Response function method

Response function method

- An analytical approach to the solution of ordinary and partial differential equations using the Laplace transform:
  - An equation in the time domain is transformed into a subsidiary equation in an imaginary space;
  - subsidiary equation is solved by purely algebraic manipulations;
  - an inverse transformation is applied to obtain the solution in the time domain of the initial problem.
- PDEs are transformed to ODEs and ODEs to algebraic equations.
- Pre-calculates system response to individual heat inputs.
- Sacrifices the non-linear, systemic and stochastic attributes.
- Consider the treatment of the transient condition flowpath:

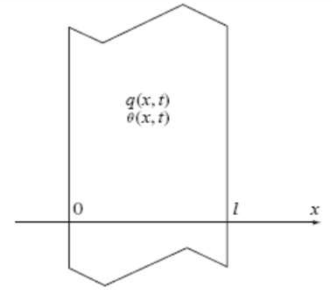


Figure 2.1: A homogeneous, isotropic element.

$$\frac{\partial \theta(x,t)}{\partial x} = -\frac{1}{k} q(x,t)$$

$$\frac{\partial q(x,t)}{\partial x} = -\rho C \frac{\partial \theta(x,t)}{\partial t}$$

$$\frac{\partial^2 \theta(x,t)}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \theta(x,t)}{\partial t}$$

$$\frac{\partial \theta(x,p)}{\partial x} = -\frac{1}{k} q(x,p)$$

$$\frac{\partial q(x,p)}{\partial x} = -\rho C p \theta(x,p) + \rho C \theta(x,0)$$

$$\frac{\partial^2 \theta(x,p)}{\partial x^2} = \frac{p}{\alpha} \theta(x,p) - \frac{1}{\alpha} \theta(x,0)\,.$$

Table 2.1: Some common Laplace transform pairs.

	f(t)	f(p)
Unit impulse	$\delta(t)$	1
Unit step	$H(t)$	$1/p$
Unit ramp	t	$1/p^2$
	t^n	$n!/p^{n+1}$; n + ve integer
Delayed unit impulse	$\delta(t-\Delta)$	$e^{-p\Delta}$
Delayed unit step	$H(t-\Delta)$	$e^{-p\Delta}/p$
	e^{-at}	$1/(p+a)$
	$e^{-a(t-\Delta)}H(t-\Delta)$	$e^{-p\Delta}/(p+a)$
	te^{-at}	$1/(p+a)^2$
	$t^n e^{-at}$	$n!/(p+a)^{n+1}$
	$\sin bt$	$b/(p^2+b^2)$
	$\cos bt$	$p/(p^2+b^2)$

$$\theta(x,p) = \cosh[(p/\alpha)^{\frac{1}{2}}x]\theta(0,p) - \frac{\sinh[(p/\alpha)^{\frac{1}{2}}x]q(0,p)}{k(p/\alpha)^{\frac{1}{2}}}$$

$$q(x,p) = -k(p/\alpha)^{\frac{1}{2}} \sinh[(p/\alpha)^{\frac{1}{2}}x]\theta(0,p) + \cosh[(p/\alpha)^{\frac{1}{2}}x]q(0,p)\,.$$

Time domain response functions

- Predetermine system response to a unit excitation relating to the boundary conditions anticipated in reality.

- A unit excitation function has a value of unity at its start and zero thereafter (1, 0, 0, 0, . . .).

- Response of a linear, time invariant equation system to this unit excitation function is termed the unit response function (URF) and the time-series representation of this URF are the response factors.

- URFs depend on design parameters and assumptions regarding thermo-physical properties.

- Number of URFs depends on the combinations of excitation function (solar radiation, outside temperature, sky longwave radiation *etc*.) and responses of interest (heating/cooling load, indoor temperature, power *etc*.).

