ME404 Numerical method – systems

Essentially, there are two approaches to explicit systems simulation – *sequential* and *simultaneous*.

In the sequential approach, plant components are replaced by an equivalent input/output relationship so that when connected to form a system, the calculated output from one component becomes the input to the next. An iterative solution method is then used to achieve solution convergence throughout the network. The algorithms that represent the individual components may be simplified (e.g. based on manufacturers' data) or detailed (e.g. based on a fundamental mathematical model). The technique has three principal advantages: different modelling methods can be applied to different plant components allowing simplified and fundamental models to coexist; a rapid prototyping approach is fostered because component models can initially be rudimentary; and the discreteness of the approach prevents new models from negatively impacting on the overall solution. Difficulties will arise, however, when control dynamics are included or where a component model requires downstream information that is, at the time of need, undetermined (e.g. where recirculation loops are present).

In the simultaneous approach, each plant component is represented by discrete finite volumes (FV) as was done on the building side. Each FV is then assigned a set of conservation equations depending on the number of phases present and the properties to be conserved (energy, mass, electrical power etc.). The matrix equation to emerge for the network of plant components may then be combined with the building and fluid flow matrix equations, control statements superimposed, and the entire equation system solved using appropriate numerical techniques. In this way the problems associated with the sequential approach are overcome and the combinatorial possibilities for systems representation are effectively without limit. Some examples of the numerical method applied to plant systems follows.

Air conditioning

The function of an air conditioning (AC) system is to deliver appropriately clean air at a given temperature and moisture content as required to offset the sensible and latent loads imposed on the conditioned space. Specific systems may be associated with one of three general types: all air, air and water and packaged.

Irrespective of its categorisation, an AC system can be built from a limited number of components: duct, mixing box, fan, heating coil, cooling coil, boiler, pump, pipe, chiller, heat pump, supply diffuser, damper and n-way diverging/converging junction. Mathematical models of these components are required at different levels of detail in order to support the range of possible design tasks – from a rapid assessment of overall performance, to an analysis of the impact of extended surface geometry on coil efficiency.

The simulation of an AC system is complicated by the fact that the working fluid comprises two phases, dry air and water vapour. Consider the rudimentary FV scheme applied to a packaged air handling unit as shown in figure 1. Outside air at temperature θ_o, humidity ratio g_o and enthalpy h_o is mixed with zone return air at temperature θ_r, humidity ratio g_r and enthalpy h_r, and passed to a chilled water cooler, a humidifier and a re-heater to achieve the required supply condition to offset the zone's sensible and latent loads. An energy balance is formulated at some arbitrary time-row, ξ:

For component 1:

$$m_o h_o + m_r h_r - m_1 h_1 + q_{e1} = \frac{d\left(\bar{\rho}_1 V_1 h_1\right)}{dt}\Big|_{t=\xi}$$

For component 2:

$$m_1h_1 - m_2h_2 - m_ch_c + q_{e2} - q_{x2} = \frac{d(\bar{\rho}_2V_2h_2)}{dt}\Big|_{t=\xi}$$

For component 3:

$$m_2h_2 + m_hh_h - m_3h_3 + q_{e3} = \frac{d(\bar{\rho}_3V_3h_3)}{dt}\Big|_{t=\xi}$$

For component 4:

$$m_3h_3 - m_4h_4 + q_{e4} + q_{x4} = \frac{d(\bar{\rho}_4V_4h_4)}{dt}\Big|_{t=\xi}$$

For component 5:

$$m_4h_4 - m_5h_5 + q_{e5} = \frac{d(\bar{\rho}_5V_5h_5)}{dt}\Big|_{t=\xi}$$

where m is the mass flow rate of the air/vapour mixture (kg/s), h the mixture specific enthalpy (J/kg), q_{ei} the heat exchange of component i with the surroundings (W), q_{x2} the cooling coil total heat transfer (W), q_{x4} the re-heater coil total heat transfer (W), ρ_i the volume weighted density of component i (kg/m^3), V_i the total volume of component i (m^3). The subscripts o and r relate to ambient and return air states respectively, c relates to the cooler moisture extract, and h relates to the humidifier moisture addition.

