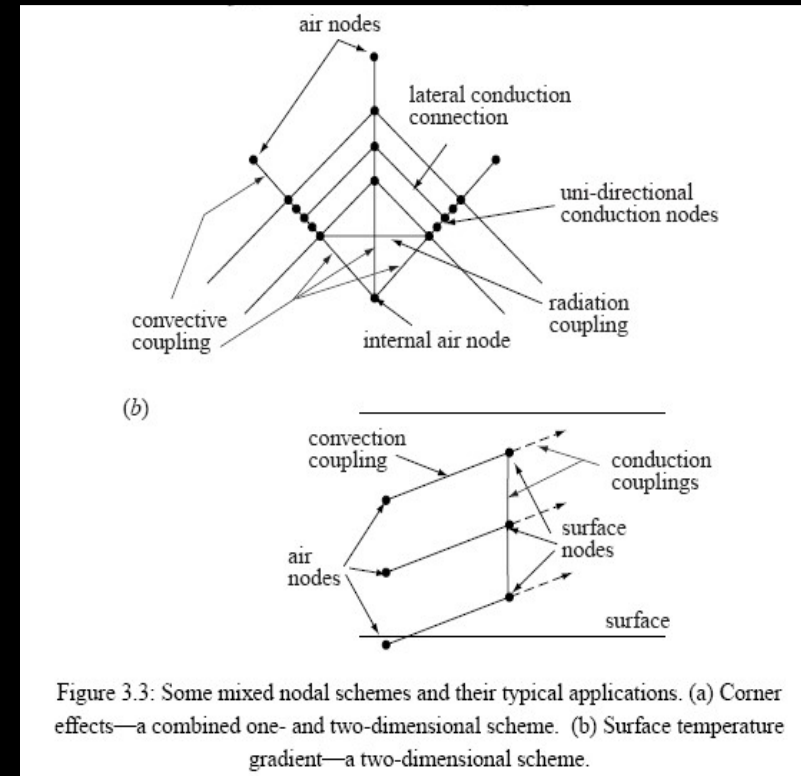
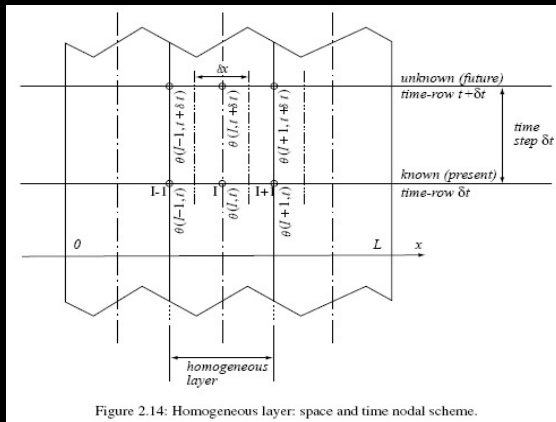
Two sources of error associated with finite differencing schemes:

- ❑ Rounding - where computations include an insufficient number of significant figures. Can be minimised by careful design of the numerical scheme and by operating in double precision.
- ❑ Discretisation - resulting from the replacement of derivatives by finite differences. Error minimised by reducing space and time increments.
  - not possible to prescribe space and time increments because they depend on modelling objectives;
  - implicit formulation attractive because it is unconditionally stable;
  - discretisation depends on factors such as surface insolation, local convection, corner effects/thermal bridges, and the shape of the capacity/insulation system.



# System discretisation

- ❑ By prescription: e.g. at least 3 nodes per homogeneous element with a time step less than 1 hour.
- ❑ Using thermal criteria:
  - Biot Number  $\ll 1$  use lumped parameter;
  - else, equal thermal capacity divisions using the dwell time.
- ❑ Mixed node schemes often required.