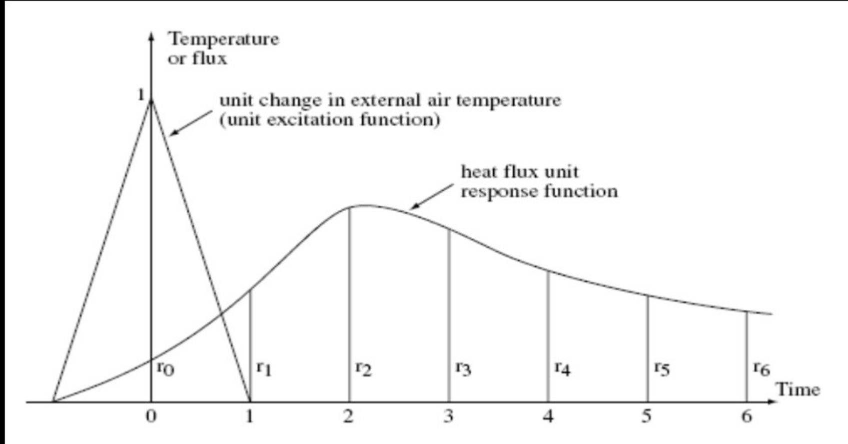


Figure 2.2: Unit excitations and response functions.

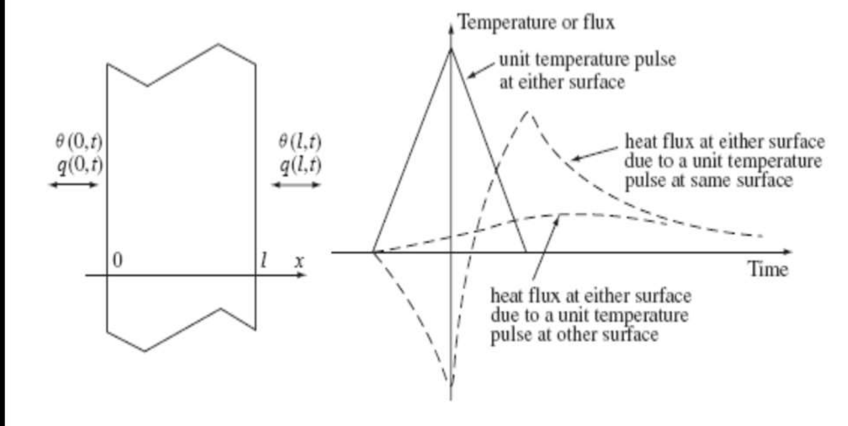


Figure 2.5: Heat flux due to a unit temperature pulse at either surface.

Time domain response function method

- ❑ Method comprises 3 steps after all URFs have been determined.

- ❑ Actual excitation functions are resolved into equivalent time-series by triangular or rectangular approximation.

- ❑ URFs are combined with a corresponding excitation function to determine the system response using the Convolution Theorem:
  - response of a linear, time invariant equation system is determined as the sum of the products of the URFs and the actual excitation after time adjustment.

- ❑ Individual responses from the different excitation functions are superimposed to give the overall response.

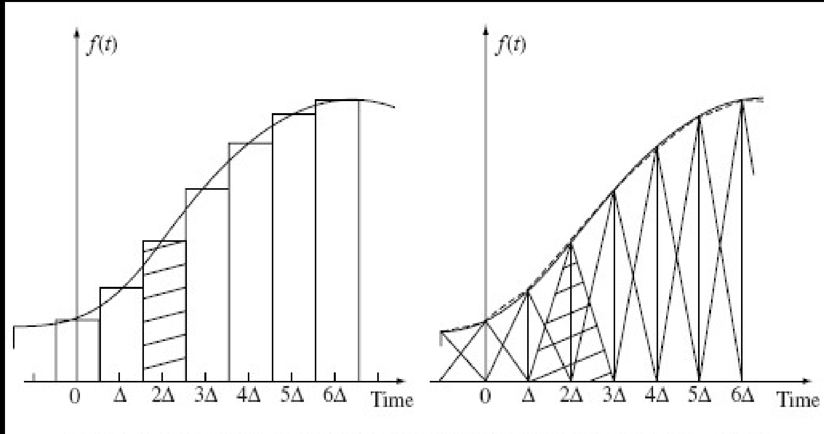


Figure 2.3: Rectangular and triangular pulse representation of a continuous function.