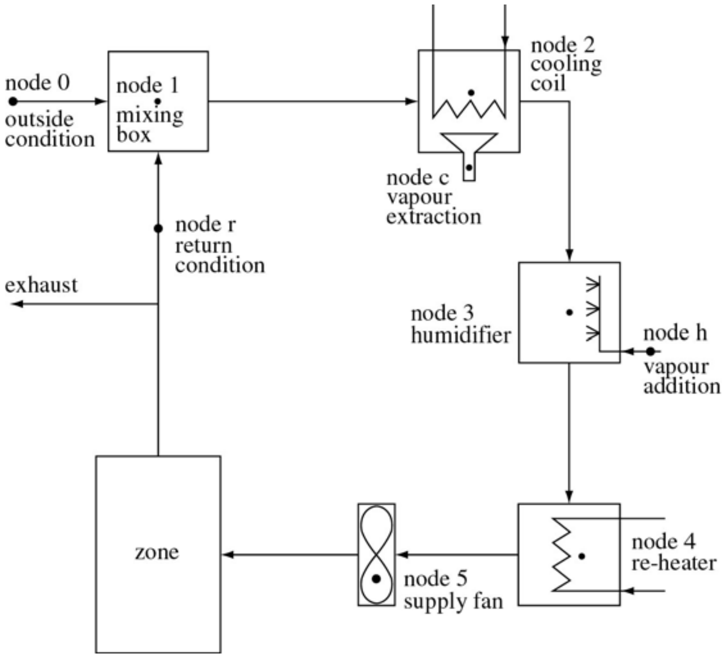

Figure 1: A simple model of a packaged air handling unit.

Since each component is represented by a single node, its thermal inertia is a function of the average thermodynamic state as represented by its single density value. This requires the use of an average density:

$$\bar{\rho}_i = \sum_{j=1}^{N}(\rho_jV_j) \Big/ \sum_{j=1}^{N}V_j$$

where N is the number of distinct intra-component regions. (A more refined approach will result from the introduction of a multi-FV representation whereby the thermal inertia of these intra-component regions is explicitly represented.

The component mass balances, for the dry air and water vapour separately, may be formulated as follows.

For component 1:

$$m_{o(d)} + m_{r(d)} - m_{1(d)} = 0\Big|_{t=\xi}$$

$$m_{o(d)}g_o + m_{r(d)}g_r - m_{1(d)}g_1 = 0\Big|_{t=\xi}$$

For component 2:

$$m_{1(d)} - m_{2(d)} = 0\Big|_{t=\xi}$$

$$m_{1(d)}g_1 - m_{2(d)}g_2 - m_c = \frac{d(\rho_L V_c)}{dt}\Big|_{t=\xi}$$

For component 3:

$$m_{2(d)} - m_{3(d)} = 0\Big|_{t=\xi}$$

$$m_{2(d)}g_2 - m_{3(d)}g_3 + m_h = \frac{d(\rho_L V_c)}{dt}\Big|_{t=\xi}$$

For component 4:

$$m_{3(d)} - m_{4(d)} = 0\Big|_{t=\xi}$$

$$m_{3(d)}g_3 - m_{4(d)}g_4 = 0\Big|_{t=\xi}$$

For component 5:

$$m_{4(d)} - m_{5(d)} = 0\Big|_{t=\xi}$$

$$m_{4(d)}g_4 + m_{5(d)}g_5 = 0\Big|_{t=\xi}$$

where $m_{i(d)}$ is the mass flow rate of dry air (kg/s) associated with component i, g the humidity ratio (kg/kg), ρ_L the density of the water remaining in the cooler or humidifier (kg/m^3), V_c the volume of this water, V_h the humidifier residual water volume, m_c the cooler vapour extraction rate (kg/s sup) and m_h the humidifier vapour addition rate (kg/s).