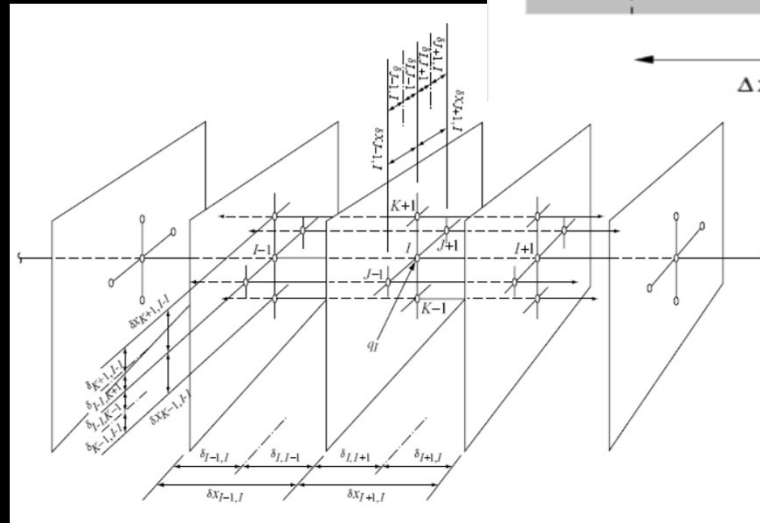
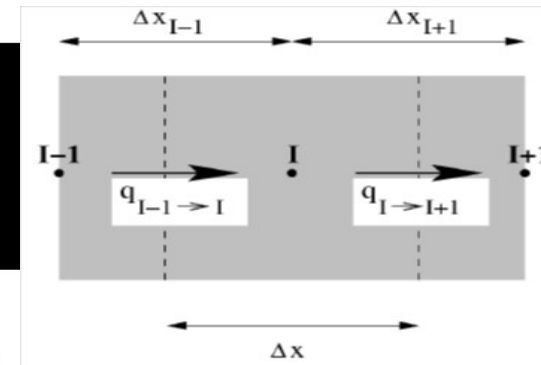
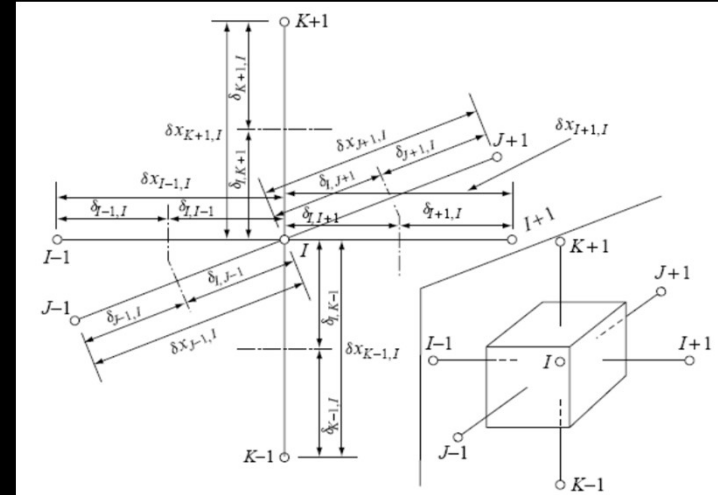
# Material energy conservation equation

$$\begin{aligned}
 & \left( 2\rho_I(t+\delta t)C_I(t+\delta t) + \frac{2\delta t k(t+\delta t)}{\delta x_I^2} \right) \theta(I, t+\delta t) \\
 & - \frac{\delta t k(t+\delta t)}{\delta x_I^2} \theta(I-1, t+\delta t) - \frac{\delta t k(t+\delta t)}{\delta x_I^2} \theta(I+1, t+\delta t) - \frac{\delta t q_I(t+\delta t)}{\delta x_I \delta x_J \delta x_K} \\
 & = \left( 2\rho_I(t)C_I(t) - \frac{2\delta t k(t)}{\delta x_I^2} \right) \theta(I, t) \\
 & + \frac{\delta t k(t)}{\delta x_I^2} \theta(I-1, t) + \frac{\delta t k(t)}{\delta x_I^2} \theta(I+1, t) + \frac{\delta t q_I(t)}{\delta x_I \delta x_J \delta x_K} .
 \end{aligned}$$

$$\begin{aligned}
 & C_s(t+\delta t)\theta(I, t+\delta t) - \sum_{i=1}^N C_{ci}(t+\delta t)\theta(i, t+\delta t) - \frac{\delta t q_I(t+\delta t)}{\delta V_I} \\
 & = C_s(t)\theta(I, t) + \sum_{i=1}^N C_{ci}(t)\theta(i, t) + \frac{\delta t q_I(t)}{\delta V_I} + \varepsilon
 \end{aligned}$$

Material node types:

- opaque intra-construction;
- transparent intra-construction;
- phase change;
- boundary between elements;
- lumped.