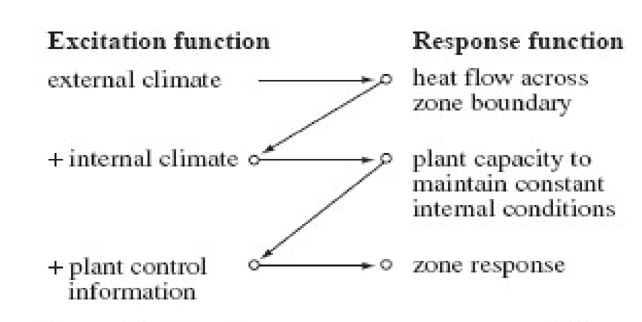


Figure 2.4: Overall zone response—sequence of steps.

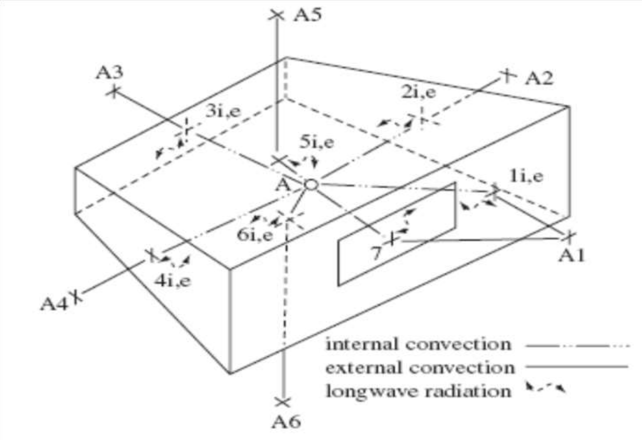
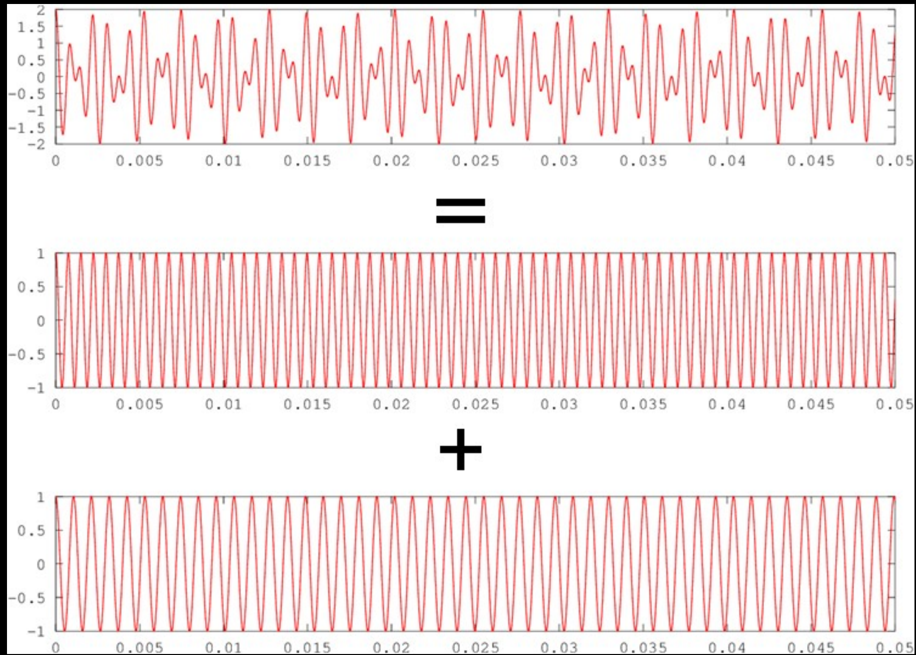


Figure 2.7: Example for zone energy balance.

Frequency domain response functions

- Underlying assumption is that time-series excitations (e.g. weather) can be represented by a steady-state term accompanied by a number of sine wave harmonics with increasing frequency and reducing amplitude.



- Each harmonic is processed separately and modified by thermal response factors appropriate to its frequency.

- The principle of superimposition is then invoked to obtain the final temperature or flux prediction by summing the cyclic contribution from each harmonic and expressing the result with respect to the mean condition.

- Employing only the 24 hour period harmonic gives rise to the *Admittance Method*, which may be applied manually.

Admittance method

- Employs three response factors:
  - decrement
  - surface factor
  - admittance
  - each possess a corresponding phase angle that determines the time difference between cause and effect.

