ME404 Radiation modelling

Heat transfer by radiation exchange may be conveniently divided into two categories: short-wave radiation emitted by the sun and long-wave radiation exchange between surfaces comprising the building or plant system.

1. Short-wave radiation

Figure 1 shows the spectral composition of the electromagnetic energy emitted by the sun. This range of wavelengths is known as the solar spectrum, which in practical terms extends from about 0.29 µm (1 µm = 10^{-6} meters) in the longer wavelengths of the ultraviolet region, through the visible region (0.4 to 0.8 µm), to about 3.2 µm in the far infrared. The majority of the solar energy comes from the visible and infrared parts of the spectrum in the form of light and heat respectively.

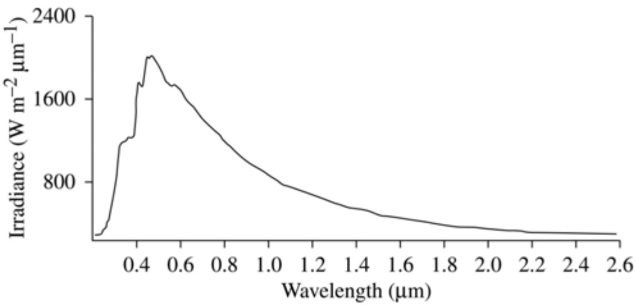
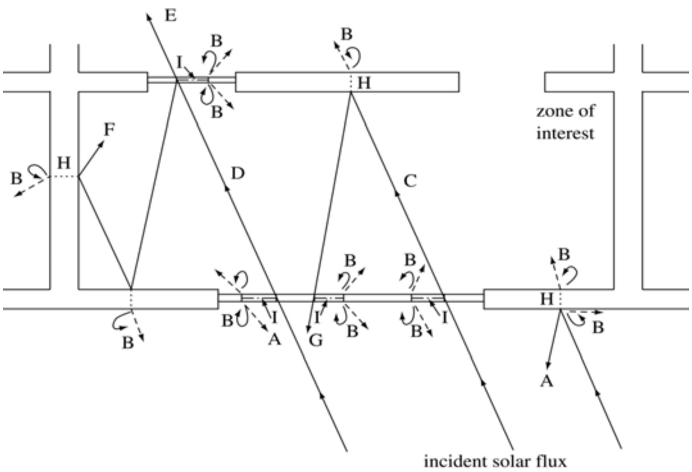


Figure 1: The solar spectrum.

To determine the terrestrial irradiance, the extraterrestrial intensity may be modified to account for the effects of atmospheric transmission. As the radiation traverses the atmosphere, scattering and absorption occurs due to the natural and anthropogenic related presence of gases, aerosols and pollutants. The result is that some portion of the solar power is lost, while the remaining portion comprises direct and diffuse components. The direct and diffuse irradiance, whether synthesised or measured, is used in the determination of the insolation of exposed locations throughout the building.

Figure 2 details the interactions between a building and the incident direct and diffuse solar radiation. The short-wave flux incident on external opaque surfaces will be partially absorbed and partially reflected, while some portion of the absorbed component may be transmitted to the corresponding interior surface, by conduction, to elevate the inside surface temperature and so enter the building via surface convection and longwave radiation exchange. Likewise, a portion of the absorbed component will cause outside surface temperature elevation and so give rise to a re-release of energy to ambient. If a multi-layered construction is opaque overall but has transparent elements located towards its outermost surface, some portion of the incident direct and diffuse radiation will also be transmitted inward until it strikes the intra-constructional opaque interface. Here, absorption and reflection will again occur, the latter giving rise to further absorptions and interface reflections as the flux travels outwards; the process continuing, essentially instantaneously, until the incident flux has been redistributed.

With windows, the direct and diffuse shortwave flux is reflected, absorbed and transmitted at each interface with the internally absorbed component being transmitted inward and outward by the processes of conduction, convection and long-wave radiation exchange. The transmitted direct beam continues onward to cause internal surface insolation as a function of the zone geometry. The subsequent treatment of this incident flux will depend on the nature of the receiving surface(s): absorption and reflection for an opaque surface, or absorption, reflection and transmission (to another zone or back to outside) in the case of a transparent surface. If the internal surface is a specular reflector then the reflected beam's onward path may be tracked by some suitable technique until diminished to insignificance. If the zone surface is a diffuse reflector then the apportioning of the reflected flux to other internal surfaces may be determined by weighting factors derived from the zone view factors. The same technique may be applied to the transmitted diffuse beam.