# Fluid energy conservation equation

$$\left( 2W_I(t + \delta t) + \frac{\delta t \sum_{i=1}^N h_{ci,I}(t + \delta t) \delta A_{i,I}}{\delta V_I} + \frac{\delta t \sum_{j=1}^M v_{j,I}(t + \delta t) \bar{\rho}_{j,I}(t + \delta t) \bar{C}_{j,I}(t + \delta t)}{\delta V_I} \right) \theta(I, t + \delta t) - \frac{\delta t \sum_{i=1}^N h_{ci,I}(t + \delta t) \delta A_{i,I} \theta(i, t + \delta t)}{\delta V_I} - \frac{\delta t \sum_{j=1}^M v_{j,I}(t + \delta t) \bar{\rho}_{j,I}(t + \delta t) \bar{C}_{j,I}(t + \delta t) \theta(j, t + \delta t)}{\delta V_I} - \frac{\delta t q_{I,t}(t + \delta t)}{\delta V_I} = \left( 2W_I(t) - \frac{\delta t \sum_{i=1}^N h_{ci,I}(t) \delta A_{i,I}}{\delta V_I} - \frac{\delta t \sum_{j=1}^M v_{j,I}(t) \bar{\rho}_{j,I}(t) \bar{C}_{j,I}(t)}{\delta V_I} \right) \theta(I, t) + \frac{\delta t \sum_{i=1}^N h_{ci,I}(t) \delta A_{i,I} \theta(i, t)}{\delta V_I} + \frac{\delta t \sum_{j=1}^M v_{j,I}(t) \bar{\rho}_{j,I}(t) \bar{C}_{j,I}(t) \theta(j, t)}{\delta V_I} + \frac{\delta t q_{I,t}(t)}{\delta V_I} + \epsilon$$

$$C_s(t + \delta t) \theta(I, t + \delta t) - \sum_{i=1}^N C_{ci}(t + \delta t) \theta(i, t + \delta t) - \frac{\delta t q_{I,t}(t + \delta t)}{\delta V_I} = C_s(t) \theta(I, t) + \sum_{i=1}^N C_{ci}(t) \theta(i, t) + \frac{\delta t q_{I,t}(t)}{\delta V_I} + \epsilon$$

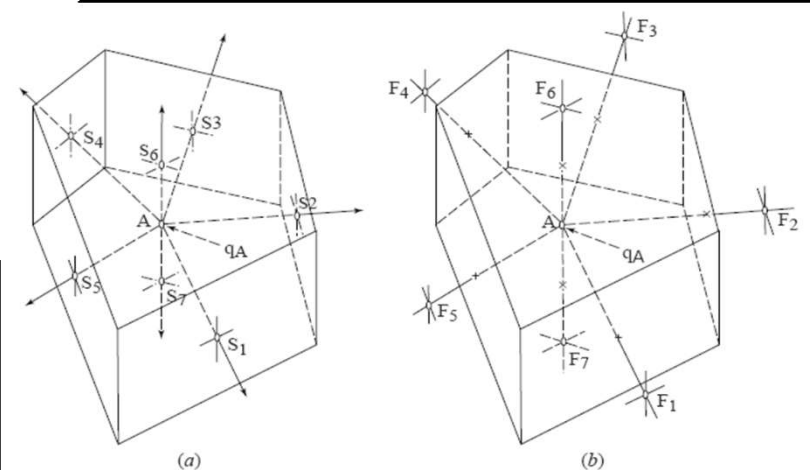
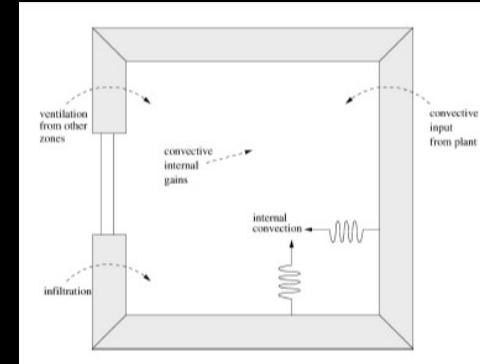


Figure 3.10: Fluid volume energy balance nodal scheme. (a) Fluid volume contained by real surfaces; convective heat transfer. (b) Fluid volumes contained by fictitious surfaces; advective heat transfer.

Surface node types:

- room air;
- construction air gaps;
- plant component fluids.

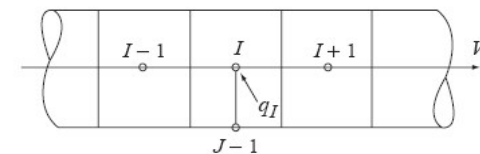


Figure 3.11: Nodal scheme for fluid flow in ducts and pipes.

# Equation structuring – building zone + contents + radiator

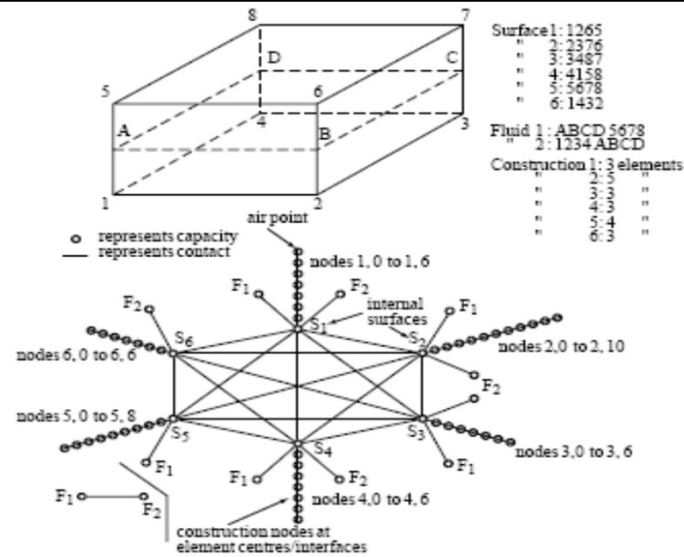


Figure 3.12: A single zone system and equivalent nodal scheme.