As with the building-side, energy conservation equations are obtained by performing an equal weighting of the explicit and implicit forms of the above equations. For example, the component 2 energy and two phase mass balance equations will then be as follows.

$$\begin{aligned}&\left[2\bar{\rho}_2(t+\delta t)V_2 + \delta t m_2(t+\delta t)\right]h_2(t+\delta t) - \delta t m_1(t+\delta t)h_1(t+\delta t) + \delta t m_c(t+\delta t)h_c(t+\delta t) \\ &- \delta t q_{e2}(t+\delta t) + \delta t q_{x2}(t+\delta t) = \left[2\bar{\rho}_2(t)V_2 - \delta t m_2(t)\right]h_2(t) + \delta t m_1(t)h_1(t) - \delta t m_c(t)h_c(t) \\ &+ \delta t q_{e2}(t) - \delta t q_{x2}(t).\end{aligned}$$

for energy

$$m_{1(d)}(t+\delta t) - m_{2(d)}(t+\delta t) = -m_{1(d)}(t) + m_{2(d)}(t)$$

for dry air

$$\begin{aligned}&m_{1(d)}(t+\delta t)g_1(t+\delta t) - m_{2(d)}(t+\delta t)g_2(t+\delta t) - m_c(t+\delta t) - 2\rho_L(t+\delta t)V_c/\delta t \\ &= -m_{1(d)}(t)g_1(t) + m_{2(d)}(t)g_2(t) + m_c(t) - 2\rho_L(t)V_c/\delta t\end{aligned}$$

for water vapour

This process may be continued for the other components and the equations to result added to the other conservation equations representing the building, other systems etc.

Active solar systems

Figure 2 shows the components of an active solar system. The flat plate collector (air or water) supplies some heating load directly or, in times of excess, delivers the heat to a thermal store. When applying the finite volume technique, some of the resulting conservation equations will be identical to those already derived for the building-side, for example, the collector back plate is the same as a wall surface, the glass cover as a window and the working fluid as the air in a room. Other components, such as the heat exchanger and thermal store, will require additional equations.

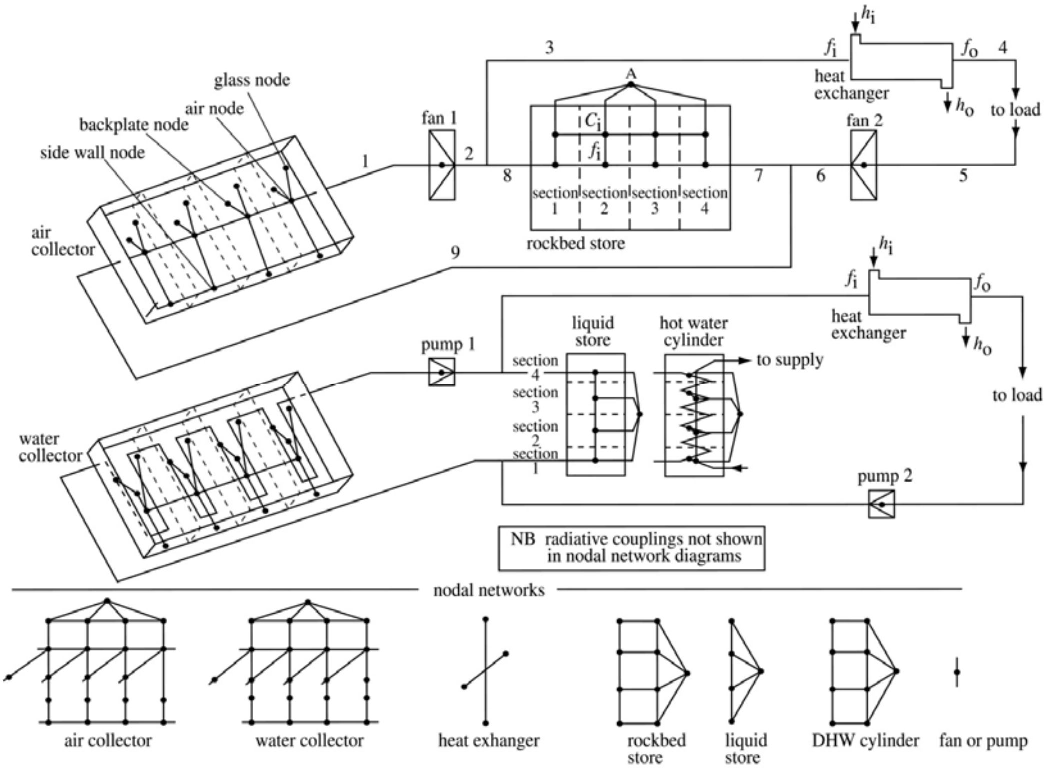

Figure 2: Active solar system components and FV scheme.

Figure 3 shows the whole system matrix equation to result for the system of figure 2.