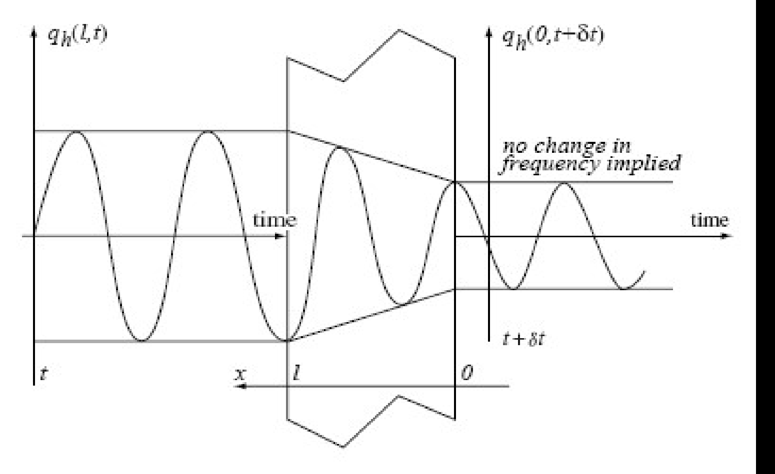


Figure 2.9: Effect of decrement factor and time lag.

- *decrement:* the ratio of the cyclic flux transmission to the steady state flux transmission

- *surface factor:* the portion of the heat flux at an internal surface that is re-admitted to the internal environmental point when temperatures are held constant.

- *admittance:* the amount of energy entering a surface for each degree of temperature swing at the environmental point.

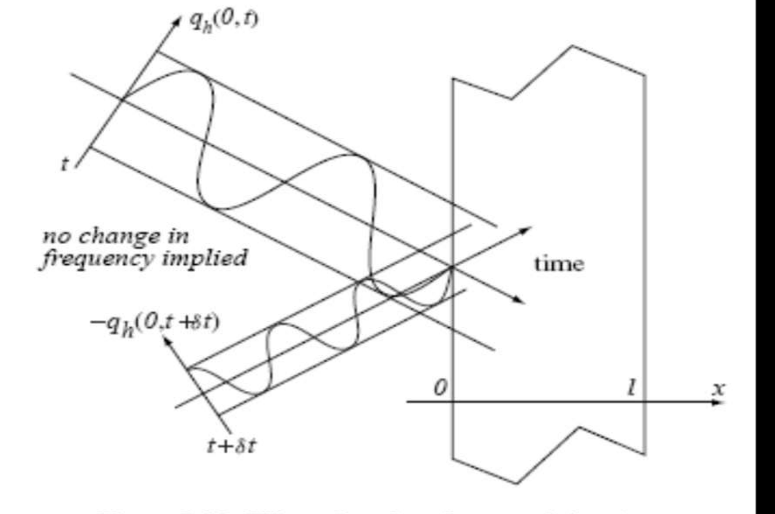


Figure 2.10: Effect of surface factor and time lag.

Admittance method: overheating assessment

Assessment process :

1. determine mean heat gains from all sources;
2. calculate mean internal temperature;
3. determine mean–to–peak swing in heat gains;
4. calculate swing in internal temperature; and
5. determine peak internal temperature as (2) + (4).

1. Mean Heat Gains: $Q'_t = Q'_s + Q'_c$

Q'_s = mean solar gain (W) = $SI' A_g$

I' = mean solar intensity (W/m^2; Table A8.1)

S = solar gain factor (Table A8.2)

A_g = area of glazing (m^2)

Q'_c = mean casual gain (W) = $\dfrac{(q_{c1}\, x\, t_1)+(q_{c2}\, x\, t_2)+etc}{24}$

q_{c1} and q_{c2} are instantaneous casual gains (W)

t_1 and t_2 are duration of individual gains (h)

2. Mean internal temperature

$Q'_t = \left(\sum A_g U_g + C_v\right)\left(t'_{ei} - t'_{ao}\right) + \sum A_f U_f \left(t'_{ei} - t'_{eo}\right)$

ΣAU = sum of products of exposed areas and U-values (W/ºC)
C_v = ventilation loss (W/ºC)
t'_{ei} = mean internal environmental temperature (ºC)
t'_{eo} = mean sol-air temperature (ºC; Table A8.3)
t'_{ao} = mean outdoor air temperature (ºC; Table A8.3)
g and f refer to glazed and opaque surfaces