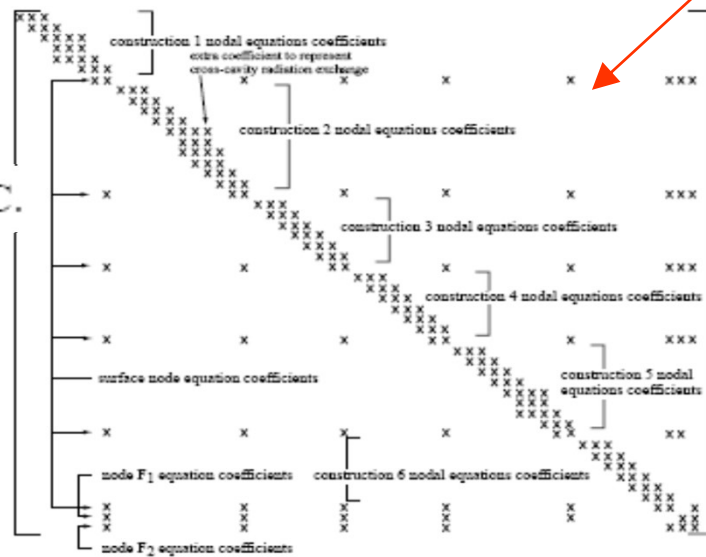


Figure 3.13: The future time-row coefficients matrix (A) of the single zone matrix equation  $A\theta_{n+1} = B\theta_n + C$ .

$$A\theta_{n+1} = B\theta_n + C.$$

$$Z = B\theta_n + C$$

$$\theta_{n+1} = A^{-1}Z.$$

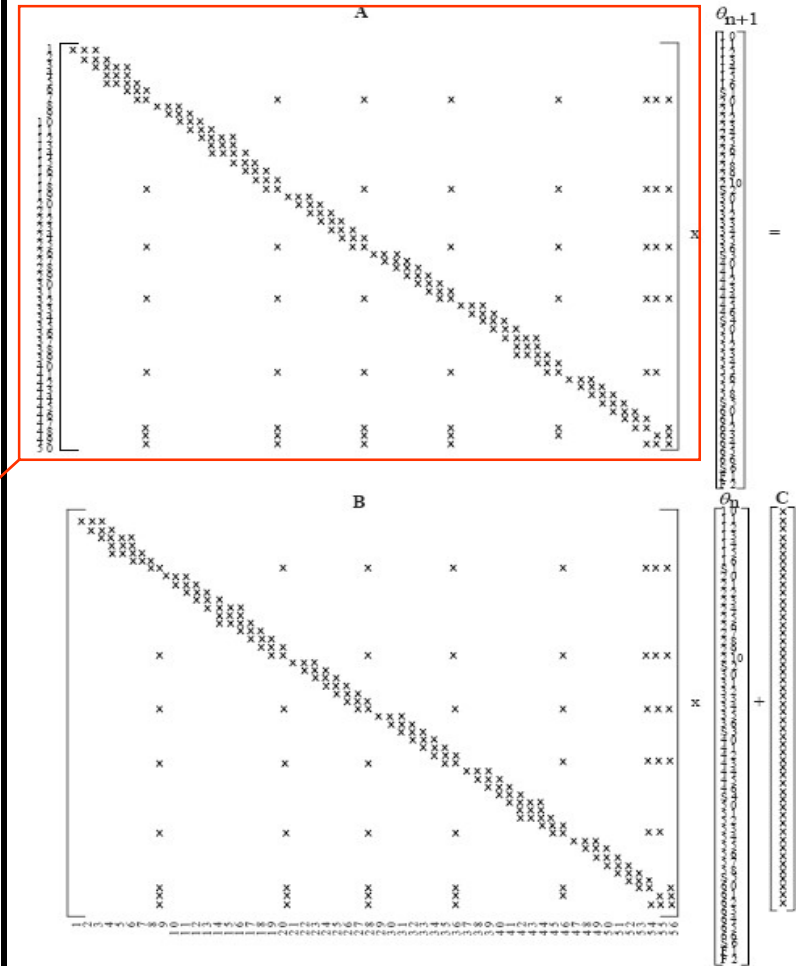


Figure 3.20: The single zone matrix equation  $A\theta_{n+1} = B\theta_n + C$ .

outputs support: energy and comfort, impact of infiltration & ventilation, short- & longwave radiation, casual gains etc.