A reflected short-wave flux;
B flux emission by convection and long-wave radiation;
C short-wave flux transmission causing opaque surface insolation;
D short-wave transmission causing transparent surface insolation;
E short-wave transmission to adjacent zone;
F enclosure reflections;
G short-wave loss;
H solar energy penetration by transient conduction;
I solar energy absorption prior to retransmission by B.

Figure 2: Building-solar interaction.

The causal effect of these short-wave processes is then represented by the energy conservation equations, given that the short-wave flux injection at appropriate finite volumes can be established at each computational time-row. The requirement therefore is to establish the time-series of shortwave flux injection for finite volumes representing external opaque and transparent surfaces, intra-constructional elements, where these are part of a transparent multi-layered construction, and internal opaque and transparent surfaces.

Solar position

As shown in figure 3, the position of the sun may be represented in terms of altitude and azimuth angles that depend on site latitude, solar declination and local solar time.

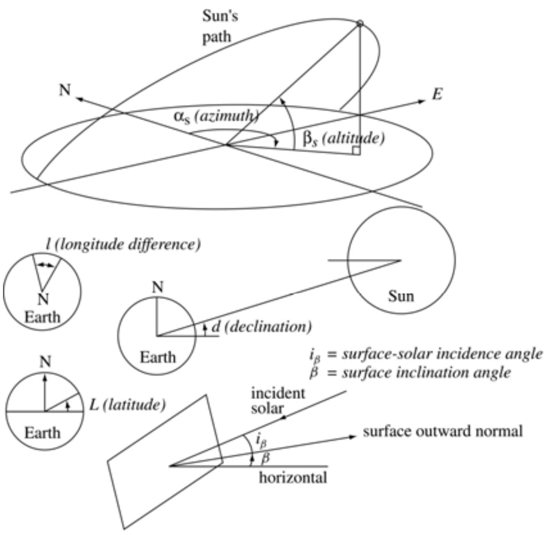


Figure 3: Solar angles.

The solar declination, d, may be determined from

$$d = 23.45 \sin(280.1 + 0.9863Y)$$

where Y is the year day number (January 1 = 1, February 1 = 32 etc.).

The solar altitude is then obtained from

$$\beta_s = \sin^{-1} [\cos L \cos d \cos \theta_h + \sin L \sin d]$$

where β_s is the solar altitude, L the site latitude (North +ve) and θ_h the hour angle, which is the angular expression of solar time and is positive for times before solar noon and negative for times thereafter:

$$\theta_h = 15\ (12 - t_s\ )$$

where t_s is the solar time (or local apparent time). This is a time scale which relates to the apparent angular motion of the sun across the sky vault, with solar noon corresponding to the point in time at which the sun traverses the meridian of the observer. Note that solar time does not necessarily coincide with local mean (or clock) time, t_m, with the difference given by

$$t_s - t_m = \pm\ l/15 + e_t + \delta$$

where l is the longitude difference (°), e_t the equation of time (hours) and δ a possible correction for daylight saving (hours). The longitude difference is the difference between an observer's actual longitude and the longitude of the mean or reference meridian for the local time zone. The difference is negative for locations to the West of the reference meridian and positive to the East. For the UK, the reference meridian is at 0° and local mean time is known as Greenwich Mean Time (GMT). For this case the previous equation becomes

$$t_s = GMT \pm l'/15 + e_t + \delta$$

where l' is the actual longitude of the observer. The equation of time makes allowance for the observed disturbances to the earth's rate of rotation:

$$e_t = 9.87\ \sin(1.978Y - 160.22) - 7.53\ \cos(0.989Y - 80.11) - 1.5\ \sin(0.989Y - 80.11).$$

The solar azimuth is given by

$$\alpha_s = \sin^{-1}\ [\cos d\ \sin \theta_h\ /\ \cos \beta_s\ ].$$

The angle of incidence of the direct beam, i_β, may be found from

$$i_\beta = \cos^{-1}\ [\sin \beta_s \cos(90 - \beta_f) + \cos \alpha_s \cos \omega \sin(90 - \beta_f)]$$

where ω is the surface-solar azimuth (= $|\alpha_s - \alpha_f|$) and α_f & β_f are the surface azimuth and elevation respectively. Note that negative values of cos i_β imply that the surface in question faces away from the sun and is therefore not directly insolated.