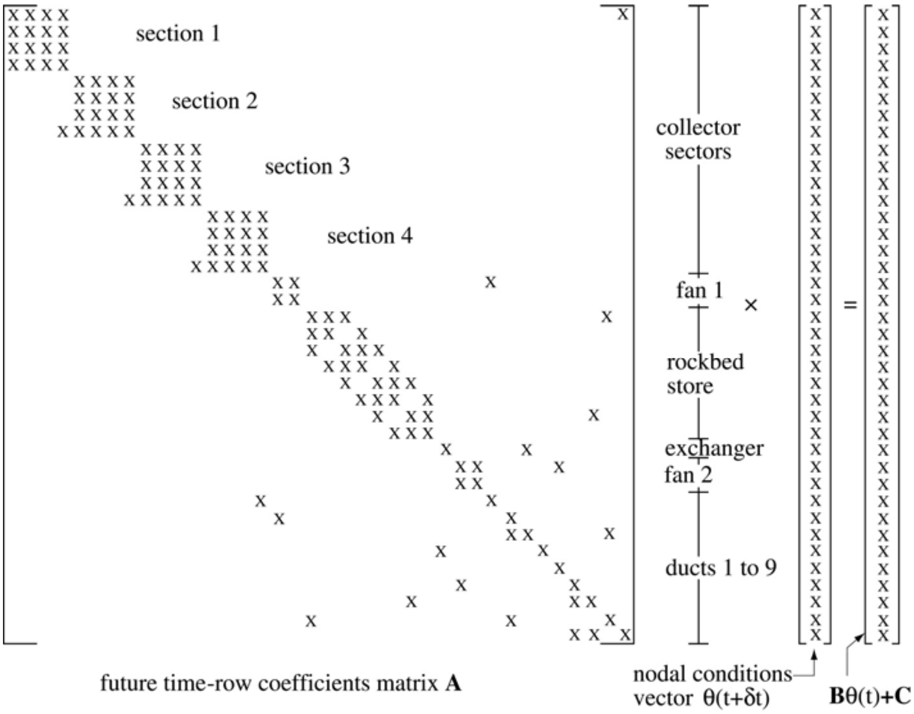

Figure 3: Active solar system energy balance matrix equation.

Wet central heating system

Figure 4 shows the components of a wet central heating system: boiler, pump, radiators and hot water cylinder all linked by distribution pipes and subjected to control action.

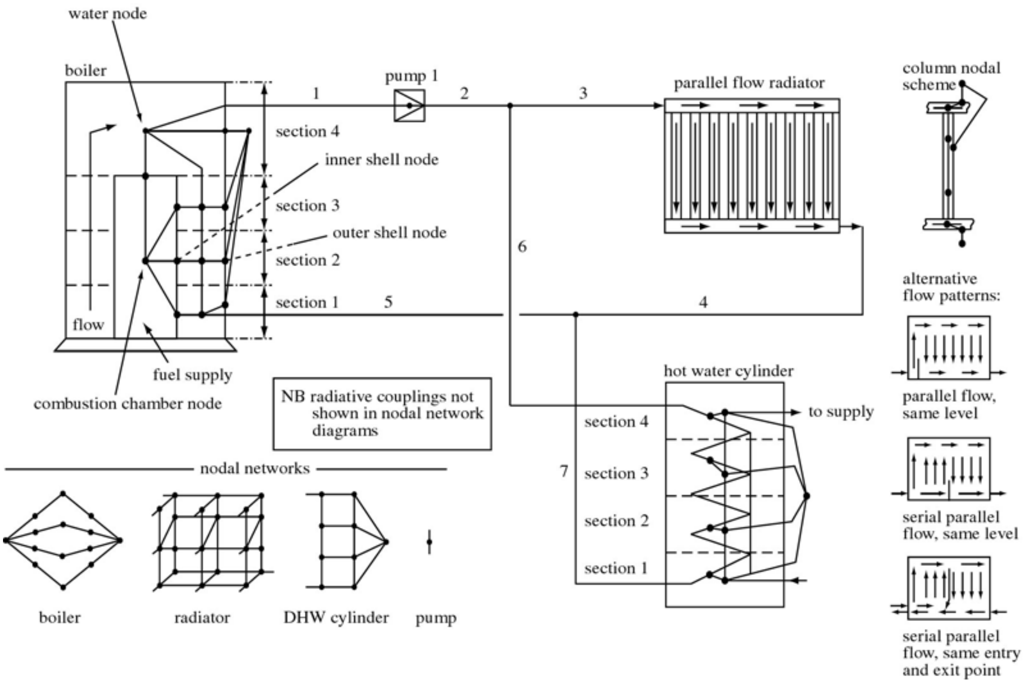

Figure 4: Wet central heating system components and FV scheme.