# Equation structuring - passive/active solar system

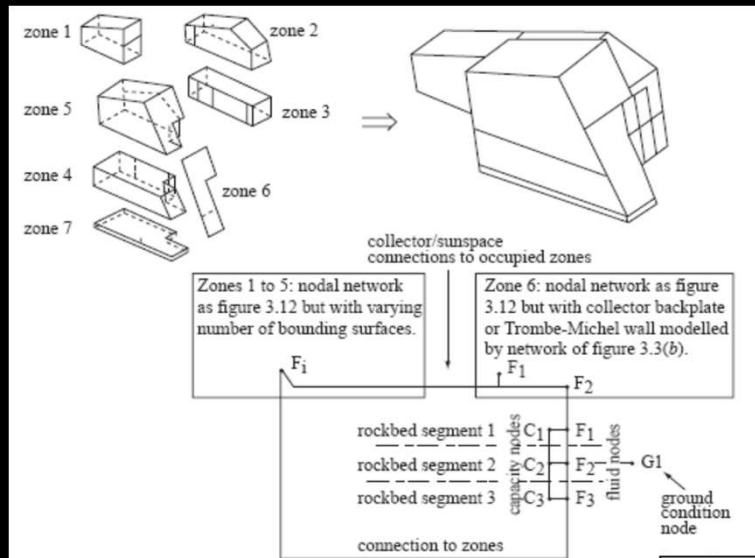


Figure 3.18: An passive/ active solar system.

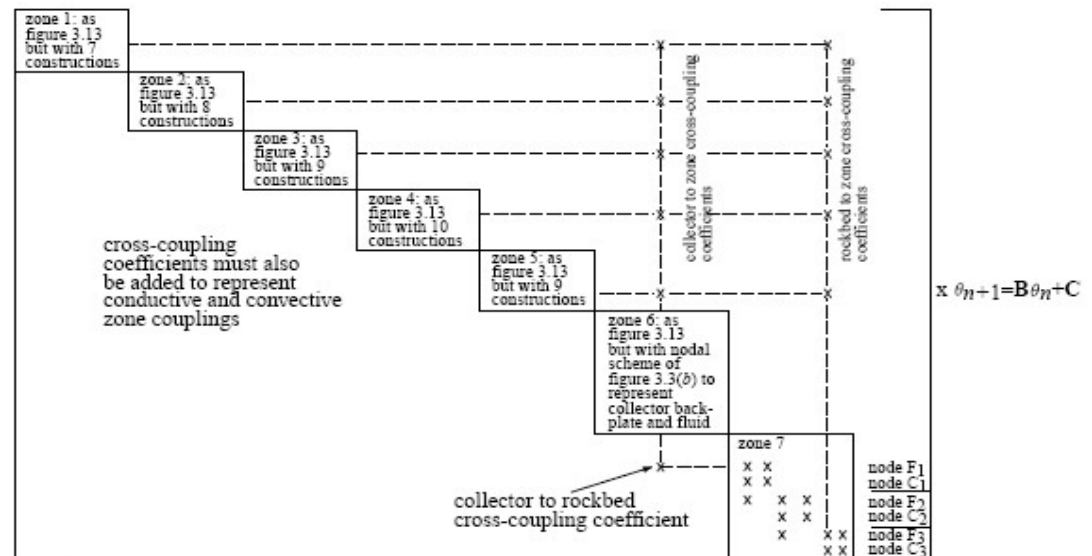


Figure 3.19: The energy balance matrix equation for the system of figure 3.18.





# Calculating equation coefficients

## Construction Conservation Equation

$$\begin{aligned}
 & \left( 2\rho_1(t+\delta t)C_1(t+\delta t) + \frac{2\delta t k(t+\delta t)}{\delta x_f^2} \right) \theta(I, t+\delta t) \\
 & - \frac{\delta t k(t+\delta t)}{\delta x_f^2} \theta(I-1, t+\delta t) - \frac{\delta t k(t+\delta t)}{\delta x_f^2} \theta(I+1, t+\delta t) - \frac{\delta t q_{II}(t+\delta t)}{\delta x_I \delta x_J \delta x_K} \\
 & = \left( 2\rho_1(t)C_1(t) - \frac{2\delta t k(t)}{\delta x_f^2} \right) \theta(I, t) \\
 & + \frac{\delta t k(t)}{\delta x_f^2} \theta(I-1, t) + \frac{\delta t k(t)}{\delta x_f^2} \theta(I+1, t) + \frac{\delta t q_{II}(t)}{\delta x_I \delta x_J \delta x_K} .
 \end{aligned}$$