Inclined surface irradiation

The total radiation incident on an exposed opaque or transparent surface of inclination β_f and azimuth α_f has three components: direct beam, surroundings-reflected and sky diffuse.

The <u>direct beam</u> is relatively straightforward to determine since it involves only angular operations on the known direct horizontal irradiance:

$$I_{d\beta} = I_{dh} \cos i_\beta / \sin \beta_s$$

where $I_{d\beta}$ is the direct intensity on the inclined surface and I_{dh} is the direct horizontal intensity (both W/m^2).

The surroundings-reflected component comprises short-wave reflections from the surfaces of surrounding buildings and the ground. The former may be estimated as a fraction of the short-wave flux incident on the corresponding face of the target building. The ground is usually considered as a, isotropic source (diffuse reflector) and representative view factors are used to associate portions of the reflected radiation with each building surface. For an unobstructed vertical surface ($\beta_f = 0$), the view factor between the surface and the ground, and between the surface and the sky is in each case 0.5 and so the radiation intensity at the surface due to ground reflection is given by

$$I_{rv} = 0.5\ (I_{dh} + I_{fh})r_g$$

where I_{fh} is the horizontal diffuse radiation and r_g the ground reflectivity. For a surface of non-vertical inclination, a simple view factor modification is introduced so that

$$I_{r\beta} = 0.5\ [1 - \cos\ (90 - \beta_f)](\ I_{dh} + I_{fh})r_g$$

where $I_{r\beta}$ is the ground reflected radiation incident on a surface of inclination β_f.

Calculating the sky diffuse component on a surface of inclination β_f is problematic because of the anisotropic nature of the sky radiance distribution. The following approach, one of several possible models, increases the intensity of the diffuse flux due to circumsolar activity and horizon brightening:

$$I_{s\beta} = I_{fh}\{0.5[1+\cos(90-\beta_f)]\}.\{1+[1-(I_{fh}^2/I_{Th}^2)]\sin^3 0.5\beta_f\}.\{1+[1-(I_{fh}^2/I_{Th}^2)]\cos^3 i_\beta \sin^3(90-\beta_f)\}$$

where I_{Th} the total horizontal radiation, $I_{dh}+I_{fh}$. When the sky is completely overcast, $I_{fh}/I_{Th}=1$ and the expression reduces to the isotropic sky case.

The foregoing equations allow the computation of direct and diffuse shortwave radiation impinging upon exposed external surfaces. These flux quantities, when multiplied by surface absorptivity, are the short-wave nodal heat generation terms, q_{SI}, of the nodal energy conservation equations.

Intra-zone short-wave distribution

The above equations permit the calculation of the direct and diffuse irradiance of exposed building surfaces. For opaque surfaces, irradiance modification by surface absorptivity and shading factors will give the short-wave heat injection to be applied to surface nodes via the excitation matrix (**C**). For a window system, the transmitted portion of the direct beam can be evaluated from

$$Q_{dt} = I_{dh}/\sin\alpha_s[\tau_{i\beta}(1-P_g)A_g.\cos i_\beta]$$

where Q_{dt} is the transmitted direct beam flux (W), $\tau_{i\beta}$ the overall transmissivity for a given flux incidence angle, P_g the window shading factor (proportion of 1) and $A_g.\cos i_\beta$ the apparent window area. If, as is often the case, more than one internal surface will share this transmitted radiation then any internal surface will receive a heat injection given by

$$q_{Si} = Q_{dt}\ P_i\ \Omega_i/A_i$$

where Ω_i is the surface absorptivity, A_i the surface area and P_i the proportion of the window direct beam transmission that strikes the surface in question (proportion of 1).