Figure 5 shows the system matrix equation to result for this example system.

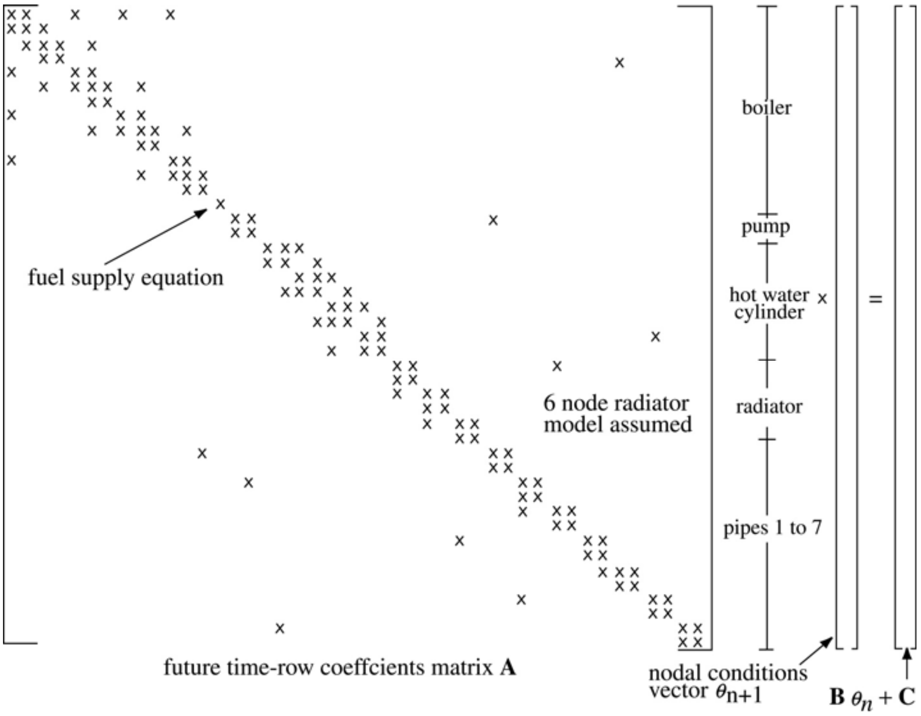

Figure 5: Wet central heating system energy balance matrix equation.

New and renewable energy conversion systems

Renewable energy (RE) systems have typically been pursued at the strategic level, with the grid connection of medium-to-large scale hydro stations, bio-gas plant and wind farms. In order to avoid problems with network balancing and power quality, it has been estimated that the deployment of RE systems with limited control possibilities should be restricted to around 25% of total installed capacity. This limitation is due to the intermittent nature of RE sources, requiring controllable, fast responding reserve capacity to compensate for fluctuations in output; and energy storage to compensate for non-availability. To achieve a greater penetration of RE, alternative deployment approaches will be required. For an equivalent installed capacity of 2000 MW (i.e. a conventional power station), table 1 gives the number of systems required to maintain the same installed capacity as the technology scale reduces. Not surprisingly, the replication extent is inversely proportional to the deployment scale. Less obvious, perhaps, is the increased opportunity for energy demand reduction and economic stimulation at the reduced scale where the ‘greening’ of society can more readily be enacted.

Table 1: The spectrum of RE deployment opportunities.

Scale	Replication extent[#]
Power station	1 @ 2000 MW
Wind farm	100 @ 20 MW
Wave power	4000 @ 0.5 MW
Micro gas turbine	40,000 @ 0.050 MW
Building-integrated RE system	200,000 @ 0.010 MW
[#] The number of RE replications would be 3-5 times greater if the requirement is to match energy production.	