## Surface Conservation Equation

$$\begin{aligned}
 & \left( 2W_I(t+\delta t) + \frac{\delta t k'_{I-1,I}(t+\delta t)}{\delta x_{I-1,I} \delta_{I,I-1}} + \frac{\delta t h_{cI+1,I}(t+\delta t)}{\delta_{I,I-1}} + \frac{\delta t \sum_{s=1}^N h_{rs,I}(t+\delta t)}{\delta_{I,I-1}} \right) \theta(I, t+\delta t) \\
 & - \frac{\delta t k'_{I-1,I}(t+\delta t)}{\delta x_{I-1,I} \delta_{I,I-1}} \theta(I-1, t+\delta t) - \frac{\delta t h_{cI+1,I}(t+\delta t)}{\delta_{I,I-1}} \theta(I+1, t+\delta t) \\
 & - \frac{\delta t \sum_{s=1}^N h_{rs,I}(t+\delta t) \theta(s, t+\delta t)}{\delta_{I,I-1}} - \frac{\delta t q_{PI}(t+\delta t)}{\delta_{I,I-1} C_{I-1,K+1} \delta_{N-1,K+1}} \\
 & = \left( 2W_I(t) - \frac{\delta t k'_{I-1,I}(t)}{\delta x_{I-1,I} \delta_{I,I-1}} - \frac{\delta t \sum_{s=1}^N h_{rs,I}(t)}{\delta_{I,I-1}} \right) \theta(I, t) \\
 & + \frac{\delta t k'_{I-1,I}(t)}{\delta x_{I-1,I} \delta_{I,I-1}} \theta(I-1, t) + \frac{\delta t h_{cI+1,I}(t)}{\delta_{I,I-1}} \theta(I+1, t) + \frac{\delta t \sum_{s=1}^N h_{rs,I}(t) \theta(s, t)}{\delta_{I,I-1}} \\
 & + \frac{\delta t [q_{PI}(t) + q_{SI}(t) + q_{RI}(t) + q_{SI}(t+\delta t) + q_{RI}(t+\delta t)]}{\delta_{I,I-1} \delta_{I-1,J+1} \delta_{K-1,K+1}} + \epsilon .
 \end{aligned}$$

## Fluid Conservation Equation

$$\begin{aligned}
 & \left( 2W_I(t+\delta t) + \frac{\delta t \sum_{i=1}^N h_{ci,I}(t+\delta t) \delta A_{i,I}}{\delta V_I} + \frac{\delta t \sum_{j=1}^M v_{j,I}(t+\delta t) \beta_{j,I}(t+\delta t) \bar{C}_{j,I}(t+\delta t)}{\delta V_I} \right) \theta(I, t+\delta t) \\
 & - \frac{\delta t \sum_{i=1}^N h_{ci,I}(t+\delta t) \delta A_{i,I} \theta(i, t+\delta t)}{\delta V_I} - \frac{\delta t \sum_{j=1}^M v_{j,I}(t+\delta t) \beta_{j,I} + \delta t \bar{C}_{j,I}(t+\delta t) \theta(j, t+\delta t)}{\delta V_I} \\
 & - \frac{\delta t q_{II}(t+\delta t)}{\delta V_I} = \left( 2W_I(t) - \frac{\delta t \sum_{i=1}^N h_{ci,I}(t) \delta A_{i,I}}{\delta V_I} - \frac{\delta t \sum_{j=1}^M v_{j,I}(t) \beta_{j,I}(t) \bar{C}_{j,I}(t)}{\delta V_I} \right) \theta(I, t) \\
 & + \frac{\delta t \sum_{i=1}^N h_{ci,I}(t) \delta A_{i,I} \theta(i, t)}{\delta V_I} + \frac{\delta t \sum_{j=1}^M v_{j,I}(t) \beta_{j,I}(t) \bar{C}_{j,I}(t) \theta(j, t)}{\delta V_I} + \frac{\delta t q_{II}(t)}{\delta V_I} + \epsilon
 \end{aligned}$$