The first reflected flux is given by

$$q_{Ri} = q_{Si}(1- \Omega_i)/ \Omega_i \ .$$

The accumulated flux reflections from each surface can now be further processed to give the final apportioning between all internal surfaces. If the usual assumption of diffuse reflections is made then apportionment can be decided on the basis of enclosure view factor information as described in the following section. For the case of specular reflections a recursive ray tracing technique can be employed.

Where the internal surface is composed of opaque and transparent portions, there will be onward transmission of incident short-wave flux to a connected zone or back to the outside. This can have a significant impact within buildings incorporating passive solar features.

The diffuse beam transmission can be determined from

$$Q_{ft} = (I_{s\beta} + I_{r\beta})A_g \cos 51 \ \tau_{51}$$

where τ_{51} is the overall transmissivity corresponding to a 51° incidence angle, representing the average approach angle for anisotropic sky conditions.

This flux quantity can now be processed by the technique described for the direct beam: internal surface distribution on the basis of specular or diffuse reflections.

2. Long-wave radiation

Heat transfer by long-wave radiation exchange between two surfaces is an important issue in energy systems modelling but one which introduces mathematical complexity due to non-linear behaviour and the spatial problems caused by complex geometries and inter-surface obstructions.

The radiation flux, q_b, emitted by a perfect black body is given

$$q_b = \sigma \ A \ \theta^4$$

where σ is the Stefan-Boltzmann constant (5.6704×10^{-8} W/m^2.K^4) and θ the body's absolute temperature (K). Real building materials do not behave as black bodies and deviate in their ability to absorb the incoming long-wave energy completely.

The radiant flux emitted by such a ‘grey’ body is given by a temperature dependent modification to the above equation:

$$q = \varepsilon \ \sigma \ A \ \theta^4$$

where ε is the surface emissivity.

Within an enclosure the radiation emitted by all surfaces will, after multiple reflections, be totally re-absorbed and, in the process, redistributed. Assuming that no energy is lost by transmission directly to an adjacent enclosure and that the surfaces are diffuse reflectors, then a recursive solution is possible. The initial fluxes emitted by each surface are tracked to first reflection and the surface absorptions determined.

For example, if four grey surfaces form an enclosure as shown in figure 4, then the flux emitted by each surface is given by

$$q_1 = \varepsilon\sigma A_1\theta_1^4 \qquad q_2 = \varepsilon\sigma A_2\theta_2^4$$

$$q_3 = \varepsilon\sigma A_3\theta_3^4 \qquad q_4 = \varepsilon\sigma A_4\theta_4^4$$

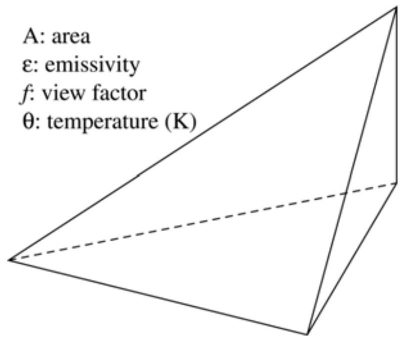


Figure 4: Four surfaces bounding an enclosure.

At first reflection, the absorption at each surface will have contributions as follows

$$a_1' = \quad + q_2 f_{2\to1}\varepsilon_1 \quad + q_3 f_{3\to1}\varepsilon_1 \quad + q_4 f_{4\to1}\varepsilon_1$$

$$a_2' = + q_1 f_{1\to2}\varepsilon_2 \quad + q_3 f_{3\to2}\varepsilon_2 \quad + q_4 f_{4\to2}\varepsilon_2$$

$$a_3' = + q_1 f_{1\to3}\varepsilon_3 \quad + q_2 f_{2\to3}\varepsilon_3 \quad + q_4 f_{4\to3}\varepsilon_3$$

$$a_4' = + q_1 f_{1\to4}\varepsilon_4 \quad + q_2 f_{2\to4}\varepsilon_4 \quad + q_3 f_{3\to4}\varepsilon_4$$

where a_i' is the total flux absorption at surface i from all surfaces after the first reflection (W), and $f_{j\to i}$ the geometric view factor between surfaces j and i. A single flux quantity can now be determined for each surface that represents the total apparent flux emission for processing to the next reflection:

$$r_i' = a_i'(1 - \varepsilon_i)/\varepsilon_i \text{ ; I =1,2,3,4}$$

where r_i' is the flux reflected at surface i after first reflection. After the second reflection, the total absorption at each surface is given by

$$a_1'' = \quad a_1' \quad + r_2' f_{2\to1}\varepsilon_1 \quad + r_3' f_{3\to1}\varepsilon_1 \quad + r_r' f_{4\to1}\varepsilon_1$$

$$a_2'' = \quad a_2' \quad + r_1' f_{1\to2}\varepsilon_2 \quad + r_3' f_{3\to2}\varepsilon_2 \quad + r_4' f_{4\to2}\varepsilon_2$$

$$a_3'' = \quad a_3' \quad + r_1' f_{1\to3}\varepsilon_3 \quad + r_2' f_{2\to3}\varepsilon_3 \quad + r_4' f_{4\to3}\varepsilon_3$$

$$a_4'' = \quad a_4' \quad + r_1' f_{1\to4}\varepsilon_4 \quad + r_2' f_{2\to4}\varepsilon_4 \quad + r_3' f_{3\to4}\varepsilon_4$$

where a_i'' is the total absorption of flux at surface i from all surfaces after the second reflection.

The flux reflections are then given by

$$r_1'' = \left(a_1'' = a_1'\right)\left(1-\varepsilon_1\right)/\varepsilon_1 \qquad r_2'' = \left(a_2'' = a_2'\right)\left(1-\varepsilon_2\right)/\varepsilon_2$$

$$r_3'' = \left(a_3'' = a_3'\right)\left(1-\varepsilon_3\right)/\varepsilon_3 \qquad r_4'' = \left(a_4'' = a_4'\right)\left(1-\varepsilon_4\right)/\varepsilon_4$$

where the absorptions and reflections at each recursive step may be determined from

$$\left.\begin{aligned} a_i^n &= a_i^{n-1} + \sum_{j=1}^{N} r_j^{n-1} f_{j\to i}\varepsilon_i \\ r_i^n &= \left(a_i^n - a_i^{n-1}\right)\left(1-\varepsilon_i\right)\Big/\varepsilon_i \end{aligned}\right] \quad \begin{aligned} &1 \le n \le \infty \\ &a_i^0 = 0 \\ &r_i^0 = q_i \\ &f_{i\to i} = 0 \end{aligned}$$

and in practice the recursive process continues until the reflected flux is reduced to insignificance. In most applications incorporating conventional building materials (high emissivity) this condition will be reached after about three recursive steps. More recursions will be required where low emissivity surfaces are present (e.g. within a low-ε window system).

View factor determination

The determination of the view factor relationship is a non-trivial task in the case of buildings with the possibility of complex geometries, inter-surface obstructions, surface openings and specular reflections. A number of approaches to view factor assessment are possible, ranging from rigorous numerical methods to published solutions for standard geometries.

Consider figure 5, which shows a ray of intensity I, contained within a solid angle dΩ, corresponding to an elemental area dA, propagated in a direction Ω, and making an angle θ with the normal, n, to the surface element.

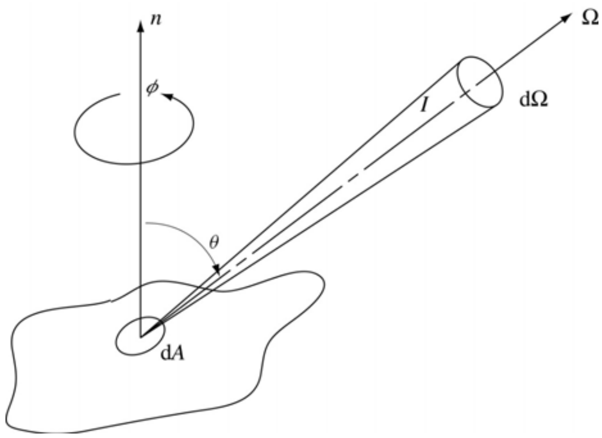


Figure 5: surface elemental ray.