Recent developments in small scale new and renewable energy systems (NRE) – in the form of photovoltaic modules, micro gas turbines, fuel cells and ducted wind turbines – have given rise to the concept of a micro power approach whereby NRE technologies are embedded within the built environment. In conjunction with demand reduction measures, the approach requires the matching of local supply potentials to optimised demand: passive solar and other technologies are used to reduce energy requirements, and active NRE systems are used to meet a significant proportion of the residual demand. Any energy deficit is met from the public electricity supply operating co-operatively with the NRE systems. For the approach to be successful, NRE systems deployment and energy efficiency measures must be pursued together. Consider the following scenario.

For a northern European climate characterised by a mean annual global horizontal solar irradiance of 150 W/m^2, the mean power production from a photovoltaic component of 13% conversion efficiency is approximately 20 W/m^2. For a mean wind speed of 5 m/s, the power produced by a micro wind turbine will be of a similar order of magnitude (but with a different profile shape). The point is that such power densities are significantly lower than a typical building's energy demand. In the UK, for example, an office building will have a demand of approximately 300 kWh/m^2.y. Assuming 250 working days in the year, this translates to 50 W/m^2 of floor area or 100 W/m^2 of facade area, i.e. 5 times the available renewable resource! This situation changes dramatically when energy efficiency measures are firstly put in place. Consider the case where aggressive measures result in a reduced demand of 70 kWh/m^2.y, which translates to approximately 24 W/m^2 of facade area. Now the supply matches the demand, quantitatively if not temporally: the challenge then is to overcome the temporal mismatch without recourse to importing/exporting from/to the public electricity supply (PES).

The goal within a micro power approach is to utilise energy efficiency and/or passive solar measures to reduce the overall energy demand and adjust the demand profiles to facilitate NRE systems integration. At times where the demand exceeds the NRE supply, the deficit is met either from local storage or from the PES. At times when the demand is less than the available NRE supply, the excess is used to charge the local storage system.

Two possibilities exist to avoid exporting to the PES when the storage system is fully charged. Either the excess can be directed to a local dump load or the RE systems can be forced to operate at lower efficiency (i.e. non-optimally) by adjusting the load conditions experienced by the micro generators. In the case of photovoltaic components, for example, this can be done by increasing the load voltage above that of the maximum power point. Such a non-exporting arrangement minimises the risk of compromising power quality and overcomes the concerns of the network operators with respect to the connection of distributed, stochastic RE systems.

The identification of the optimum mix of demand reduction measures and NRE systems for deployment is a task ideally suited to the integrated simulation approach. For example, simulations can be performed to assess the performance of different energy efficiency measures when deployed jointly and severally. This might involve the assessment of the potential of daylight utilisation to reduce electricity consumption for lighting, transparently insulated facade components to off-set heating loads, advanced glazings to reduce heat loss/gain, and smart control to enforce minimum heating/lighting set-points. In this way the energy demand profile may be reshaped to temporally accommodate the introduction of NRE systems. After the required level of demand reduction has been achieved, simulations can be performed to identify appropriate NRE technology combinations to meet the residual temporal demand, eliminate inherent conflicts (e.g. between a facade-integrated photovoltaic system and daylight utilisation) and minimise the capacity of the required energy storage system.

Because NRE components typically straddle the thermal and electrical domains – that is, they interact with a building's fabric, plant, lighting and control systems – they cannot be modelled in isolation. Instead, a coupled modelling approach is necessary: this requires the establishment of a power flow model of the building's electrical network and power producing/consuming equipment.

Facade-integrated photovoltaic components

Consider the facade-integrated photovoltaic component shown in Figure 6.

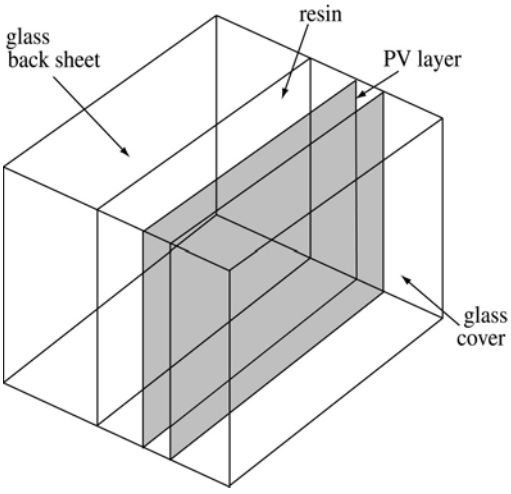

Figure 6: A façade-integrated photovoltaic component.