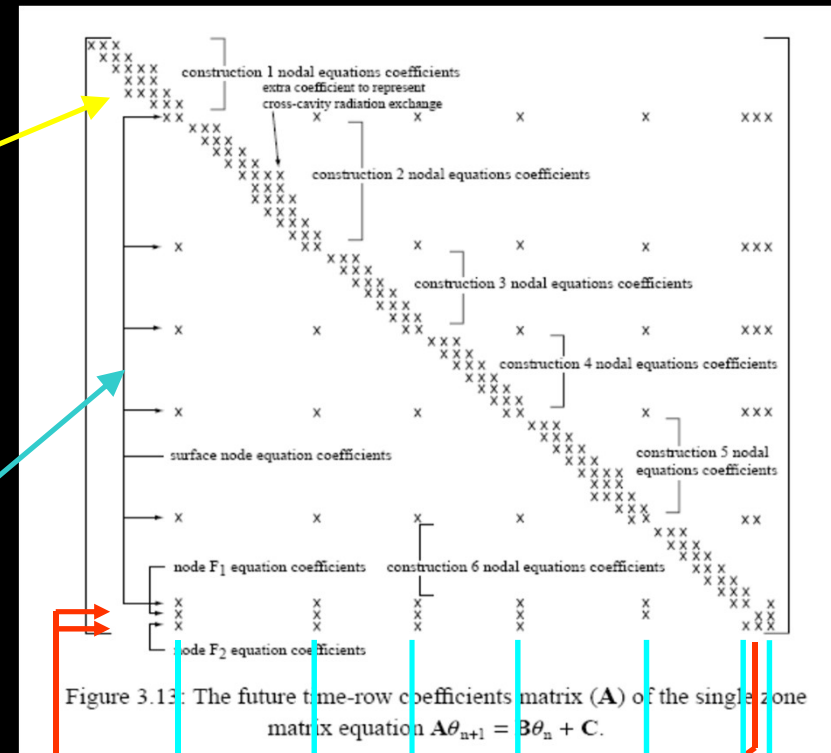


Figure 3.13: The future time-row coefficients matrix ( $A$ ) of the single zone matrix equation  $A\theta_{n+1} = B\theta_n + C$ .

needs flow estimation

needs radiation and convection estimation

# Imposing control

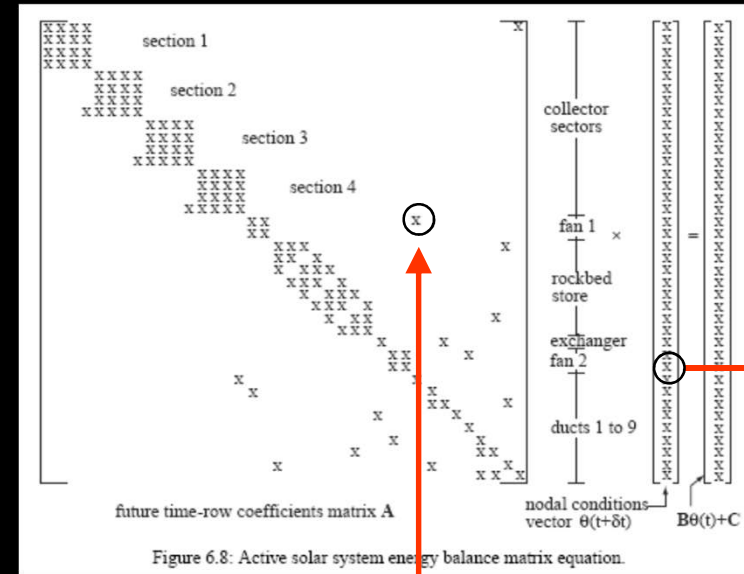
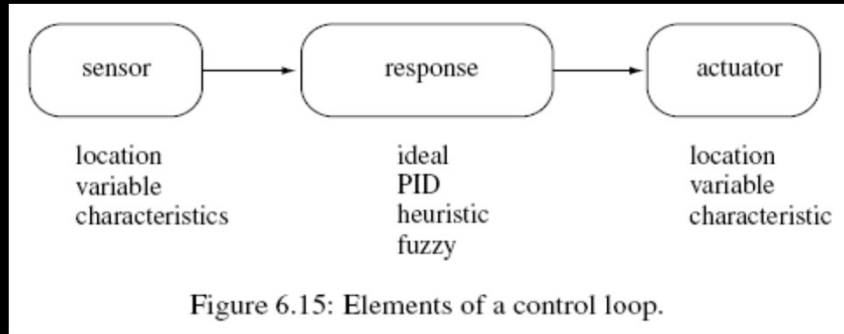


Table 6.8: Examples of control loop elements.

<i>Sensed/actuated parameters</i>	<i>Regulation law</i>	<i>Sensed/actuated parameters</i>	<i>Regulation law</i>
time of day/year	ideal	CO <sub>2</sub> level	duty cycling
climate	PID combinations	ventilation rate	load shedding
various temperatures	optimum start/stop	room air velocity	adaptive
glare and illuminance	weather compensation	hygro-thermal properties	fuzzy logic
luminaire status	cascade	humidity	neural network
occupancy	enthalpy cycle		