The emitted flux contained within dΩ is

$$dq = I \cos \theta \, d\Omega$$

with the flux contained in the solid angle over the entire hemisphere obtained by integration:

$$q = \int \cos \theta \, d\Omega \, .$$

Now since dΩ = sin θ dθ dφ, where φ is the azimuth angle illustrated in figure 5, this gives

$$q = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I \cos\theta \sin\theta d\phi = \pi I .$$

Consider figure 6, which shows two planar elemental areas, dA_i and dA_j, separated by a distance r and with polar angles, θ_i and θ_j, between r and normals n_i and n_j respectively.

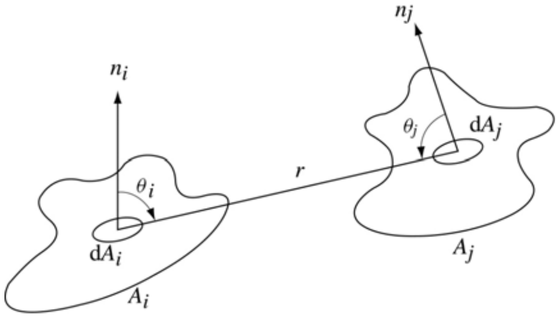


Figure 6: Two communicating elemental areas.

The radiative flux leaving dA_i that arrives at dA_j is from the above

$$dq_i = I_i \cos\theta_i \, d\Omega_{ij}$$

and since $d\Omega_{ij} = dA_j \cos\theta_j / r^2$, it follows that

$$dq_i = I_i \cos\theta_i \, dA_j \cos\theta_j / r^2.$$

Introducing the elemental view factor, $df_{dAi \to dAj}$, which is the ratio of the radiative flux leaving dA_j that arrives directly at dA_j to the total radiative flux leaving dA_j:

$$df_{dAi \to dAj} = dq_i/q = \cos\theta_i \, dA_j \cos\theta_j / \pi r^2.$$

In a similar manner, the point view factor $f_{dAi \to Aj}$ can be defined as the fraction of the radiative energy leaving dA_i that is received by surface A_j:

$$f_{dA_i \to A_j} = \int_{A_j} df_{dA_i \to dA_j} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} dA_j$$

Now, the flux emitted by A_j is $I_j A_j$ and the amount received by dA_i is $I_j \int^{Aj} df_{dAj \to dAi} \, dA_j$ and therefore $f_{Aj \to dAi}$ is given by

$$f_{A_j \to dA_i} = I_j \int_{A_j} \frac{df_{dA_j \to dA_i}}{I_j A_j} dA_j = \frac{1}{A_j} \int_{A_j} df_{dA_j \to dA_i} dA_j$$

and so

$$f_{A_i \to dA_j} = \frac{dA_i}{A_j} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} dA_j.$$

The reciprocity relationship follows by equating the last two equations:

$$dA_i \, f_{dA_i \to A_j} = A_j \, f_{A_j \to dA_i}.$$

Lastly, the flux emitted by A_i is $I_i A_i$ and the amount received by A_j is $I_i \int^{Ai} f_{dAi \to Aj} dA_i$ and therefore the area-to-area view factor, $f_{Ai \to Aj}$, is given by

$$f_{A_i \to A_j} = \frac{1}{A_i} \int_{A_i} f_{dA_i \to A_j} dA_i = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2}$$

with, as before

$$A_i f_{A_i \to A_j} = A_j f_{A_j \to A_i}.$$

The determination of the view factor relationship by double integration over the extents of two directly communicating surfaces can be achieved by a finite difference representation. A surface polygon is subdivided into a number of elemental areas and then a unit hemisphere is established above the centre point of each area to represent dA_i as shown in figure 7.

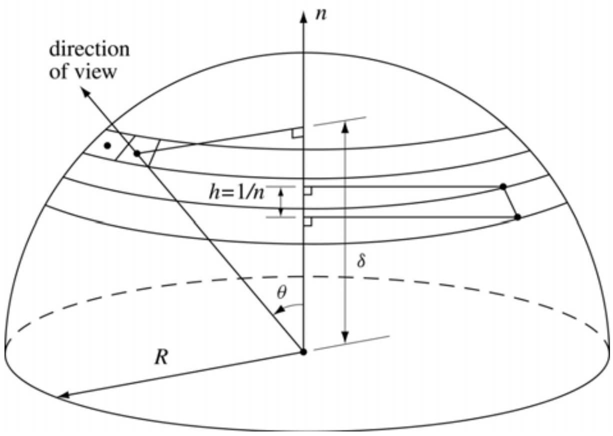


Figure 7: Strip and patch subdivision of a unit hemisphere.