Solar radiation is transmitted to the surface of the photovoltaic layer through the glass cover. Before this flux is applied to the corresponding FV conservation equation its magnitude is reduced to reflect the fact that not all the absorbed solar radiation, α_i, will be converted to heat since a proportion is converted to electrical energy:

$$\alpha_i^{'} = \alpha_i - q_{ei}$$

where $\alpha_i^{'}$ is the actual absorption and q_{ei} is the PV power output (W), which may be determined from:

$$q_{ei} = nc\left[V_i I_g \left(1 - \exp\left\{ \frac{eV_i}{\lambda \sigma \theta_i} \right\} \right) - \frac{ViTsc\alpha_i^{'}}{\alpha_{i(ref)}^{'}} \right]$$

where θ_i is the temperature (K) of the PV material node (as determined from the conduction model), V_i the node voltage, I_g the light generated current, I_{sc} the short circuit current, λ the electron charge (1.6 x 10^{-19} Coulombs), n the number of series connected cells, c the number of parallel connected cells, and σ the Stefan-Boltzmann constant (1.38 x 10^{-23} W/m^{2}K^4). The light generated current is calculated as a function of the solar energy absorbed in the PV layer, α_i, when referenced to the solar absorption, $\alpha'_{i(ref)}$, corresponding to the standard test condition.

Ducted wind turbines

The power output, P_w, from a conventional wind turbine may be characterised by

$$P_w = 0.5\, C_P\, \rho\, \pi\, R^2\, v^3$$

where v is the wind speed (m/s), R the rotor radius (m), ρ the air density (kg/m^3) and C_P a power coefficient that represents the effectiveness of the turbine rotor. For a given turbine, C_P is a function of the tip speed ratio t_s:

$$t_s = \omega R/v$$

where ω is the angular velocity of the rotor.

Ideas for wind energy conversion in an urban environment have ranged from the notion of placing wind turbines on roofs, to building-integrated ducted wind turbines. With reference to figure 7, when the wind blows at right angles to the face of a tall building, stagnation will occur at about two-thirds of the total height. Below this level, a rolling vortex is formed; above it, the air rises to pass over the roof. If the roof is flat, it will separate from the upwind edge, possibly reattaching some distance downstream. A ducted wind turbine (DWT) is designed to draw air from the high-pressure region on the upwind face of the building and exhaust it into the low-pressure region above the flat roof. The device will operate efficiently over a 60° range of wind directions, with the ducting serving to dampen any turbulence in the air stream, which can be a serious problem when other buildings are in close proximity.

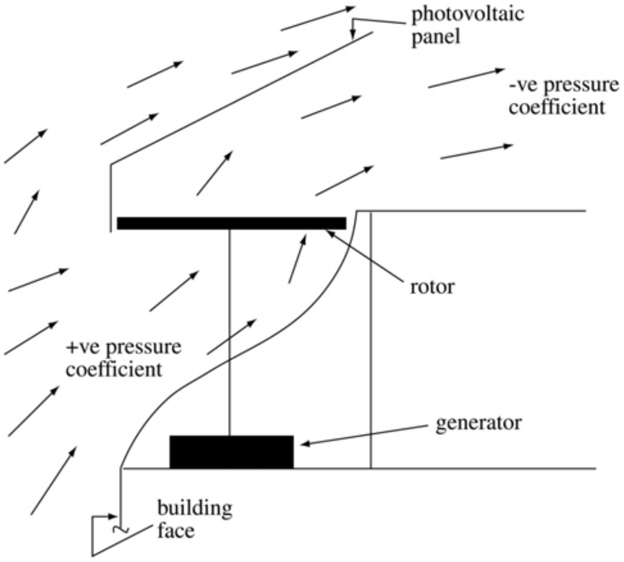

Figure 7: A building-integrated ducted wind turbine.

Because most conventional grid-connected wind turbines are constrained to turn at constant angular velocity, as the wind speed varies the value of t_s (and hence C_P) will change. A DWT, being a small unit for autonomous use, is unlikely to be connected to the electrical grid and so can operate in variable-speed mode. Maintaining a constant value for t_s allows the DWT to operate at a constant (optimum) value of C_P. Combining the last two equations gives:

$$P_w = T\omega = 0.5C_P\rho\pi R^2\omega R/ t_s^3 = K\omega^3$$

where K is a constant and the torque T at the rotor shaft follows a quadratic characteristic, $T = K\omega^2$. In practice it is a simple matter to control the generator to produce a quadratic torque characteristic at the rotor shaft, and so a near-constant value of C_P may be achieved.