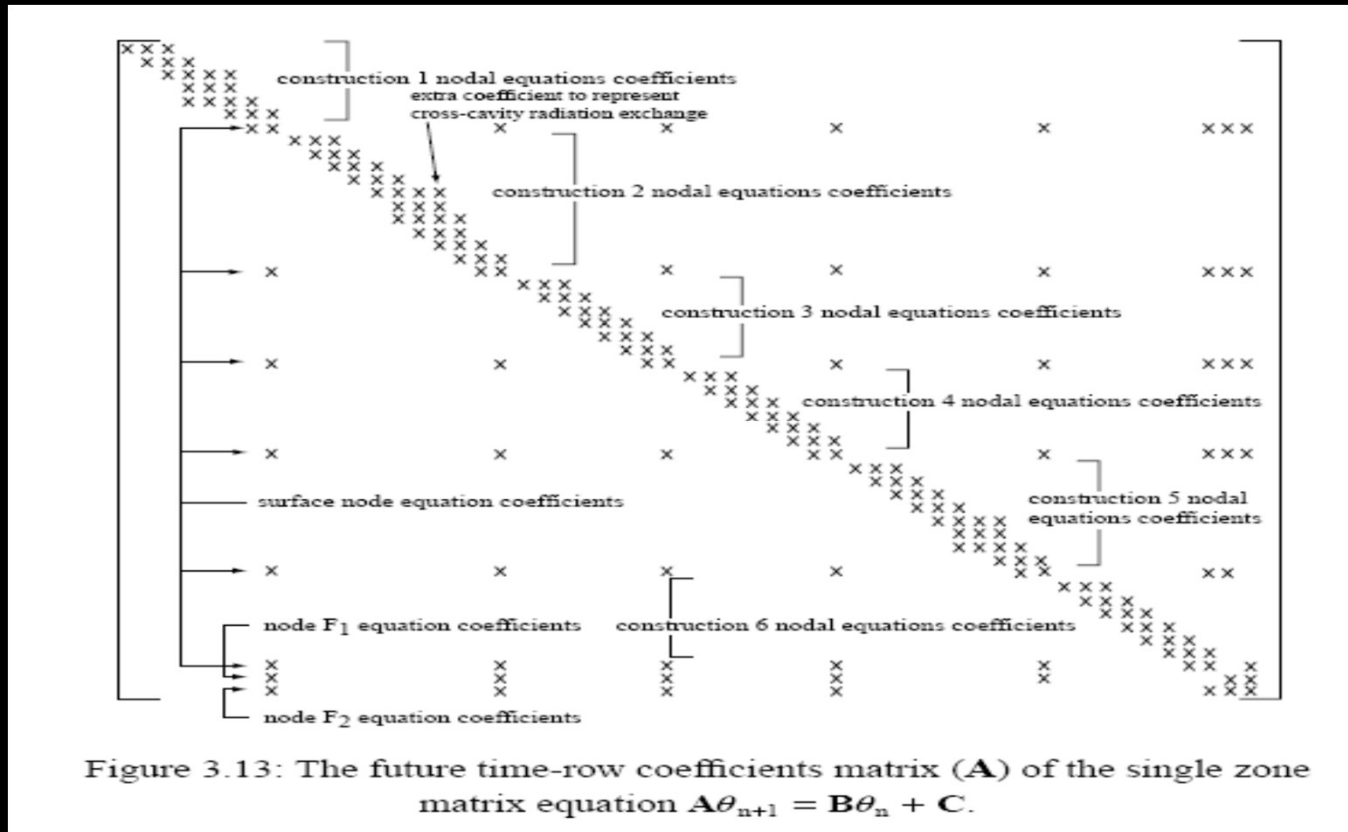
Table 6.9: Some examples of simulation-assisted control.

<i>Control focus</i>	<i>Optimised parameter</i>	<i>Control focus</i>	<i>Optimised parameter</i>
HVAC operation	start/stop time	district heating	match to load
night cooling	hours of operation	under-floor heating	period of operation
night set-back	set-back temperature	mixed ventilation	avoid overheating
boiler sequencing	heating system eff.	ice store charging	hours of operation
load shedding	energy consumption	ground heat pump	thermal storage
CHP	hours of operation		

## Equation-set solution

- ❑ Equations are linked => simultaneous solution required.
- ❑ Equation-set is sparse and populated by clusters of equations relating to components with different time constants => special solver required to minimise the computational effort.
- ❑ Partitioning techniques often used allowing different clusters to be processed at different frequencies depending on the related time constant.
- ❑ This allows control decisions to be made, and problem parameters recomputed, more frequently for an item of plant requiring a computational time-step of, say, 1 minute, than for a heavyweight construction requiring, say, 60 minutes.
- ❑ Two main equation solving approaches: iterative and direct.

## Equation-set extension



Equation coefficients can be extended to improve the modelling resolution in relation to:

- short-wave and long-wave radiation;
- air movement;
- casual gains;
- surface convection;
- variation in fundamental parameters.

Technique can be similarly applied to all energy systems.