Each unit hemisphere is then subdivided into a number of equal solid angles by 'strip and patch' subdivision as shown. Every solid angle is then projected until intersection occurs with another surface so that a 'viewed' polygon is associated with each hemispherical patch to account for all radiation emitted by dA_i. The point view factor is then determined by summing the contributions of each viewed polygon. The view factor between two finite surfaces A_i and A_j is then found from

$$f_{A_i \to A_j} = \frac{1}{A_i} \int_{A_i} f_{dA_i \to A_j} dA_i$$

Such an algorithm guarantees that

$$\sum_j f_{i \to j} = 1$$

and that reciprocity will prevail.

Exchange between external surfaces

The net long-wave radiation exchange at an external building surface will depend on the temperature of surrounding objects. If the surroundings are represented by an equivalent temperature, θ_e then the net long-wave radiation exchange can be expressed as

$$q = A_s \, \varepsilon \, \sigma \, (\theta_e^4 - \theta_s^4)$$

where A_s is the surface area (m^2), ε the surface emissivity, σ the Stefan-Boltzmann constant ($W/m^2.K^4$) and θ_s the absolute temperature of the surface (K). The equivalent temperature is a function of the temperatures of the sky, ground and surroundings:

$$\theta_e^4 = f_1\,\theta_{sky}^4 + f_2\,\theta_{grd}^4 + f_3\,\theta_{sur}^4$$

where f_1, f_2 and f_3 are the view factors to the sky, ground and surroundings respectively.

The need, then, is to estimate these three temperatures from known weather data.

The sky temperature under non-cloudy conditions can be determined from

$$R_s = 5.31x10^{-13}\,\theta_{sc}^6$$

where R_s is the sky radiation (W/m_2) and θ_{sc} the screen air temperature (K). If the assumption is made that the clear sky behaves as a black body then

$$R_s = \sigma\,\theta_{sky}^4$$

and so

$$\theta_{sky} = 0.05532\,\theta_{sc}^{1.5}.$$

In the presence of clouds, the mean sky temperature increases and an alternative expression has been proposed:

$$R_s' = (1-C)R_s + C\,\varepsilon_c\,\sigma\,\theta_{sc}^4$$

where R_s' is cloudy sky radiation (W), C the cloud cover factor (proportion of 1), and ε_c the emissivity of the cloud base given by

$$\varepsilon_c = (1-0.84C)\left(0.527+0.161e^{[8.45(1-273/\theta_{sc})]}+0.84C\right)$$

Substitution of this equation in the previous one and assuming that $R_s' = \sigma\,\theta_{sky}^4$ gives the final expression for the effective sky temperature under cloudy conditions:

$$\theta_{sky} = \left\langle 9.365574x10^{-6}(1-C)\theta_{sc}^6 + \theta_{sc}^4 C(1-0.84C)\left[\left(0.527+0.161e^{[8.45(1-273/\theta_{sc})]}\right)+0.84C\right]\right\rangle^{0.25}.$$

The simplest method for ground temperature estimation is to use the concept of sol-air temperature so that

$$\theta_{grd} = \theta_A + (\alpha_g\,I_{gh} - q_{lw})/R_{so}$$

where θ_A is the air temperature (°C), α_g the ground absorptivity, I_{gh} the total solar irradiance (W/m^2), q_{lw} the net longwave radiation exchange (W/m^2) and R_{so} the combined convective/radiative ground surface layer resistance (m^2 °C/W^{-1}). Application of this expression will require, firstly, that the longwave exchange term be evaluated. This, in turn, will require knowledge of the temperatures of the sky and obstructions.

In the absence of a detailed model of the surroundings for use in an explicit simulation exercise, a pragmatic approach is to evaluate the surroundings temperature as a weighted function of the immediate past temperatures of the surfaces of the target building.