For air change rates <2/h; $C_v = 0.33\ NV$
For air change rates >2/h; $1/C_v = (1/0.33NV) + (1/4.8A)$
N = rate of air interchange (1/h; Table A8.4)
V = enclosure volume (m^3)
A = total area bounding the enclosure (m^2)

Sol-air temperature: $t'_{eo} = t'_{ao} + R_{so}\ (\alpha I_t + \varepsilon I_1)$
R_{so}= outside surface resistance (m^2K/W)
I_t = total solar radiation intensity (W/m^2)
I_1 = net longwave radiation (W/m^2)

3. Swing from mean–to–peak in heat gains (influenced by variation in solar gain, structural heat gain, casual heat gain and ventilation gain):

a. Swing in solar gain (W): $Q_{\sim s} = S_a A_g (I_p - I')$
   S_a = alternating solar gain factor (Table A8.6)
   I_p = peak intensity of solar radiation (W/m^2)

b. Swing in structural heat gain (W): $Q_{\sim f} = f\,AU\,(t'_{ei} - t'_{eo})$
   f = decrement factor of constructions (taking account of time lags)

c. Swing in casual heat gain: $Q_{\sim c} = Q_c - Q'_c$
   $Q_c = q_{c1} + q_{c2} + q_{c3} + \ldots..$

d. Swing in heat gain to air (W) : $Q_{\sim a} = (A_g U_g + C_v)\, t_{\sim ao}$
   $t_{\sim ao}$ = swing in outside air temperature (ºC; Table A8.3)

Total swing in heat gains: $Q_{\sim t} = Q_{\sim s} + Q_{\sim f} + Q_{\sim c} + Q_{\sim a}$

4. Swing in internal temperature:

$$t_{\sim ei} = \frac{Q_{\sim t}}{\sum A.a + C_v}$$

$\Sigma A.a$ = sum of product of area and admittance for all internal surfaces (W/°C)

5. Peak internal temperature:

$t_p = t'_{ei} + t_{\sim ei}$

Worked example

Calculate the internal environmental temperature likely to occur at 16h00 on a sunny day in August in a south facing office as described by the following data.

Latitude: 51.7°N.
Internal dimensions: $7m \times 5m \times 3m$ high.
External wall: $7m \times 3m$, light external finish.
Window: 3. $5m \times 2m$, open during day, closed at night, internal (white) venetian blind.
Occupancy: 4 persons for 7 hours at 85W (sensible) per person.
Lighting: $20W/m^2$ of floor area, ON 08h00-17h00.
Computers: $5W/m^2$ of floor area, ON continuously.

Construction details:

Component	U-value (W/m^2C)	Admittance (W/m^2C)	Decrement (-)	Time Lag (h)	Comment
External wall	0.59	0.91	0.3	8	220mm brickwork, 25mm cavity, 25mm insulation, 10mm plasterboard.
Window	2.9	2.9	-	-	Double glazed, 12mm air gap, normal exposure (ignore frame).
Internal walls	1.9	3.6	-	-	220mm brickwork, 13mm light plaster finish.
Floor	1.5	2.9	-	-	150mm cast concrete, 50mm screed, 25mm wood block finish.
Ceiling	1.5	6.0	-	-	As floor but reversed.

Also, calculate the effect of leaving the window open at night.

Mean environmental temperature

i) Mean solar heat gain:

$Q'_s = S' \, I'_T \, A_g = 0.46 \times 175 \times (3.5 \times 2) = 563.5$ W

where Q'_s is the mean solar gain (W), A_g the sunlit area of glazing (m^2), I'_T the mean total solar irradiance (W/m^2; see Table A8.1) and S' the mean solar gain factor (see Table A8.2).

ii) Mean casual gain:

$Q'_c = 1/24 \, (g_{c1} \times t_1 + g_{c2} \times t_2 + ...) = 1/24 \, (4 \times 85 \times 7 + 7 \times 5 \times 20 \times 9 + 7 \times 5 \times 5 \times 24) = 536.7$ W

where Q'_c is the mean casual gain (W), g_{c1}, g_{c2}, ... are the instantaneous casual gains (W) and t_1, t_2, ... are the durations of g_{c1}, g_{c2}, ... (hours).

iii) Total mean heat gain:

$Q'_t = 563.5 + 536.7 = 1100.2$ W

iv) Mean internal environmental temperature:

$$Q'_t = (\Sigma A_g U_g + C_v)(T'_{ei} - T'_{ao}) + \Sigma A_f U_f \, (T'_{ei} - T'_{eo}) \qquad (1)$$

where *g* and *f* refer to glazed and opaque surfaces and C_v is the ventilation conductance evaluated from

$$1/C_v = 1/(0.33 \, NV) + 1/4.8 \, \Sigma A) = 1/(0.33 \times 3 \times\!\sim 105) + 1/(4.8 \times 142) \Rightarrow C_v = 90.2 \text{ W/K}$$

where N is the ventilation rate (h^{-1}; from Table A8.4), V the room volume (m^3) and ΣA the total internal surface area (m^2).

From Table A8.3, the mean outside air temperature, T'_{ao}, is 16.5ºC and the mean sol-air temperature, T'_{eo}, is 19.5ºC. Therefore from eqn (1):

$$1100.2 = (3.5 \times 2 \times 2.9 + 90.2)[(T'ei - 16.5) + (7 \times 3 - 7) \times 0.59] \, (T'ei - 19.5) \Rightarrow T'_{ei} = 26ºC$$

Swing in environmental temperature

i) Swing in effective solar heat gain:

$Q^*_s = S_a A_g (I_p - I') = 0.46(3.5 \times 2)(375 - 175) = 644$ W

where Q^*_s is the swing in effective heat gain due to solar radiation, S_a the alternating solar gain factor (from Table A8.6) and I_p the peak intensity of solar radiation (W/m^2) occurring (here) 1 hour before the peak to account for any lag.

ii) Swing in structural conduction gain:

$Q^*_s = f\,A\,U\,(T_{eo} - T'_{eo}) = 0.3 \times (7 \times 3 - 7) \times 0.59 \times (18.0 - 19.5) = -3.7$ W

Q^*_f is the swing in effective heat input due to structural gain, f the decrement factor, Φ the time lag associated with the decrement factor, T_{eo} the sol-air temperature at time of peak less time lag (here 8 hours) and T'_{eo} the mean sol-air temperature.

iii) Swing in casual gain:

$Q^*_c = Q_c - Q'_c = 4 \times 85 + 7 \times 5 \times 20 + 7 \times 5 \times 5 - 536.7 = 678.3$ W

where Q_c is the casual gain value at the peak hour (= $g_{c1} + g_{c2} + ...$).

iv) Swing in gain, air-to-air

$Q^*_a = (\Sigma A_g U_g + C_v)\, T^*_{ao} = [(3.5 \times 2 \times 2.9) + 90.2] \times 5 = 552.5$ W (2)

where Q^*_a is the swing in effective heat input due to the swing in outside air temperature and T_{ao} is swing in outside air temperature (from Table A8.3 = 21.5-16.5 = 5ºC).

v) Total flux swing:

$Q^*_t = 644 - 3.7 + 678.3 + 552.5 = 1878.5$ W

Swing in internal environmental temperature

$Q^*_t = (\Sigma AY + C_v)\, T^*_{ei}$ (3)

with the sum of the product of surface area and admittance given by

Component	AY
Floor	35 x 2.9 = 101.5
Ceiling	35 x 6.0 = 210
Window	7 x 2.9 = 20.3
External Wall	14 x 0.91 = 12.74
Internal Walls	51 x 3.6 = 183.6
	ΣAY = 528.1 W/K

$\Rightarrow T^*_{ei} = 1878.5/(528.1 + 90.2) = 3$ºC

Peak internal environmental temperature

$T''_{ei} = T'_{ei} + T^*_{ei} = 26 + 3 = 29$ºC

Effect of night ventilation

Use $C_v = 229.7$ (i.e. an air change rate of 10 h^{-1}) in Eqn (1) to obtain $T'_{ei} = 20.9$ºC

Substitute C_v in Eqn (2) to obtain new value of Q^*_a and therefore Q_t as 2576 W.

Substitute this Q^*_t and the recalculated ΣAY (651.3 W/K) in Eqn (3) to obtain $T^*_{ei} = 2.1$ºC.

The resulting peak internal environmental temperature is 23ºC, which is acceptable.