This mode of operation can be abandoned in high wind speeds, where it is necessary to limit the power produced. Here, the electrical load on the generator may be increased, reducing the rotor speed, stalling its blades and reducing C_P to the required value. At low wind speeds (below about 5 m/s), the power output will fall below the values indicated by the above equation as a result of disproportionate mechanical losses and low generator efficiency.

Control systems

Within the numerical approach, a control system is conceived as a collection of control loops, with each loop serving to manipulate a related model parameter such as room temperature, boiler fuel supply rate, luminaire voltage, window opening extent or shading device position. In fact, any model parameter may be 'actuated', whether it corresponds to a real entity, such as a dimmer switch, or an abstract entity, such as room flux input/extract. This is a useful feature in that ideal control regimes can be established at an early design stage in order to constrain simulations in support of design exploration. The imposition of the desired temperature profile on a room in order to explore design options is a simple example. Real control system response characteristics can then be included at a later stage.

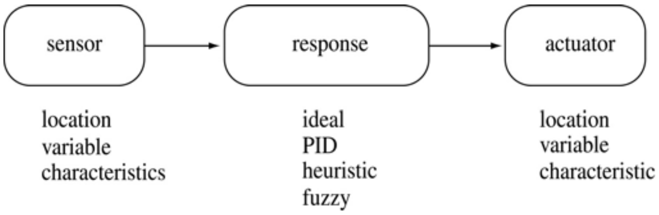

Figure 8: Elements of a control loop.

As shown in Figure 8, a control loop comprises three basic elements: a sensor to detect a model parameter or group of parameters, an actuator to deliver the control action and a regulation law to represent the characteristics of the sensor and actuator devices (delay, hysteresis, off-set etc.) and determine the control action as a function of the sensed condition. Table 2 gives some typical examples of each element.

Table 2: Examples of control loop elements.

Sensed/actuated parameters	Regulation law
time of day/year	ideal
climate	PID combinations
various temperatures	optimum start/stop
glare and illuminance	weather compensation
luminaire status	cascade
occupancy	enthalpy cycle
$CO sub 2$ level	duty cycling
ventilation rate	load shedding
room air velocity	adaptive
hygro-thermal properties	fuzzy logic
humidity	neural network

Once established, the control system may be imposed on the solution of the coupled equation-sets representing the domains comprising the integrated model. At each time-step, the condition detected by the sensor is fed to some algorithm representing controller response. This algorithm acts to modify one or more model parameters prior to final matrix equation solution or reformulation. In this way, the sensed condition(s) may be controlled as a function of a prescribed state (ideal regulation), a deviation (proportional control), a deviation rate of change (derivative control) or a deviation past history (integral control). Controller types can be combined (proportional + integral action etc.) and the effects of response rates or learning algorithms incorporated.

Essentially control algorithms may be regarded as comprising three elements: spatial, temporal and logical. Spatial elements relate to the distributed parameters that may be controlled (such as temperature and flow rate), temporal elements relate to the scheduling of control actions (including event management and controller response time), and logical elements relate to the control system characteristic and capability (such as hysteresis and optimum start control). Utilising these concepts, it is possible to place simulation at the core of the supervisory element of a building energy management system in order to compare the efficacy of alternative control actions before enacting the best one. Of the many possible control strategies, table 3 lists a few examples that might be enabled by a simulation assisted control approach.

Table 3: Some examples of simulation-assisted control.

Control focus	Optimised parameter
HVAC operation	start/stop time
night cooling	hours of operation
night set-back	set-back temperature
boiler sequencing	heating system efficiency
load shedding	energy consumption
CHP	hours of operation
district heating	match to load
under-floor heating	period of operation
mixed ventilation	avoid overheating
ice store charging	hours of operation
ground heat pump	thermal storage

The approach facilitates the creation of complex time varying control regimes to allow the examination of plant operation and the determination of optimum start times, night set-back temperatures and so on. In this way idealised control and/or real plant interactions can be intermixed to facilitate the design of an efficient and effective HVAC system.